The no-trade interval of Dow and Werlang: some clarifications

Alain Chateauneuf, Caroline Ventura

Université Paris 1 Panthéon-Sorbonne
Ecole doctorale d'Economie Panthéon-Sorbonne
EBIM - Economic Behavior and Interaction Models

chateauneuf@univ-paris1.fr and caroline.ventura@malix.univ-paris1.fr

Abstract

The aim of this paper is two-fold: first, to emphasize that the seminal result of Dow and Werlang [3] *remains valid under weaker conditions, and this even if non-positive prices are considered, or equally that the no-trade interval result is robust when considering assets which can yield non-positive outcomes. Second, to make precise the weak uncertainty aversion behavior characteristic of the existence of such an interval.*

1. Motivation

In a seminal paper, Dow and Werlang dealt with the basic portfolio problem under uncertainty. They proved that for an uncertainty averse Choquet expected utility decision maker (CEU DM), endowed with a convex capacity v and a C^2 concave increasing utility function u, there exists an interval of prices $[I(X),\ -I(-X)]$ within which this agent neither buys nor sells an asset X. In this paper, we prove that the no-trade interval result is robust when considering assets which can yield non-positive outcomes and that it remains valid under weaker hypotheses, requiring only that the capacity satisfy super-additivity at certainty, that is requiring that $v(A)+v(A^c)\leq 1$ for any event A instead of being convex and that u is C^1 instead of being C^2. We also obtain the converse implication and so the equivalence between the no-trade interval $[I(X),\ -I(-X)]$ and the following two conditions: 1) v is super-additive at certainty and 2) u is concave.

We furthermore make precise the weak uncertainty aversion behavior of the agent, characteristic of the existence of such an interval, by proving that the previous conditions 1) and 2) are actually equivalent to 3) attraction by perfect hedging and 4) preference for comonotone diversification.

Finally, we show that for a CEU DM endowed with an increasing utility function u, the existence of the no-trade interval proves to be equivalent both to aversion to some specific increase of uncertainty and to subjective increasing risk.

2. Definitions and notations

-v is a (normalized) capacity on the measurable space $(\Omega,\mathcal{A})$ if $v:\ \mathcal{A}\rightarrow[0,1]$ is such that $v(\emptyset)=0$, $v(\Omega)=1$ and $\forall A,\ B\in\mathcal{A},\ A\subset B\Rightarrow v(A)\leq v(B)$.

-The Choquet integral of $X\in\mathcal{B}_\infty$ with respect to the capacity v is $I(X)=\int X dv$ where
$\int X dv=\int_{-\infty}^{0}(v(X\geq t)-1)\,dt+\int_{0}^{+\infty}v(X\geq t)dt$.

-The core of a capacity v is defined by
$C(v)=\{(finitely\ additive)\ probabilities\ P:\ P(A)\geq v(A)\ \forall A\in\mathcal{A}\}$.

-We say that a DM satisfies the CEU model if his (her) preferences can be represented through an increasing utility function $u:\ \mathbb{R}\rightarrow\mathbb{R}$, and a capacity v on $(\Omega,\mathcal{A})$ which models his (her) personal evaluation of the likelihood of events. The representation of his (her) preferences is given by $I\circ u:\ X\in\mathcal{B}_\infty\mapsto\int_\Omega(u\circ X)\,dv$.

3. The result of Dow and Werlang

In the CEU framework, Dow and Werlang consider a measurable space $(\Omega,\mathcal{A})$, a convex capacity v on $\mathcal{A}$ and a utility function u assumed to be C^2 and such that $u'>0$ and $u''\leq 0$.

Theorem (Dow and Werlang): A risk averse (resp. risk neutral) investor with certain wealth $W>0$, who is faced with an asset which yields a present value X per unit, whose price is $p>0$ per unit, will buy the asset if $p<I(X)$ (resp. $p\leq I(X)$). He (she) will sell the asset if $p>-I(-X)$ (resp. $p\geq -I(-X)$).

This theorem is very intuitive and offers an appealing interpretation of the uncertainty aversion in terms of pessimism since, according to a well-known theorem of Schmeidler which says that v is convex if and only if $C(v)\neq\emptyset$ and $I(X)=Min_{P\in C(v)}\,E_P[X]$, the agent views as possible the set of probabilities above the convex capacity v and will evaluate all assets X by $Min_{P\in C(v)}\,E_P[X]$. Thus, the DM will have no position on the asset X if and only if its price p is between $I(X)$ and $-I(-X)$.

4. Generalization and extension of the result of Dow and Werlang

Let $(\Omega,\mathcal{A})$ be a measurable space such that $\mathcal{A}$ contains at least one non-trivial event. Let $\mathcal{B}_\infty$ be the set of bounded $\mathcal{A}$-measurable mappings from Ω to $\mathbb{R}$. We consider a CEU DM with a C^1 utility function $u:\ \mathbb{R}\rightarrow\mathbb{R}$ such that $u'>0$ and a capacity v on $\mathcal{A}$ which is non trivial in the sense that there exists at least one event $A\in\mathcal{A}$ such that $0<v(A)<1$.

Theorem: The two following assertions are equivalent:

(a) For any $X\in\mathcal{B}_\infty,\ I(X)\leq -I(-X)$. Furthermore, in any situation of feasible trade the DM has no position in the asset X on the range of prices $[I(X),-I(-X)]$, and he (she) buys a positive amount of the asset X at prices below $I(X)$, and holds a short position at prices higher than $-I(-X)$.

$(b)\begin{cases}(1)\ v(A)+v(A^c)\leq 1\ for\ all\ A\ in\ \mathcal{A}\\(2)\ u\ is\ concave\end{cases}$

5. Uncertainty aversion behavior

Theorem: A CEU DM will exhibit the no-transaction interval of Dow and Werlang if and only if:

(3) He (she) is attracted by perfect hedging
(i.e. $[X,Y\in\mathcal{B}_\infty,\ X\succsim Y,\ \alpha\in[0,1],\alpha X+(1-\alpha)Y=a1_\Omega\ a\in\mathbb{R}]\Rightarrow a1_\Omega\succsim Y$).

and

(4) He (she) exhibits preference for comonotone diversification
(i.e. $X,Y\in\mathcal{B}_\infty,\ X\ and\ Y\ comonotone,\ X\sim Y\Rightarrow\alpha X+(1-\alpha)Y\succsim Y\ \forall\alpha\in[0,1]$).

6. Aversion to increasing uncertainty and subjective increasing risk

Definition: *A CEU DM is symmetrical monotone uncertainty averse (SMUA) if for all* $X,\ Y\in\mathcal{B}_\infty,\ X\succsim_{SM}Y\Rightarrow X\succsim Y$ *where* $X\succsim_{SM}Y$ *means that there exists* $Z\in\mathcal{B}_\infty$, Z *comonotone with* X *such that* $I(Z)=I(-Z)$ *and* $Y=X+Z$.

Y represents a monotone symmetrical increase of uncertainty in relation to X. So, a DM is symmetrical monotone uncertainty averse if he (she) doesn't like the monotone symmetrical increase of uncertainty i.e. if he (she) always prefers X to Y.

Definition: *Let* $A,\ B\in\mathcal{A}$ *and* $x_1,\ x_2,\ y_1,\ y_2\in\mathbb{R}$ *such that* $x_1\leq x_2$ *and* $y_1\leq y_2$. *We say that* $Y=y_1 1_{B^c}+y_2 1_B$ *is a binary subjective mean-preserving spread (SMPS) of the binary act* $X=x_1 1_{A^c}+x_2 1_A$ *if* $y_1\leq x_1,\ x_2\leq y_2$, $v(A)=v(B)$ *and* $y_1(1-v(B))+y_2v(B)=x_1(1-v(A))+x_2v(A)$.

Definition: *A CEU DM is averse to binary SMPSs if he (she) prefers a binary act to every one of its binary SMPSs.*

Theorem: Let a CEU DM be endowed with a C^1 increasing utility mapping $u:\ \mathbb{R}\rightarrow\mathbb{R}$ and a non trivial capacity v on $\mathcal{A}$, then the two following assertions are equivalent:

$(1)\begin{cases}(a)\ The\ DM\ is\ SMUA\\(b)\ The\ DM\ is\ averse\ to\ binary\ SMPS\end{cases}$

$(2)\begin{cases}(a)\ v(A)+v(A^c)\leq 1\ for\ all\ A\ in\ \mathcal{A}\\(b)\ u\ is\ concave\end{cases}$

7. Concluding comments

After recalling the pioneering result of Dow and Werlang on the no-trade interval of a CEU DM, we generalize this result by allowing for negative prices while merely weakening convexity of the capacity into super-additivity at certainty. We also prove that a DM will exhibit this no-trade interval if and only if he (she) is attracted by perfect hedging and has preference for comotone diversification or equivalently if he (she) presents some kind of uncertainty aversion. Finally, we show that for a CEU DM endowed with an increasing utility function, the existence of the no-trade interval is equivalent both to aversion to some specific increase of uncertainty and to subjective increasing risk. We generalize the result of Dow and Werlang, who restricted themselves to the case of positive initial wealth and prices, by allowing for non-positivity. Our goal was achieved under the assumption that borrowing was excluded. We intend in a future study to examine the robustness of our results when this restriction is removed.

References

[1] Abouda, M. and Chateauneuf, A., Positivity of bid-ask spreads and symmetrical monotone risk aversion, Theory and Decision, 2002, 52, 149-170.
[2] Chateauneuf, A. and Tallon, J.M., Diversification, convex preferences and non-empty core in the Choquet expected utility model, Economic Theory, 2002, 19, 509-523.
[3] Dow, J. and Werlang, Da Costa S.R.,Uncertainty aversion, risk aversion, and the optimal choice of portfolio, Econometrica, 1992, 60 number 1, 197-204.
[4] Schmeidler, D., Subjective probability and expected utility without additivity, Econometrica, 1989, 57 (3), 571-587.