

3.2.2 Ambiguity and Risk in Finance and Economics

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The aim of this project is to further explore the foundations of non-expected utility theory for economics and finance on the one hand. This is done by non-expected utility theory, which is based upon Choquet and multiple prior models, or by several extensions of classical mean- variance preferences. On the other hand, we will provide and investigate several applications in economics and finance with special emphases on optimal stopping problems (as they arise from American Options, e.g.) and on comparative static analyses for mean-variance preferences (in portfolio selection problems, insurance problems or issues of public economics). In addition, we will explore possible uses of dynamic ambiguity models in Game Theory.

A: Nonexpected Utility Theory in Finance

Topic and State of the Art

In recent years, several fields have begun exploring and extending the so-called expected utility hypothesis and have developed axiomatic systems and other approaches that use multiple priors or non-expected utility models as a basis for decision making.

In Decision Theory, Schmeidler (1989) and Gilboa & Schmeidler (1989) have axiomatized Choquet and multiple prior models in which the decision maker uses either a non-additive probability (a capacity) or a set of probability measures to account for the inherent uncertainty in the model. This model has recently been extended to dynamic settings by Epstein & Schneider (2003). It has been further developed to so called variational utility models by Maccheroni, Marinacci & Rustichini (2006a) and others. Maccheroni, Marinacci & Rustichini (2006b) extend variational utility to dynamic settings.

In Mathematical Finance, a literature on monetary risk measurement emerged in the last ten years based on the seminal paper by Artzner, Delbaen, Eber & Heath (1999). Based on axioms for coherency of risk measures, these authors provide an alternative foundation for the use of multiple priors in economics and finance. The theory has been extended to dynamic settings by Riedel (2004), Detlefsen & Scandolo (2005), Artzner, Delbaen, Eber, Heath & Ku (2002), and others. A corresponding general version that is mathematically equivalent to variational utility theory has been developed by Föllmer & Schied (2002). The extension to dynamic settings is due to Föllmer & Penner (2006).

In Macroeconomics, Anderson, Hansen & Sargent (2003) and others have developed a theory of Robust Macroeconomics where they account for the inherent model uncertainty by using multiple priors or so called penalty functions like relative entropy.

Although conceptually different, all three approaches use multiple prior models to study robust versions of the classical expected utility model.

Own Previous Work

In Riedel (2004), dynamic coherent risk measures are axiomatized. The main problem is identified as being time consistency. When one deviates from the Bayesian world, Sarin & Wakker (1998) have shown that one typically uses the property of time-consistency. Time consistency corresponds to the principle of dynamic programming and it would be very difficult to develop sensible economic theories of decision making in dynamic context if one does not have this property. Riedel (2004) shows that time consistency for dynamic coherent risk measures is satisfied if and only if the set of priors satisfies a certain stability condition. The same stability condition plays a crucial role in the work of Epstein & Schneider (2003). Stability of the set of priors ensures that the non-additive expectation operator defined by the set of priors satisfies a law of iterated expectations.

The paper by Riedel (2007) provides the foundations for the theory of optimal stopping under time consistent multiple priors and dynamic coherent risk measures. In this paper, Riedel develops a suitable theory of multiple prior martingales and uses this theory to extend the classical theory of optimal stopping to the framework of time-consistent multiple priors. Applications are given for American Options in discrete time, house-selling problems in Operations Research, and the famous parking problem.

Chateauneuf, Kast & Lapied (1996) show how certain non-additive probabilities can be used for pricing assets in financial markets. Applications of Choquet-expected utility models to Risk Sharing and General Equilibrium Theory can be found in Chateauneuf, Dana & Tallon (2000). The paper Chateauneuf & Rebillé (2004) provides in a discrete infinite horizon, a tractable formula for dynamic consumption choice which aims at taking into account complementarities across successive periods.

Expected Line of Progress

The aim of this project is three-fold. On the one hand we want to explore further the foundations of non-expected utility theory especially in dynamic contexts. On the other hand the project will study applications of non-expected utility theory and dynamic risk measures to important questions in economics and finance.

As far as the foundations of non-expected utility models are concerned, we are going to investigate the possibility of using models based on Choquet expected utility models. Sarin and Wakker have shown that the strong form of dynamic time-consistency does not hold true with Choquet models in general. However, the recent approach by Klibanoff & Hanany (2007) opens the way for studying Choquet models with weaker notions of time-consistency. We would like to explore further how this approach can be used in concrete investment and control problems in economics and finance. The relationship with the earlier work by one of the researchers of the project remains to be explored (Chateauneuf, Dana & Tallon (2001)).

We are going to extend the theory of optimal stopping under multiple priors to so-called variational utility as developed in Maccheroni, Marinacci & Rustichini (2006b). A second project consists in extending the discrete time version provided by Riedel (2007) to continuous time. This of course will require a lot technical extensions of existing work and will be a whole dissertation in its own right. An application of the theory of optimal stopping to multiple priors will be concerned with the so called 'secretary problem', also known as the best choice problem. Further applications are concerned with exotic American options in financial markets, e. g. barrier options or so-called shout options. The last two topics are directly concerned with the use of non-expected utility models, the first one would generalize in continuous time the evaluation formula proposed in [Some characterizations of non-additive multi-period models, by A. Chateauneuf and Y. Rébillé, in *Mathematical Social Sciences* 48 (2004) 235-250] and would explore how to fit this model to the framework of Coherent measures of risk, the second one would aim at extending previous results of Shalit and Yitzhaki "Derivation of the mean-Gini efficient portfolio frontier" (2002) by building upon the Yaari's Dual theory of choice under risk (*Econometrica* 55, p 95-105, 1987), thus offering simpler analytic derivations of the mean-Gini efficient frontier, an alternative solution to the mean-variance model.

Last not least, this project aims to make first steps in applying non-expected utility models to dynamic games. So far, very few models in Game Theory use non-expected utility models because Game Theory typically makes heavy use of expected utility. However, the development of time-consistent dynamic models as outlined above makes it possible to study typical economic games in the framework of ambiguity. For example, in the first funding period, a doctoral student (Jan-Henrik Steg) studies irreversible investment problems in duopoly. He has developed an interesting theory that allows to treat singular control problems involved in this interaction model with new methods. A natural project for the second funding period, would be to include ambiguity in these models. For example, fixed costs for new projects are relatively hard to estimate in general. Therefore, ambiguity or model robustness is an important topic in this context.

B: Mean-Variance and Mean-Variance-Skewness Preferences in Economics and Finance

Topic

The aim of this project is on the one hand to study the properties of mean-variance-skewness utility functions and their relation to existing concepts of risk aversion and their refinements. On the other hand we investigate optimal portfolio selection, insurance demand and redistributive taxation and carry out comparative static analyses of increases in mean, variance, skewness and the dependence structure of multiple risks when agents' preferences are represented by either mean-variance or mean-variance-skewness utility functions.

State of the Art and Own Prior Work

Another approach to representing preferences under uncertainty is the mean-variance approach which goes back to the pioneering works of Tobin (1958) and Chipmann (1973). Quite surprisingly, since the groundbreaking paper of Meyer (1987), the literature has been silent regarding the mean-variance approach but it has experienced a renaissance in recent years. In general, it need not be consistent with the expected utility approach, but as clarified by Meyer (1987) it is indeed consistent if all feasible distributions differ only by location and scale parameters. The location-scale condition is satisfied in univariate settings when the agent's choice variable depends linearly on the random variable. It holds in multivariate settings (Chamberlain (1983), Owen & Rabinovitch (1983)) if and only if random variables are elliptically distributed. In the literature two different procedures are suggested: either assuming that expected utility becomes a function of mean and variance, and then deriving equivalencies between mean-variance utility functions and von Neumann-Morgenstern utility functions – or taking the mean-variance framework as an approach which stands on its own.

Mean-variance analysis has been highly popular both in economic theory and in finance. Its main merits are expository simplicity and an intuitive interpretation: all effects can be couched in terms of risks and returns, and mean-variance models remain two-dimensional even with multi-dimensional risks or choice variables.

In the recent literature Lajeri & Nielson (2000), Wagener (2002) and Eichner & Wagener (2003b) transfer the concept of prudence Kimball (1990) into the mean-variance framework. Lajeri-Chaherli (2002) translates the concept of properness Pratt & Zeckhauser (1987) to the mean variance approach, Lajeri-Chaherli (2005) translates the concept of standardness Kimball (1993) and Eichner (2008a) transfers the notion of risk vulnerability Gollier & Pratt (1996) into mean-variance preferences.

Comparative static analyses in models with mean-variance preferences are scant. For the so-called standard portfolio problem with one safe and one risky asset Ormiston & Schlee (2001) and Lajeri-Chaherli (2003) derive sufficient conditions such that the demand for the risky asset increases in its expected rate of return and decreases in its variance. Finally, Eichner & Wagener (2003a) , Eichner (2005) and Eichner (2008b) characterize the comparative static effects in the quasi-linear decision model¹ Bigelow & Menezes (1995) extended by a dependent background risk. More precisely, Eichner (2008b) applies duality theory and employs a Slutsky equation in terms of mean and variance and characterizes the corresponding income and substitution effects.

A straightforward extension of two-moment utility functions are three-moment utility functions which include next to mean and variance the skewness of the distribution function as argument of

¹This class of linear decision models encompasses the analysis of a competitive firm under price uncertainty Sandmo (1971), the standard portfolio problem Fishburn & Porter (1976) and insurance demand Ehrlich & Becker (1972).

the utility function. While still in its infancy, there are two promising papers which analyze general² three-moment utility functions (Chiu (2005) Chiu (2008)). More specifically, Chiu (2008) proves that the three-moment utility function is increasing in the skewness if preferences exhibit increasing downside risk aversion Menezes, Geiss & Tressler (1980) which in turn is equivalent to a von Neumann-Morgenstern utility function with a convex derivative presupposing expected utility and three-moment approach are compatible. To the best of our knowledge comparative static analyses in three-moment decision models with general utility functions do not exist.

Expected Line of Progress

A first line of future work studies mean-variance preferences. In a first step we want to further elaborate the comparative static effects of changes in the risks' dependence structure measured either by the covariance or the Pearson correlation coefficient in the quasi-linear decision model Bigelow & Menezes (1995). Next, we turn to portfolio selection problems with two risky assets. Here, we aim at answering the following questions: How does an investor changes the investment in an asset

- if the mean return of that asset increases?
- if the mean return of another asset increases?
- if the variance of that asset increases?
- if the variance of another asset increases?
- if the dependence structure changes?

In addition, applying the technique of Eichner (2008b) we want to elaborate a Slutsky equation which may allow for characterizing income and substitution effects in the portfolio selection problem. It is worth mentioning that these effects are on the research agenda for more than thirty years (Levy (1973), Aivazian (1977)). Starting with a two portfolio choice problem, it is natural to ask whether the results are robust to an extension to n assets. Finally, observe that the portfolio selection problem is part of the capital asset pricing model (CAPM). Since the portfolio selection determines the agents' demand for assets, we want to shed further light on the market demand function in CAPM (see also Bottazzi, Hens & Löffler (1998)) and on the existence and uniqueness of equilibria in CAPM (Nielsen (1990), Dana (1999) , Hens, Laitenberger & Löffler (2002)).

A second line of future work is concerned with three-moment utility functions. To prepare for the comparative static analysis of skewness changes, the properties (especially the sign) of the second and third partial derivative of the three-moment utility function with respect to skewness and the cross derivatives which involve the skewness must be studied and must be given a plausible and economic meaningful interpretation. In addition, we aim at pointing out whether these derivatives are related to the concepts of prudence, standardness, properness or risk vulnerability (mentioned in the preceding section). Next, we study the comparative static effects of changes in means, variances, skewness and dependence structure in the quasi-linear decision problem Bigelow & Menezes (1995) and in the portfolio problem with two risky assets. The main innovative issues are to point out the new effects tracing back to the inclusion of the skewness into the analysis. As a by-product, it would be interesting to establish conditions for the compatibility of expected utility and three-moment approach.

Finally, we want to apply three-moment utility functions to issues in public economics. Sinn (1995) has shown how the welfare state, i.e. social insurance that works through redistributive taxation, should respond to increases in risk. Here, we modify Sinn (1995) model such that agent's and government's have three-parameter instead of two-parameter preferences and elaborate the implications for income taxation. The skewness may be interpreted as a parameter which reflects the society's poverty.

Dissertation Projects

1. Optimal Stopping for Dynamic Variations Preferences

²In studies of skewness preference to date (except for Chiu (2005) Chiu (2008)) either a cubic utility function is assumed or a Taylor approximation of the expected utility function is taken. While cubic utility functions have some unattractive properties, truncated Taylor series can be a reasonable approximation only for small risks.

2. Optimal Stopping with Multiple Priors: the Continuous-time Case
3. Exotic American Options in Incomplete Markets
4. Robust Best Choice Problems
5. A non-additive multi-period model; the Continuous-time Case
6. Mean-Gini efficient portfolio and Yaari's model
7. Portfolio selection with mean-variance preferences
8. Increases in risks and skewness with three-moment preferences
9. The theory of the welfare state when governments dislike poverty

Postdoc Project: Ambiguity in Dynamic Games

The postdoc project is about introducing dynamic ambiguity models into Game Theory. A possible research topic is, as indicated above, the introduction of dynamic multiple prior models into dynamic oligopoly games, or, more generally, repeated games with ambiguous payoffs.

References

- AIVAZIAN, V. (1977): "The demand for assets under conditions of risk: Comment", *Journal of Finance*, 32, 927–929.
- ANDERSON, E., L. HANSEN & T. SARGENT (2003): "A Quartet of Semigroups for Model Specification, Robustness, Prices of Risk, and Model Detection", *Journal of the European Economic Association*, 1(1), 68–123.
- ARTZNER, P., F. DELBAEN, J.-M. EBER & D. HEATH (1999): "Coherent Measures of Risk", *Mathematical Finance*, 9, 203–228.
- ARTZNER, P., F. DELBAEN, J.-M. EBER, D. HEATH & H. KU (2002): "Multiperiod Risk and Coherent Multiperiod Risk Measurement", Mimeo, ETH Zürich.
- BIGELOW, J. P. & C. F. MENEZES (1995): "Outer risk aversion and the comparative statics of increasing risk in quasi-linear decision models", *International Economic Review*, 36, 643–673.
- BOTTAZZI, J. M., T. HENS & A. LÖFFLER (1998): "Market demand functions in the capital asset pricing model", *Journal of Economic Theory*, 79, 192–206.
- CHAMBERLAIN, G. (1983): "A characterization of the distributions that imply mean-variance utility functions", *Journal of Economic Theory*, 29, 185–201.
- CHATEAUNEUF, A., R. DANA & J. TALLON (2000): "Optimal Risk-sharing Rules and Equilibria with Choquet-expected-utility", *Journal of Mathematical Economics*, 34, 191–214.
- (2001): "Conditioning Capacities and Choquet Integrals: The Role of Comonotony", *Theory and Decision*, 51, 367–386.
- CHATEAUNEUF, A., R. KAST & A. LAPIED (1996): "Choquet Pricing for Financial Markets with Frictions", *Mathematical Finance*, 6, 323–330.
- CHATEAUNEUF, A. & Y. REBILLE (2004): "Some Characterizations of Non-Additive Multi-Period Models", *Mathematical Social Sciences*, 48(3), 235–250.
- CHIPMANN, J. S. (1973): "The ordering of portfolios in terms of mean and variance", *Review of Economic Studies*, 40, 167–190.
- CHIU, W. H. (2005): "Skewness preference, risk aversion and the precedence relations on stochastic changes", *Management Science*, pp. 1816–1828.

- (2008): “Skewness preference, risk taking and expected utility maximization”, *discussion paper*.
- DANA, R. A. (1999): “Existence, uniqueness and determinacy of equilibrium in CAPM with a riskless asset”, *Journal of Mathematical Economics*, 32, 167–175.
- DETLEFSEN, K. & G. SCANDOLO (2005): “Conditional and Dynamic Convex Risk Measures”, *Finance and Stochastics*, 9(5), 539–561.
- EHRlich, J. & G. S. BECKER (1972): “Market insurance, self-insurance and self-protection”, *Journal of Political Economy*, 80, 623–648.
- EICHNER, T. (2005): “Comparative statics of properness in two-moment decision models”, *Economic Theory*, 25, 1005–1012.
- (2008a): “Mean variance vulnerability”, *Management Science*, 54, 586–593.
- (2008b): “Slutsky equations and substitution effects of risks in terms of mean-variance preferences”, *Theory and Decision, in press*.
- EICHNER, T. & A. WAGENER (2003a): “Variance vulnerability, background risk and mean variance preferences”, *Geneva Papers of Risk and Insurance*, 28, 173–184.
- (2003b): “More on parametric characterizations of risk aversion and prudence”, *Economic Theory*, 21, 895–900.
- EPSTEIN, L. & M. SCHNEIDER (2003): “Recursive Multiple Priors”, *Journal of Economic Theory*, 113, 1–31.
- FISHBURN, P. C. & R. B. PORTER (1976): “Optimal portfolios with one safe and one risky asset: effects of changes in rate of return and risk”, *Management Science*, 22, 1064–1073.
- FÖLLMER, H. & I. PENNER (2006): “Convex Risk Measures and The Dynamics of Their Penalty Functions”, *Statistics and Decisions*, 24, 61–96.
- FÖLLMER, H. & A. SCHIED (2002): “Convex Measures of Risk and Trading Constraints”, *Finance and Stochastics*, to appear.
- GILBOA, I. & D. SCHMEIDLER (1989): “Maxmin Expected Utility with Non-Unique Prior”, *Journal of Mathematical Economics*, 18, 141–153.
- GOLLIER, C. & J. W. PRATT (1996): “Risk vulnerability and the tempering effect of background risk”, *Econometrica*, 64, 1109–1123.
- HENS, T., J. LAITENBERGER & A. LÖFFLER (2002): “Two remarks on the uniqueness of equilibria in the CAPM”, *Journal of Mathematical Economics*, 37, 123–132.
- KIMBALL, M. S. (1990): “Precautionary saving in the small and in the large”, *Econometrica*, 58, 53–73.
- (1993): “Standard risk aversion”, *Econometrica*, 61, 589–611.
- KLIBANOFF, P. & E. HANANY (2007): “Updating Preferences with Multiple Priors”, *Theoretical Economics*, 2, 261–298.
- LAJERI, F. & L. T. NIELSON (2000): “Parametric characterizations of risk aversion and prudence”, *Economic Theory*, 15, 469–476.
- LAJERI-CHAHERLI, F. (2002): “More on properness: The case of mean-variance preferences”, *Geneva Papers on Risk and Insurance Theory*, 27, 49–60.
- (2003): “Partial derivatives, comparative risk behavior and concavity of utility functions”, *Mathematical Social Sciences*, 46, 81–99.

- (2005): “Proper and standard risk aversion in two-moment decision models”, *Theory and Decision*, 57, 213–225.
- LEVY, H. (1973): “The demand for assets under conditions of risk”, *Journal of Finance*, 28, 79–96.
- MACCHERONI, F., M. MARINACCI & A. RUSTICHINI (2006a): “Ambiguity Aversion, Robustness, and the Variational Representation of Preferences”, *Econometrica*, 74, 1447–1498.
- (2006b): “Dynamic Variational Preferences”, *Journal of Economic Theory*, 128, 4–44.
- MENEZES, C., C. GEISS & J. TRESSLER (1980): “Increasing downside risk”, *American Economic Review*, 70, 921–932.
- MEYER, J. (1987): “Two-moment decision models and expected utility maximization”, *American Economic Review*, 77, 421–430.
- NIELSEN, L. T. (1990): “Existence of equilibrium in CAPM”, *Journal of Economic Theory*, 52, 223–231.
- ORMISTON, M. B. & E. SCHLEE (2001): “Mean variance preferences and investor behavior”, *Economic Journal*, 111, 849–861.
- OWEN, J. & R. RABINOVITCH (1983): “On the class of elliptical distributions and their applications to the theory of portfolio choice”, *Journal of Finance*, 38, 745–752.
- PRATT, J. W. & R. J. ZECKHAUSER (1987): “Proper risk aversion”, *Econometrica*, 55, 143–154.
- RIEDEL, F. (2004): “Dynamic Coherent Risk Measures”, *Stochastic Processes and Their Applications*, 112, 185–200.
- (2007): “Optimal Stopping under Ambiguity”, IMW Discussion Paper No. 390, Bielefeld University.
- SANDMO, A. (1971): “On the theory of the competitive firm under price uncertainty”, *American Economic Review*, 61, 65–73.
- SARIN, R. & P. WAKKER (1998): “Dynamic Choice and Nonexpected Utility”, *Journal of Risk and Uncertainty*, 17, 87–119.
- SCHMEIDLER, D. (1989): “Subjective probability and Expected Utility without Additivity”, *Econometrica*, 57, 571–587.
- SINN, H.-W. (1995): “A theory of the welfare state”, *Scandinavian Journal of Economics*, 97, 495–526.
- TOBIN, J. (1958): “Liquidity preference as behavior towards risk”, *Review of Economic Studies*, 25, 65–86.
- WAGENER, A. (2002): “Prudence and risk vulnerability in two-moment decision models”, *Economic Letters*, 74, 229–235.