The competitive firm and information about uncertain factor prices

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1. Introduction

Starting with the seminal work of Sandmo (1971) much has been written on the theory of the competitive firm under price uncertainty. These studies replace the classical assumption that the firm knows the price at the time the production decision is made by the assumption that only the probability distribution is known and that the firm is averse to risk. A number of studies not only incorporate uncertainty in the firm's decision problem but also discuss the effects of a change in the degree of uncertainty. This is operationalized via changing higher moments of the probability distribution or concepts like stochastic dominance. These techniques lead to a number of valuable results but cannot be regarded as entirely satisfactory. The reason is that the stochastic nature of the models is purely exogenous since the causes of the change in riskiness are not explicitly modeled. There is no rigorous foundation for normative conclusions and policy recommendations.

But how can the degree of uncertainty be controlled? A decision maker's exposure to uncertainty depends essentially on the available information about the distribution of the existing risks. Broll and Eckwert (2006) study a firm in a framework where price uncertainty due to a random exchange rate is to some extent caused by imperfect information. This information is represented by a signal that can be observed before taking the production decision. The risk may be reduced by the revelation of more precise information and is therefore, at least in part, endogenous. They show that the effect of better information in the sense of Blackwell (1951) on the output decision depends on the curvature of the inverse marginal cost function.

The focus of this work is the role of more reliable information about random input instead of output prices on the production decision and performance (ex ante expected volumes of factor use, production, cost and profit) of a competitive firm in such a framework. A more general notion of informativeness as mentioned above will be used.

2. The Model

A single competitive firm is considered, which produces a homogeneous good from a single input factor and sells it at a given price equal to one. At the time the production decision has to be taken the price of the input factor is represented by a real-valued random variable $\tilde{z}$. However, prior to making its decision, the firm observes an information signal y, which is correlated with the realization of the future factor price z.

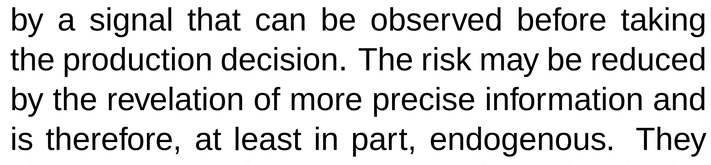

Figure 1: *The timing of events.*

In addition, the firm has access to a futures market which allows it to hedge the factor price risk. A futures contract pays one unit of the factor. Let $h \in \mathbb{R}$ denote the forward commitment of the firm. It is assumed that the terms of forward contracting are unbiased, which implies that the futures market clears at price

$$z(y) := \mathsf{E}\left[\tilde{z}|y\right]. \tag{1}$$

Both the payoff and the purchase price of the contract fall due in the second period.

3. Production Decision

The firm has to make its production decision after observing the signal y but before the realization of the factor price. It buys h contracts on the futures market to hedge the risk and its random profit is given by

$$\tilde{\Pi} = f(x) - \tilde{z}x + h(\tilde{z} - z(y)). \tag{2}$$

It is assumed that the production function f is strictly increasing, strictly concave and thrice continuously differentiable. Let U be the firm's von Neumann-Morgenstern utility function and assume that U is strictly increasing, strictly concave and twice continuously differentiable. The firm maximizes expected utility by solving

$$\max_{h,x} \mathsf{E}[U(\tilde{\Pi})|y]. \tag{3}$$

The unique optimal factor use x^* and futures commitment h^* are given by

$$f'(x^*) = z(y), \tag{4}$$
$$h^* = x^*. \tag{5}$$

4. Information

The observation of the realization of the information signal $\tilde{y}$ is used to update the prior belief. Most likely, different realizations will lead to different posterior beliefs and, hence, different conditional expected factor prices. Intuitively, changing the signal realization will have a higher impact on the posterior if the signal is more reliable.

Definition 1 *Signal $\tilde{y}$ is more informative than signal $\tilde{y}'$ if the distribution of $z(\tilde{y})$ is a mean preserving spread of the distribution of $z(\tilde{y}')$.*

This notion of informativeness is a characterization of Blackwell's sufficiency criterion as well as many other concepts. It reflects the intuition that if the signal reveals more information the posterior distribution becomes more sensitive with respect to the observation.

5. Transparency, Production and Profit

It turns out that more transparency leads on average to more (less) factor use, if the marginal product is convex (concave). This can be linked to the trend of the strength of the law of diminishing marginal returns.

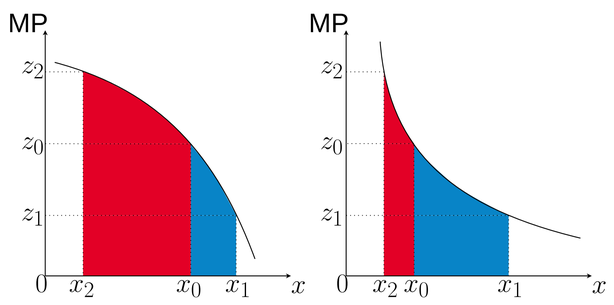

Figure 2: *Optimal factor use and production change.*

Ex ante expected production may increase or decrease depending on the relation of the curvatures of total and marginal product. This relation compares the strength of two (potentially opposing) effects. The former is the loss in average production caused by a spread in optimal factor inputs due to diminishing marginal returns. The latter one is caused by the change in average factor use, which may be positive or negative as just mentioned. In a number of cases average production may decrease although average factor use increases.

The cost from ex ante perspective may also go up or down. Here the magnitude of more or less average factor use is interacting with an efficiency gain by better decisions due to more informative signal observations. Under certain conditions it occurs that the efficiency gain is big enough so that average expenditure for the factor decreases although the demand increases on average.

By contrast, ex ante profit always increases if more reliable information about the future factor price is available, independent of the specific type of production function. This means that the efficiency effect always dominates the effect caused by diminishing marginal returns.

All described effects can be quantified using the following two measures for the production function's curvature:

$$R_f(x) := -\frac{f''(x)}{f'(x)}, \qquad P_f(x) := -\frac{f'''(x)}{f''(x)}. \tag{6}$$

Their relation characterizes different strengths of decreasing concavity.

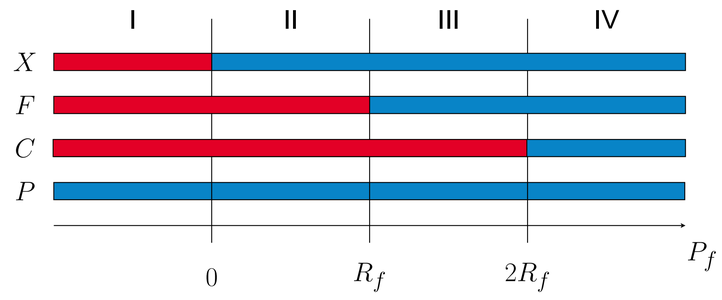

Figure 3: *Impact of higher transparency.*

6. Welfare

More reliable information leads in the presented model not only to better decisions but on the other hand also destroys risk sharing opportunities. The overall effect on welfare therefore depends on the decision maker's level of risk aversion. For small levels of risk aversion welfare increases with more transparency as the former effect is dominating and for high levels welfare is decreasing because the latter one is bigger.

References

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