Fiscal policy in an endogenous growth model with human capital and heterogenous agents

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Abstract

This paper studies effects of fiscal policy in an endogenous growth model with human capital and heterogenous agents. Two types of households are considered. One household acquires human capital or skills through education while the other household remains unskilled. Sustained growth is the result of human capital accumulation which is a function of the existing human capital employed in the educational sector and of public spending for education. Aggregate production is given by a function with physical capital and labor as input factors, where total labor input is modelled by a CES function with skilled and unskilled labor as arguments. The paper studies effects of fiscal policy as concerns long-run growth and the distribution of income as well as concerns welfare of the two households.

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1 Introduction

One force of sustained per capita growth in endogenous growth models is human capital. The seminal papers in this respect are the contributions by Uzawa (1965) and by Lucas (1988). While in the model presented by Lucas the representative individual decides how much of his available time is spent for the formation of human capital, the original contribution by Uzawa assumes that human capital is built in an educational sector with labor as the only input factor, which can be interpreted as teaching staff. If one takes the original approach as presented by Uzawa and if one assumes a decentralized economy, the question arises how educational spending is financed. While in the USA private financing of human capital plays the major role, in many European countries most of the expenditures for the formation of human capital are undertaken by the government sector.

In the economics literature one can find both the approach where human capital formation is only financed by the private sector and studies where only the public sector spends resources for the formation of human capital. In addition, there also exist contributions where human capital formation is the result of both public and private expenditures. For example, Glomm and Ravikumar (1992) and Blankenau and Simpson (2004) assume that human capital accumulation results from both private and public services. Glomm and Ravikumar present an OLG model with heterogenous agents where human capital accumulation is the result of formal schooling. They demonstrate that public education leads to a faster decline of income inequality whereas private education may lead to higher per-capita incomes. Blankenau and Simpson present an endogenous growth model with both private and public inputs in the process of human capital accumulation. They demonstrate that the response of growth to public education depends on the tax structure, on the level of government spending and on parameters of the production function. However, they do not allow for heterogenous agents but assume homogenous agents.

On the other side, Ni and Wang (1994) and Beauchemin (2001) present models where human capital is the result of public spending alone. Ni and Wang also assume homogenous agents and present an OLG model where human capital is the result of public education which is financed by an income tax. Using calibration exercises they derive that the optimal income tax rate is in the range of six to ten percent. Beauchemin presents a political-economic OLG model of growth and human capital accumulation where human capital accumulation is the result of public education. The paper demonstrates that a sufficiently rapid population growth may generate economic stagnation.

An early contribution that studies optimal fiscal policy in an endogenous growth model with human capital and productive public spending is the paper by Corsetti and Roubini (1996). These authors present a general framework where public spending may either enter the production of final goods or the production function of human capital formation. The goal of their paper is to derive optimal tax rates that can replicate the first best optimum. They show that in optimum tax rates are positive so that the externality related rents are taxed away and no public debt is necessary to attain the first-best solution. If there are restrictions as concerns the available tax instruments, the optimal policy may be obtained only if the governments borrows or lends in order to smooth distortions over time.

In this paper we will present an endogenous growth model with human and physical capital where investment in human capital is undertaken by the government, similar to the approach by Corsetti and Roubini (1996). Thus, we will assume that human capital formation is the result of public spending for teachers and of spending for teaching material. However, in contrast to Corsetti and Roubini, we consider an economy with two different types of households. One household supplies skilled labor, due to human capital formation, whereas the other household supplies unskilled labor but benefits from human capital accumulation through spill-over effects. Further, we posit that aggregate production is a function of physical capital and of total labor input, with labor given by a CES function with skilled and unskilled labor as arguments, in contrast to Corsetti and Roubini who consider a Cobb-Douglas production function.

Thus, our paper integrates heterogenous agents in an endogenous growth model, where the government plays an active role by fostering human capital accumulation and by taxing labor and capital income, which has not been done up to now as far as we know. The goal of the paper, then, is to derive growth effects of fiscal policy for the model on the balanced growth path and exemplarily for the model taking into account transition dynamics. In addition, we study effects of fiscal policy as concerns income inequality between the two types of households and we analyze how fiscal policy affects welfare of the households.

As concerns the empirical relevance of human capital, the survey by Krueger and Lindahl (2001) shows that there is strong evidence that education is positively correlated with income growth at the microeconomic level and the positive correlation seems to be quite robust. However, this does not necessarily hold for the macroeconomic level where the findings are more fragile. But this may be due to measurement errors and Krueger and Lindahl demonstrate that cross-country regressions show a positive correlation with economic growth if measurement errors are taken into account. It should also be pointed out that Levine and Renelt (1992) have demonstrated that human capital, measured by the secondary enrollment rate, is a robust variable in growth regressions. Because of that, building endogenous growth models with human capital as the engine of sustained growth is certainly justified.

The rest of the paper is organized as follows. In the next section, we present the structure of our growth model and analyze its dynamics. In section 3, we derive growth effects of fiscal policy as well as distributional and welfare implications and section 4, finally, concludes.

2 The structure of the growth model

Our economy consists of three sectors: A household sector which receives labor income and income from its saving, a productive sector and the government. First, we describe the productive and the household sector.

2.1 The productive sector and the household sector

The productive sector in the economy is assumed to be competitive where the representative firm produces with a production function of the following form.

$$Y(t) = K(t)^{1-\alpha} I_L(t)^{\alpha}, \tag{1}$$

with $Y(t)$ output at time t[1] and $0 < \alpha < 1$ the labor share and $(1-\alpha)$ denotes the physical capital share. I_L denotes overall labor input which is given by the following CES function, $I_L = \left[(h_c L_d)^{(\sigma-1)/\sigma} + (\xi h_c N)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}$, with σ denoting the elasticity of substitution between skilled labor, L_d, and unskilled labor, N. The variable h_c gives per-capita human capital and the parameter $0 < \xi < 1$ determines the strength of the external effect of human capital in the production function. This implies that a larger stock of human capital does not only increase the productivity of skilled labor but also that of unskilled labor due to spill-over effects. However, we exclude full spill-overs, $\xi < 1$, which seems to be reasonable because it is primarily the person who acquires human capital that profits most of the acquired human capital.

The representative firm maximizes static profits with respect to capital, K, and with respect to skilled labor and to unskilled labor. The maximization problem of the firm is written as

$$\max_{K,L_d,N} (Y - rK - w_L L_d - w_N N) \tag{2}$$

Profit maximization yields

$$w_L = \alpha Y W^{-1} (L_d)^{-1/\sigma} \tag{3}$$

$$w_N = \alpha Y W^{-1} \xi^{(\sigma-1)/\sigma} N^{-1/\sigma} \tag{4}$$

$$r = (1-\alpha) K^{-1} Y, \tag{5}$$

with W defined as $W = (L_d)^{(\sigma-1)/\sigma} + (\xi N)^{(\sigma-1)/\sigma}$. The variable r denotes the return to capital which is taken as given by the firm just as the wage rate for a unit of skilled and unskilled labor, w_L and w_N.

[1] In the following, we omit the time argument t if no ambiguity arises.

The variable L_d gives demand of the firm for skilled labor and makes a certain part of the overall amount of skilled labor, denoted by L. The rest of the skilled labor force is hired by the government as teachers whom it pays the competitive wage rate for skilled people. We assume that the government decides about the part of the active labor force employed in the schooling sector and, thus, determines the part of the labor force employed in the final goods sector. We denote by u, $0 < u < 1$ that part of human capital or skilled labor employed in the production of the final good. Consequently, $1 - u$ is employed by the government as teachers. Thus, in equilibrium, we get $L_d = u\,L$.

The household sector is composed of two types of households. The first household supplies skilled labor, which is employed either in the production of the final good or in the educational sector, while the second household supplies unskilled labor. We assume that both households behave as immortal families corresponding to finite-lived individuals who are connected via intergenerational transfers that are based on altruism. Thus, although individuals have finite lives each family is considered as a dynasty where the decision maker behaves as if he had an infinite time horizon (cf. Barro and Sala-i-Martin, 1995, chapter 2.1).

The overall number of skilled people is composed of a stock of students, S, and of a stock of employees, L, who constitute the active labor force and produce goods or are hired as teachers. At each point of time a certain number of students, which is determined exogenously, enters the stock of students and a certain number of students becomes employees. We assume that the number of students becoming employees just equals the number of new students so that the overall stock of students is constant. Further, the number of students becoming employees equals the number of employees leaving the active labor force, so that the active labor force and, thus, the total stock of skilled labor is constant, too, just like the number of unskilled labor.

The skilled household sector is represented by one household which maximizes the discounted stream of utility resulting from consumption, C_L, over an infinite time horizon subject to its budget constraint. The utility function is assumed to be logarithmic, $U(C_L) = \ln C_L$, and labor, L, is inelastically supplied. The maximization problem of the skilled household, then, can be written as

$$\max_{C_L} \int_0^\infty e^{-\rho t} \, \ln C_L \, dt, \tag{6}$$

subject to

$$(1 - \tau_K) r K_L + (1 - \tau_L) w_L L = \dot{K}_L + C_L. \tag{7}$$

ρ is the subjective discount rate and K_L denotes the capital stock owned by the skilled household. $0 < \tau_K < 1$ is the capital income tax rate and $0 < \tau_L < 1$ is the labor income tax rate of skilled labor. The dot gives the derivative with respect to time and we neglect depreciation of physical capital.

The unskilled household supplies N units of labor inelastically and maximizes an inter-temporal optimization problem which is given by

$$\max_{C_N} \int_0^\infty e^{-\rho t} \, \ln C_N \, dt, \tag{8}$$

subject to

$$(1 - \tau_K) r K_N + (1 - \tau_N) w_N N + P = \dot{K}_N + C_N, \tag{9}$$

where the subscript N gives corresponding variables of the unskilled household and P stands for transfer payments received by the unskilled household. Further, we assume $K_N < K_L$ stating that the capital stock owned by the unskilled household is smaller than that of the skilled household. Additionally, we require that the limiting transversality conditions $\lim_{t\to\infty} e^{-\rho t} K_i / C_i = 0$, $i = L, N$, hold.

The solution of the optimization problems of the two households gives the growth rates of consumption of the households as

$$\frac{\dot{C}_i}{C_i} = -\rho + (1 - \tau_K) r, \; i = L, N. \tag{10}$$

Using $C_L + C_N = C$, the growth rate of aggregate consumption is given by

$$\frac{\dot{C}}{C} = \frac{\dot{C}_L}{C_L}\frac{C_L}{C} + \frac{\dot{C}_N}{C_N}\frac{C_N}{C} = (-\rho + (1 - \tau_K)\, r) \left(\frac{C_L}{C} + \frac{C_N}{C} \right), \tag{11}$$

with $C_L/C + C_N/C = 1$.

2.2 Human capital formation

Human capital in our economy is produced in the schooling sector where an exogenously given number of students is educated. As mentioned above, the government hires $(1-u)$ of the active labor force as teachers. Additionally, the government uses public resources for education in the schooling sector, like expenditures for books and other teaching material, which is an input in the process of human capital formation, too. Thus, the input in the schooling sector is composed of teachers and of schooling expenditures and we assume decreasing returns to scale to each input but constant returns to both inputs. The evolution of per capita human capital, then, is a function of teachers per student and of school spending per student.

It should be noted that human capital, which is embodied in students, becomes available to the whole active labor force in the economy, once students become employees. The reason for this assumption is to be seen in spill-over effects of knowledge, which leads to a diffusion of knowledge among the labor force. At first sight, this seems to be a strong assumption. But if one takes into account that in reality newly hired employees interact with existing staff and both learn from each other, this assumption becomes comprehensible.

As concerns the production function for human capital formation we assume a Cobb-Douglas specification. The differential equation describing the change in human per capita capital can be written as

$$\dot{h}_c = \epsilon((1-u)h_c L)^{\psi}(I_E)^{1-\psi}/S, \tag{12}$$

with I_E public resources used in the schooling sector, $\epsilon > 0$ a technology parameter and $0 < \psi < 1$ is the elasticity of human capital formation with respect to teachers.

2.3 The government

The government in our economy receives tax revenues from capital and labor income taxation it then uses for the remuneration of the teachers, for public spending in the schooling sector, for transfer payments, P, and for non-productive spending, G. The budget of the government is balanced at each point in time. Thus, the period budget constraint of the government is given by

$$T = I_E + (1-u)w_L L + P + G, \tag{13}$$

with T denoting tax revenue. As concerns non-productive public spending, G, we assume that this variable makes a certain part of the tax revenue, i.e. $G = (1-\kappa)T$, with $0 < \kappa < 1$.

We also assume that transfers to the unskilled household can be expressed relative to the tax revenue, with p that part of the tax revenue used for transfers, but that not all of the capital income tax revenue is used for transfers. Thus, transfers are written as $P = p\,(\tau_N w_N N + \tau_L w_L L + \mu\,\tau_K r K)$, with μ that part of the capital income tax revenue which is used for transfers. We make this assumption because often governments resort to the tax revenue from wage incomes to finance transfers rather than the tax revenue from capital income. Especially in European countries it is the wage income which serves as a basis for financing social transfers.

2.4 Equilibrium conditions and the balanced growth path

An equilibrium allocation is defined as an allocation such that the firm maximizes profits implying that factor prices equal their marginal products (equations (3), (4) and (5)), the households solve (6) subject to (7) and (8) subject to (9), respectively, and the budget constraint of the government (13) is fulfilled and the limiting transversality conditions hold.

The economy-wide resource constraint in this economy is obtained by combining the budget constraint of private households, equations (7) and (9), with the budget constraint of the government (13) as

$$\frac{\dot{K}}{K} = \frac{Y}{K} - \frac{C}{K} - \frac{I_E}{K} - (1-\kappa)\frac{T}{K}, \tag{14}$$

where I_E is given by $I_E = \kappa T - (1-u)w_L L - p(\tau_N w_N N + \tau_L w_L L + \mu\tau_K r K)$.

Aggregate consumption evolves according to equation (11) with r given by (5) so that the growth rate of aggregate consumption can be written as

$$\frac{\dot{C}}{C} = -\rho + (1-\tau_K)(1-\alpha)\left(\frac{h_c}{K}\right)^{\alpha} W^{\alpha\sigma/(\sigma-1)} \tag{15}$$

Human capital, finally, grows according to

$$\frac{\dot{h}_c}{h_c} = (\epsilon/S)((1-u)L)^{\psi}\left(\frac{I_E}{h_c}\right)^{1-\psi}. \tag{16}$$

Thus, the economy is completely described by equations (14), (15) and (16) plus the limiting transversality conditions of the households and initial conditions with respect to the capital stocks.

A balanced growth path (BGP) is defined as a path on which all endogenous variables grow at the same constant rate, i.e. $\dot{K}/K = \dot{C}/C = \dot{h}_c/h_c = g > 0$ holds, with $g = constant$. To analyze our economy around a BGP we define the new variables $c \equiv C/K$ and $h \equiv h_c/K$. Differentiating these variables with respect to time yields a two dimensional system of differential equations given by $\dot{c}/c = \dot{C}/C - \dot{K}/K$ and $\dot{h}/h = \dot{h}_c/h_c - \dot{K}/K$. Using equation (14)-(16) this system can be written as follows,

$$\begin{aligned} \dot{c} &= c\left(h^{\alpha}W^{\alpha\sigma/(\sigma-1)}((1-\tau_K)(1-\alpha)+\Omega-1) - \rho + c + (T/K)(1-\kappa)\right) && (17)\\ \dot{h} &= h\left((\epsilon/S)((1-u)L)^{\psi}h^{(\alpha-1)(1-\psi)}W^{(1-\psi)\alpha\sigma/(\sigma-1)}\Omega^{1-\psi} + c + h^{\alpha}W^{\alpha\sigma/(\sigma-1)}(\Omega-1)+\right.\\ &\quad \left.(T/K)(1-\kappa)\right) && (18) \end{aligned}$$

with Ω given by $\Omega = (\tau_K(1-\alpha)(\kappa - p\mu) + W^{-1}(\kappa-p)\tau_L\alpha u^{-1/\sigma}L^{(\sigma-1)/\sigma} + W^{-1}(\kappa - p)\tau_N\alpha(\xi N)^{(\sigma-1)/\sigma} - W^{-1}(1-u)\alpha u^{-1/\sigma}L^{(\sigma-1)/\sigma})$.

A solution of $\dot{c} = \dot{h} = 0$ with respect to h, c gives a BGP for our model and the corresponding ratios $h^\star, c^\star$ on the BGP.[2] It should be noted the tax revenue T grows at

[2]The * denotes BGP values and we exclude the economically meaningless BGP $h^\star = c^\star = 0$.

the same rate as capital K on the BGP, because the return to capital r is constant and the wages grow at the same rate as capital. Thus, T/K is constant along the BGP.

Proposition 1 gives results as concerns existence and stability of the BGP showing that the model is both globally and locally determinate.

Proposition 1 *Assume that educational investment I_E is positive. Then, there exists a unique balanced growth path for the model economy described by equations* (14)-(16) *which is saddle point stable.*

Proof: See appendix.

Before we study growth effects of fiscal policy in the next section we briefly summarize the economic structure of our model economy. Sustained per-capita growth in our economy results from human capital formation which raises labor productivity of skilled labor. Due to spill-over effects unskilled labor benefits from human capital accumulation, too, which increases its wage rate. Human capital is formed in the educational sector where the government hires a certain fraction of skilled labor as teachers and supplies additional input, financed by the tax revenue. As concerns the dynamics this economy is characterized by a unique BGP which is a saddle point.

3 Growth effects of fiscal policy

In this section we analyze how variations of policy parameters affect economic growth. First, we do this for the model on the balanced growth path and, then, taking into account transition dynamics. In the latter case, we assume that the economy is on the BGP when a parameter is changed and study the adjustment path to the new BGP.

3.1 Results for the model on the BGP

We begin with an analysis of changes in the labor income tax. The balanced growth rate is given by equation (15) showing that neither labor income tax rate directly affects the balanced growth rate. This results from our assumption that both labor supplies are given exogenously. So, variations in the labor income tax rates affect the balanced growth rate only through the effect on the return to capital r. The following proposition shows under which conditions an increase in the labor income tax rates raises the balanced growth rate.

Proposition 2 *An increase in the labor income tax rates raises (reduces) the balanced growth rate if* $\kappa > (<)\ p$ *holds.*

Proof: Differentiating (15) with respect to τ_i, $i = L, N$, shows that the sign of the derivative is equal to the sign of $\partial h^\star/\partial \tau_i$, $i = L, N$. In the appendix we define the function $q(h, \cdot)$ and a h such that $q(\cdot) = 0$ gives a BGP. The derivative $\partial h^\star/\partial \tau_i$, $i = L, N$, is obtained by implicit differentiation from $q(\cdot) = 0$ as $\partial h^\star/\partial \tau_i = (\kappa - p) const_1/(-\partial q/\partial h)$, with $const_1 > 0$ a positive constant depending on parameters. In the appendix it is shown that $\partial q/\partial h < 0$ so that the proposition is proved. □

This proposition shows that a higher labor income tax rate leads to a higher balanced growth rate if the share of productive public spending is sufficiently large. More concretely, the growth rate rises if more of the additional tax revenue is used for educational spending than for transfers. It should also be mentioned that for $\mu = 1$, i.e. if the whole capital income tax revenue is used as a basis for transfers, sustained growth would not be feasible for $\kappa < p$ because then educational spending, I_E, would be negative. So, $\kappa < p$ may generate ongoing growth only for a sufficiently small value of μ, implying that a large fraction of the capital income tax revenue is not used for financing transfer payments but, instead, for the formation of human capital.

Now, assume that $\kappa < p$ holds and μ is sufficiently small so that the economy generates sustained per-capita growth. For this case, proposition 2 states that increases in labor income tax rates reduce the balanced growth rate. This is not so obvious because one could expect that an increase in labor taxes positively affects the growth rate since the tax revenue, which is partly used for productive spending, rises and since labor supply is an exogenous variable. But when $\kappa < p$ holds, the part of the labor tax revenue used for productive educational spending is negative implying that a certain part of the capital income tax revenue is removed from educational spending and used for non-productive transfers instead. Therefore, the balanced growth rate declines when the labor income tax rates are raised because this implies that the part of the capital income tax revenue used for productive public spending becomes still smaller while that part used for non-productive spending rises.

Next, we compare growth effects of raising the labor income tax rate on skilled labor to an increase in the labor income tax rate on unskilled labor. Defining the wage sum as the product of the wage rate times the number of labor, we can derive the following corollary to proposition 2.

Corollary 1 *A rise in the labor income tax rate on skilled labor has a stronger (smaller) effect on the balanced growth rate than a rise in the labor income tax rate on unskilled labor if the wage sum of skilled labor relative to unskilled labor is larger (smaller) one.*

Proof: To prove that corollary, we compute the expression $|\partial g/\partial \tau_L| - |\partial g/\partial \tau_N|$, with g denoting the balanced growth rate. The latter expression is positive (negative) for $w_L L/w_N N \geq (\leq)\ 1$ which is equivalent to $u^{-1/\sigma} L^{(\sigma-1)/\sigma} \geq (\leq)\ (\xi N)^{(\sigma-1)/\sigma}$. □

If we eliminate scale effects by setting $L = N$, this corollary shows for example that raising the tax rate on skilled labor raises the balanced growth rate more than increasing the tax rate on unskilled labor[3] if the wage premium is larger one, which is expected to be the normal case. The wage premium is defined as the wage rate of skilled labor relative to unskilled labor. The economic mechanism behind this result is obvious. If the wage rate of skilled labor exceeds that of unskilled labor the additional tax revenue, due to a higher labor tax, from taxing skilled labor is higher than that resulting from taxing unskilled labor. As a consequence, taxing skilled labor leads to a stronger increase of educational spending and, thus, to a stronger rise in $h^\star$ and in the growth rate.

[3]Provided the balanced growth rate rises at all when the labor income tax rate is increased.

Next, assume that $L \neq N$ holds and that the amount of labor is varied. Then, a rise in the number of unskilled labor reduces the wage sum of skilled labor relative to the wage sum of unskilled labor if $\sigma > 1$ holds.[4] Thus, economies with a large number of unskilled labor, given a certain amount of skilled labor, may be able to raise the long-run growth rate stronger when the tax rate on unskilled labor is increased compared to a rise in the tax rate on skilled labor, even if the wage premium is larger one. This holds because for a sufficiently high elasticity of substitution, i.e. for $\sigma > 1$, raising the number of unskilled labor implies that the wage sum of skilled labor relative to unskilled labor declines. So, if the number of unskilled labor is sufficiently high, the wage sum of skilled labor relative to the wage sum of unskilled labor is smaller one so that variations in the labor income tax rate on unskilled labor show stronger growth effects than variations in the labor income tax rate on skilled labor. For $\sigma < 1$, that is for a small elasticity of substitution, the reverse holds. In this case, a decline in the number of unskilled labor reduces the wage sum of skilled labor relative to the wage sum of unskilled labor, given a certain supply of skilled labor. Consequently, taxing unskilled labor stronger than skilled labor generates larger growth effects in the long-run for a number of N sufficiently small in this case. When the number of unskilled labor is fixed and skilled labor is varied, the same argumentation applies inversely.

Next, we study growth effects of varying the capital income tax rate and the fraction of skilled labor employed in the educational sector. When the capital income tax rate is increased there is always a negative growth effect resulting from the disincentive to invest. On the other hand, a higher capital income tax rate raises human capital which tends to increase the balanced growth rate. Consequently, the balanced growth rate is maximized when the positive effect just equals the negative one.

As concerns an increase in the fraction of skilled labor employed in the educational sector the analysis is more complex. On the one hand, raising skilled labor in the educa-

[4]This is immediately seen from the proof of the corollary.

tional sector exerts a negative direct growth effect since it reduces the marginal product of capital because less skilled labor is employed in the productive sector. On the other hand, more skilled labor in the educational sector tends to raise human capital relative to physical capital and, thus, economic growth. But it must also be taken into account that educational spending, I_E, declines because more public spending for skilled labor implies less educational spending due to the government budget constraint. If human capital relative to physical capital rises as more skilled labor is employed in the educational sector there exists a growth maximizing value for the share of skilled labor in education. The growth maximizing value is obtained when the positive effect of education on human capital formation just equals the negative.

If there exists a growth maximizing share for the fraction of skilled labor in the educational sector, a situation may exist where too much skilled labor is employed in that sector. In this case, reducing skilled labor in the education sector and shifting it to the productive sector raises the balanced growth rate even if human capital relative to physical capital declines. This holds because the positive direct effect of more skilled labor in the productive sector dominates the negative effect of less human capital so that the return to capital rises and, thus, the incentive to invest. In such a case, we may speak of over-education in an economy. We summarize our considerations in proposition 3.

Proposition 3 *Raising the capital income tax rate raises (reduces) the balanced growth rate if* $\eta_{h,\tau_K} \geq (\leq)\, \tau_K/(\alpha(1-\tau_K))$, *with* $\eta_{h\tau_K}$ *the elasticity of the human to physical capital ratio with respect to* τ_K.

Assume that raising skilled labor in the educational sector increases human relative to physical capital. Then, a rise in the share of skilled labor in the education sector raises (reduces) the balanced growth rate if $\eta_{h,u} \geq (\leq)\, (\sigma/(1-\sigma))(\partial W/\partial u)(u/W)$.

Proof: Differentiating the balanced growth g, given by (15), with respect to τ_K yields

$$\partial g/\partial \tau_K = -(1-\alpha)(h^\star)^\alpha W^{\alpha\sigma/(\sigma-1)} + (1-\tau_K)(1-\alpha)W^{\alpha\sigma/(\sigma-1)}\alpha(h^\star)^{\alpha-1}(\partial h^\star/\partial \tau_K).$$

Defining $\eta_{h,\tau_K} = (\partial h^\star/\partial \tau_K)(\tau_K/h^\star)$ gives the result in the proposition. Note that $I_E > 0$ guarantees $(\partial h^\star/\partial \tau_K) > 0$ where this derivative is obtained by implicitly differentiating $q(\cdot) = 0$, with $q(\cdot)$ as defined in the appendix.

Differentiating (15) with respect to u gives,

$$\partial g/\partial u \geq (\leq)\ 0 \leftrightarrow \eta_{h,u} \geq (\leq)\ (\sigma/(1-\sigma))(\partial W/\partial u)(u/W).$$

Since $(\partial W/\partial u)(\sigma/(1-\sigma)) < 0$ holds the proposition is proved. □

This proposition suggests that there exist growth maximizing values for the capital income tax rate and for the share of labor employed in education. In order to gain additional insight we resort to a numerical example. We take the following parameter values as benchmark. $\rho = 0.1$, $\alpha = 0.6$, $\sigma = 1.5$, $u = 0.95$, $\xi = 0.1$, $\psi = 0.5$, $\tau_K = 0.1$, $\tau_L = 0.3$, $\tau_N = 0.25$, $p = 0.05$, $\kappa = 0.9$, $\mu = 0.5$. The discount ρ seems to be large but is justified if one assumes that one time period comprises several years. $u = 0.95$ means that 95 percent of skilled labor is employed in the productive sector and 5 percent in the educational sector. Further we normalize both skilled and unskilled labor to one, $L = 1$, $N = 1$, and we set $S = 0.1$, $\epsilon = 0.05$.

With these parameter values the growth maximizing capital income tax rate is 45 percent. This high value results from the fact that the elasticity of aggregate production with respect to human capital is equal to the labor share which is 60 percent. From other studies we know that in endogenous growth models with productive public spending the growth maximizing capital income tax rate equals the elasticity of aggregate production with respect to the public capital stock (cf. Futagami et al., 1993) if the government does not have any other types of revenues and expenditures. In our model, the growth maximizing capital income tax rate is smaller than the elasticity with respect to human capital, which is the result of public spending, because the government has also revenues from labor income taxation.

Setting the parameters to their benchmark values and varying the share of skilled labor employed in the education sector one realizes that there also exists a growth maximizing value for u. The share of skilled labor in the educational sector, $(1-u)$, that maximizes growth in our example is about 20 percent. It should be mentioned that the ratio of human to physical capital monotonically rises as $(1-u)$ is increased but the growth rate does not. The latter holds because the positive growth effect of an increase in $h^\star$ is dominated by the negative effect resulting from a decline in W as $(1-u)$ is increased beyond its growth maximizing value.

3.2 Transition dynamics

In this subsection we study effects of varying fiscal policy parameters for our model taking into account transition dynamics. We assume that our model economy is on the BGP path when a parameter is changed and work out the adjustment to the new BGP. We will exemplify the adjustment using a phase diagram where we consider an increase in the capital income tax rate leading to a higher value of the human capital to physical capital ratio. Figure 1 shows the effect in the $(c-h)$ phase diagram.

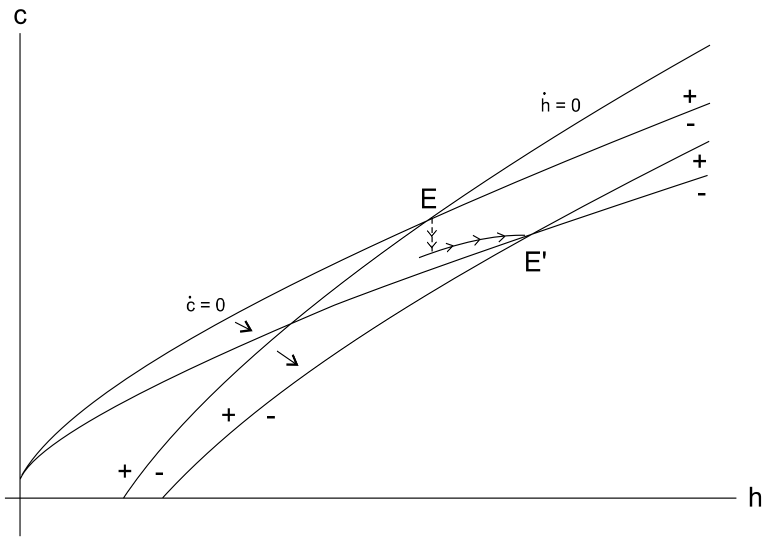

Figure 1: Adjustment to the new BGP in the phase diagram.

Originally, the economy is on the BGP in point E. A rise in the capital income tax rate, for example, shifts both the $\dot{c} = 0$ and the $\dot{h} = 0$ isoclines to the right. Since both human and physical capital are fixed at $t = 0$, when the tax rate is increased, but not consumption, the ratio of consumption to human capital jumps down to the stable manifold, as indicated by the vertical arrows in figure 1. Over time both c and h rise until the new BGP is reached at point E'.

In figure 2 we show the growth rates of human capital, of consumption and of physical capital on the transition path for our numerical example from the last subsection where we increase the capital income tax rate from $\tau_K = 0.1$ to $\tau_K = 0.15$ at $t = 0$ which raises the balanced growth rate from 0.096 to 0.099. The effects on the growth rates on the transition path are derived from equations (16), (15) and (14) where the values for $c(t)$ and $h(t)$ are obtained from the linear approximation to system (17)-(18) as $c(t) = c^\star + (h(0) - h^\star)e^{\mu_1 t}v_{11}/v_{12}$ and $h(t) = h^\star + (h(0) - h^\star)e^{\mu_1 t}$, where μ_1 is the negative eigenvalue of the Jacobian and v_{11}, v_{12} are the elements of the eigenvector corresponding to μ_1.

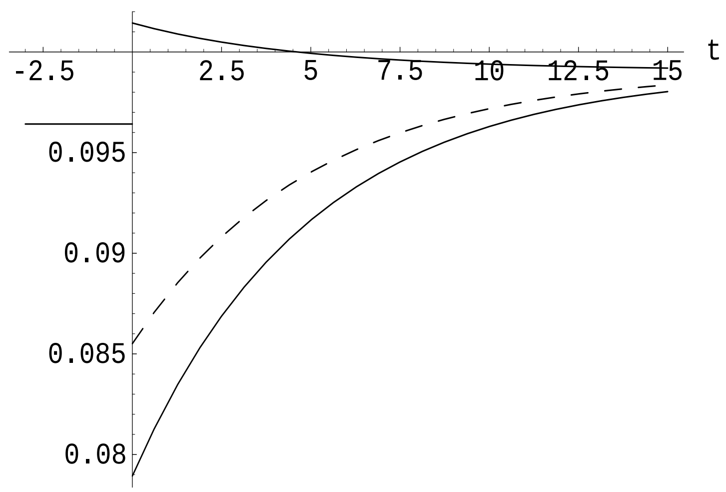

Figure 2: Growth rates of human capital, of consumption and of physical capital on the transition path.

The growth rate of human capital at $t = 0$ jumps up as a consequence of the increase in the capital income tax rate. This holds because a higher tax rate implies higher public spending for education, I_E. The growth rate of consumption is given by the dotted line. This growth rate declines when the capital income tax rate is increased because at $t = 0$ both human and physical capital are fixed so that only the negative growth effect of the higher tax rate affects growth of consumption. Over, time $h(t)$ rises and, thus, the growth rate of consumption. Finally, the growth rate of physical capital also jumps down at $t = 0$ due to the higher tax rate leading to a crowding-out of the private sector. As the ratio h rises over time the growth rate of physical capital increases and approaches the new balanced growth rate.

This subsection has demonstrated that growth effects on the transition path may be quite different from long-run effects. In particular, we could show that a higher capital income tax rate, which generates a higher balanced growth rate, may lead to a transitional crowding-out of the private sector leading to a lower growth rate of the physical capital stock on the transition path.

4 Distributional aspects and welfare effects of fiscal policy

In the last section we have analyzed growth effects of fiscal policy. In this section, we study how fiscal policy affects the distribution of income of skilled labor relative to unskilled labor and how fiscal policy affects welfare on the BGP.

4.1 Distributional aspects

When one analyzes distributional effects of fiscal policy one has to take into account that fiscal policy affects, on the one hand, the income of the households both directly (for example by the capital income tax rate) and indirectly by changing factor prices. On the other hand, fiscal policy also affects the distribution of wealth, that is the capital stock of the skilled household relative to that of the unskilled household, and, thus, the distribution of the income. However, it turns out that it is not possible to work out the latter effect for the general model. Therefore, we first analyze our analytical model assuming that the distribution of the capital stocks is fixed and, then, we study the model numerically where we also allow for an endogenous distribution of the capital stocks.

In the following we denote by θ that part of the total capital stock which is owned by the household supplying skilled labor, i.e. $\theta = K_L/K$. The ratio of income of skilled to unskilled labor after taxes, i.e. the after-tax income differential, is given by

$$\frac{Y_L}{Y_N} = \frac{r\theta K(1-\tau_K) + w_L L(1-\tau_L)}{r(1-\theta)K(1-\tau_K) + w_N N(1-\tau_N) + p(w_N N\tau_N + w_L L\tau_L + \mu r K\tau_K)} \tag{19}$$

Dividing the numerator and the denominator by rK, multiplying the numerator and the denominator by $(1-\alpha)$ and using the marginal productivity rules (3)-(5), the after-tax income differential can be rewritten as,

$$\frac{Y_L}{Y_N} = \frac{\theta(1-\tau_K)(1-\alpha) + W^{-1}\alpha(uL)^{(\sigma-1)/\sigma}(1-\tau_L)/u}{(1-\theta)(1-\tau_K)(1-\alpha) + W^{-1}\alpha(\xi N)^{(\sigma-1)/\sigma}(1-\tau_N) + \Pi}, \tag{20}$$

with $\Pi = p\left(\tau_N W^{-1}\alpha(\xi N)^{(\sigma-1)/\sigma} + \tau_L W^{-1}\alpha(uL)^{(\sigma-1)/\sigma}(1-\tau_L)/u + \mu\tau_K(1-\alpha)\right)$

As concerns the effects of variations in the capital income tax rate, τ_K, and in the share of skilled labor employed in the educational sector, $(1-u)$, the effects are not so obvious. Proposition 4 gives the result of varying τ_K.

Proposition 4 *Assume that transfers are equal to zero. Then, a rise in the capital income tax rate reduces (raises) the after-tax income of skilled relative to unskilled labor if the capital income of the skilled household relative to the capital income of the unskilled before taxes is larger (lower) than the ratio of the after-tax wage sums.*

Proof: Differentiating (20) with respect to τ_K leads to

$$\frac{\partial(Y_L/Y_N)}{\partial\tau_K} = \frac{-\theta(1-\alpha)\,(De) - (p\mu(1-\alpha) - (1-\alpha)(1-\theta))(Nu)}{(De)^2},$$

with $Nu > 0$ standing for the numerator of equation (20) and $De > 0$ for the denominator. The sign of the derivative is equal to the sign of the term $-\theta(1-\alpha)W^{-1}\alpha(\xi N)^{(\sigma-1)/\sigma}(1-\tau_N) + (1-\alpha)(1-\theta)W^{-1}\alpha(uL)^{(\sigma-1)/\sigma}(1-\tau_L)/u - \theta(1-\alpha)p\Pi - p\mu(1-\alpha)(Nu)$. Thus, for $p = 0$ we get

$$\frac{\partial(Y_L/Y_N)}{\partial\tau_K} < (>)\ 0 \leftrightarrow \frac{\theta}{1-\theta} > (<)\ \frac{(uL)^{(\sigma-1)/\sigma}(1-\tau_L)/u}{(\xi N)^{(\sigma-1)/\sigma}(1-\tau_N)}$$

This is equivalent to

$$\frac{\partial(Y_L/Y_N)}{\partial\tau_K} < (>)\ 0 \leftrightarrow \frac{rK_L}{rK_N} > (<)\ \frac{w_L L(1-\tau_L)}{w_N N(1-\tau_N)}.$$

□

That proposition demonstrates that a higher capital income tax rate reduces the after-tax income differential if the capital income of the skilled household relative to that of the unskilled before taxes exceeds the after-tax wage sum differential, for the case of no transfer payments to the unskilled household. The wage sum differential is defined as the ratio of the wage sum of skilled labor relative to the wage sum of unskilled labor. This implies that in economies with a more unequal distribution of capital, i.e. with a higher θ, a rise in the capital income tax rate is more likely to reduce income inequality.

If capital is equally distributed (θ equal to 0.5), a rise in the capital income tax rate always raises the after-tax income inequality if the after-tax wage sum of skilled labor exceeds that of unskilled labor. In this case, the capital income of both households is reduced to the same degree so that the income differential rises because the wage income of the skilled household exceeds the wage income of the unskilled household.

Further, for a given distribution of capital, a rise in the capital income tax rate is the more likely to reduce income inequality the smaller the after-tax wage sum differential. The reason behind this result is obvious. If the after-tax wage sum differential is small, a higher capital income tax rate reduces total income of the skilled household, which gets a higher capital income, stronger than total income of the unskilled household. It should also be mentioned that for positive transfers to unskilled labor a higher capital income tax rate is also more likely to reduce income inequality. From an economic point of view, this is obvious and is easily seen from the proof of proposition 4 when we set $p > 0$.

As concerns the effects of human capital in education on the income distribution, these are more complicated and concrete results can only be obtained for a special case where transfers are equal to zero and where the whole capital stock is owned by the household supplying skilled labor. In this case, an increase of human capital in the educational sector reduces the after-tax income differential if the elasticity of substitution between skilled and unskilled labor is sufficiently large. Sufficiently large means that the elasticity of substitution minus one must be greater than the ratio of the labor share to the capital share multiplied by a term which positively depends on the capital income tax rate and on the share of human capital employed in education and negatively on the skilled labor income tax rate, concretely, $(\sigma-1) > \alpha(1-\tau_L)/(u(1-\alpha)(1-\tau_K))$ must hold. For example, with the parameter values of our numerical example from the last section, a rise of human capital in education reduces (raises) after-tax income inequality for $\sigma >$ ($<$) 2.23.

The mechanism behind this result is that a rise of human capital employed in the educational sector always raises the wage sum differential of skilled to unskilled labor. The ratio of capital income of skilled labor relative to the wage income of unskilled labor, however, rises (declines) for $\sigma <$ ($>$) 1. Therefore, the after-tax income differential always rises for a sufficiently small elasticity of substitution between skilled and unskilled labor. But if the elasticity of substitution is sufficiently high, the capital income of skilled labor relative to unskilled labor income declines and outweighs the increase in the wage sum differential so that the after-tax income differential declines.

If one takes into account that the distribution of the capital stocks is endogenous along the BGP, one has to express θ as a function of the parameters underlying the model. In the appendix, it is demonstrated that θ can be computed as,

$$\theta = \frac{K_L}{K} = \frac{(C_L/C)\, c^{\star} - (1-\tau_L)\alpha(uL)^{-1/\sigma}L(h^{\star})^{\alpha}W^{-1+\alpha\sigma/(\sigma-1)}}{\rho}, \tag{21}$$

with $c^{\star}$ and $h^{\star}$ the values of c and h on the BGP which are obtained from solving $\dot{h} = \dot{c} = 0$.

In order to find the effects of varying the capital income tax rate on the distribution of incomes we take the numerical example from section 3.1, following proposition 3, as a benchmark.[5] Table 1 and table 2 show the ratio of the income of the household with skilled labor relative to that supplying unskilled labor, Y_L/Y_N, and the ratio of the capital stock owned by the skilled household to the total capital stock, $\theta = K_L/K$, for different values of the capital income tax rate, where we set $K_L(0)/K(0) = 0.75$ in table 1 and $K_L(0)/K(0) = 0.35$ in table 2.

Table 1. Income distribution and $\theta = K_L/K$ on the BGP for different capital income tax rates with $K_L(0)/K(0) = 0.75$.

τ_K	0.1	0.2	0.3	0.4	0.55
Y_L/Y_N	2.43	2.45	2.46	2.47	2.49
θ	0.75	0.76	0.77	0.78	0.81

Table 2. Income distribution and $\theta = K_L/K$ on the BGP for different capital income tax rates with $K_L(0)/K(0) = 0.35$.

τ_K	0.05	0.15	0.25	0.35	0.55
Y_L/Y_N	1.14	1.11	1.09	1.07	1.04
θ	0.37	0.33	0.29	0.25	0.1

Both tables show that a higher income tax rate improves the position of the household with the higher initial capital stock. In table 1, the household supplying skilled labor has a higher initial capital stock than the household supplying unskilled labor. If the income tax rate is increased, this leads to a further increase of the capital stock of the skilled household, relative to that of the unskilled household and relative to the total capital stock, which, then raises the relative income of that household, too. If the household

[5] With the exception of p which is set equal to zero now, i.e. $p = 0$.

supplying unskilled labor has a higher initial capital stock, an increase in the capital income tax rate raises the position of that household relative to the household supplying skilled labor, both as concerns income as well as concerns wealth, i.e. the capital stock. It should also be mentioned that in table 2, the income of the skilled household exceeds that of the unskilled household although the latter disposes of a higher capital stock. This is due to the higher human capital of the skilled household. But, of course, if the initial capital stock of the unskilled household relative to that of the skilled household was set at still a higher value, the income of the unskilled household would exceed that of the skilled household.

In the next subsection we study welfare effects of fiscal policy.

4.2 Welfare effects of fiscal policy

In this subsection we will study welfare effects of fiscal policy for the economy on the BGP. In particular, we are interested in the question of whether growth and welfare maximization are equivalent goals. Proposition 5 gives a general result and shows that this is not necessarily the case.

Proposition 5 *Assume that the economy is on the balanced growth path. Then, a fiscal policy leading to a higher balanced growth rate may yield smaller welfare if the adjustment in the level of consumption is sufficiently large.*

Proof: Welfare on the BGP is given by $\int_0^\infty e^{-\rho t} ln(C_i(0)e^{gt})dt = (g + \rho \ln C_i(0))\rho^{-2}$, with $i = L, N$ and g denoting the balanced growth rate. $C_i(0)$, $i = L, N$, are obtained from (7) and (9) on the BGP as,

$$
\begin{aligned}
C_L(0) &= K_L(0)\left(r(1-\tau_K) - g + w_L L(1-\tau_L)/K_L(0)\right) \\
C_N(0) &= K_N(0)\left(r(1-\tau_K) - g + w_N N(1-\tau_N)/K_N(0) + P/K_N(0)\right)
\end{aligned}
$$

Using the marginal productivity rules (3)-(5), the definition of P and taking g from (15), these variables can be rewritten as,

$$\begin{aligned}
C_L(0) &= \rho\, K_L(0) + (1-\tau_L)\alpha\, K(0)h(0)^\alpha W^{-1+\alpha\sigma/(\sigma-1)}(uL)^{(\sigma-1)/\sigma}/u \\
C_N(0) &= \rho\, K_N(0) + (1-\tau_N)\alpha\, K(0)h(0)^\alpha W^{-1+\alpha\sigma/(\sigma-1)}(\xi N)^{(\sigma-1)/\sigma} + p \cdot \\
&\quad K(0)h(0)^\alpha W^{-1+\alpha\sigma/(\sigma-1)}\left(\alpha\tau_L(uL)^{(\sigma-1)/\sigma} + \alpha\tau_N(\xi N)^{(\sigma-1)/\sigma} + \tau_K\mu(1-\alpha)W\right)
\end{aligned}$$

□

Proposition 5 is quite general and shows that a fiscal policy which raises the balanced growth rate may nevertheless imply smaller welfare. The reason for this outcome is that an increase in the balanced growth rate is the result of a higher investment share and a decline in the consumption share, which implies a decrease in the level of consumption at $t = 0$ when the fiscal policy is enacted. A decline in the level of consumption negatively affects welfare whereas a higher growth rate has a positive effect on welfare.

To illustrate proposition 5 we consider welfare effects of variations in the labor income tax rates. We know from proposition 2 that a rise in the labor income tax rates always increases the balanced growth rate for $\kappa > p$. In contrast to that, corollary 2 demonstrates that there may exist welfare maximizing labor income tax rates.

Corollary 2 *A rise in the labor income tax rate of the household supplying skilled labor raises (reduces) welfare of this household if $C_L(0)\,(\partial g/\partial\tau_L) > (<) -\rho\,(\partial C_L(0)/\partial\tau_L)$. A rise in the labor income tax rate of the household supplying unskilled labor raises (reduces) welfare of this household if $C_N(0)\,(\partial g/\partial\tau_N) > (<)\ -\rho\,(\partial C_N(0)/\partial\tau_N)$.*

Proof: Welfare on the BGP of the skilled and unskilled household is given by $J_i = (g + \rho \ln C_i(0))\rho^{-2}$, $i = L, N$. Differentiating J_i with respect to τ_i and setting the resulting expression $> (<)\, 0$ gives, $\partial J_i/\partial\tau_i > (<)\, 0 \leftrightarrow C_i(0)\,(\partial g/\partial\tau_i) > (<) -\rho(\partial C_i(0)/\partial\tau_i)$, $i = L, N$. Taking $C_i(0)$, $i = L, N$, from the proof of proposition 5, the derivatives $\partial C_i(0)/\partial\tau_i$,

$i = L, N$, can be computed as

$$\frac{\partial C_L(0)}{\partial \tau_L} = -\alpha\, K(0) h(0)^{\alpha} W^{-1+\alpha\sigma/(\sigma-1)} (uL)^{(\sigma-1)/\sigma}/u < 0$$
$$\frac{\partial C_N(0)}{\partial \tau_N} = -\alpha\, K(0) h(0)^{\alpha} W^{-1+\alpha\sigma/(\sigma-1)} (\xi N)^{(\sigma-1)/\sigma}(1-p) < 0$$

□

This corollary suggests that there may exist a welfare maximizing value for the labor income tax rate of each household. It is obtained when the positive growth effect of a higher tax rate multiplied by the level of consumption just equals the decline in consumption multiplied by the discount rate. However, boundary solutions cannot be excluded. In order to demonstrate that an interior welfare maximizing labor income tax rate may indeed exist, we compute this value for our numerical example of the last section.

It should be noted that welfare of the household with skilled (unskilled) labor always increases if the income tax rate on unskilled (skilled) labor rises, for $\kappa > p$. This holds because a higher labor income tax rate leads to a higher balanced growth rate and the labor income tax rate on skilled (unskilled) labor does not affect consumption of the unskilled (skilled) household. So, for example raising the labor income tax rate on skilled labor always raises welfare of the household with unskilled labor while it may lead to higher or lower welfare of the household supplying skilled labor.

As to our numerical example we use the same parameter values as in the example in section 3.1, following proposition 3. In addition, we normalize $K_0 = 1$ and we set $K_L(0) = 0.75$. With these parameter values welfare of the household with skilled labor is maximized for $\tau_L = 0.39$. Table 3 gives welfare J_L as a function of the labor income tax rate τ_L.

Table 3. Welfare of the household supplying skilled labor.

τ_L	0.1	0.2	0.3	0.39	0.5
J_L	-5.912	-5.102	-4.677	-4.557	-4.719

Setting $\tau_L = 0.3$, as in the last section, welfare of the unskilled household is maximized for $\tau_N = 0$, i.e. there is a boundary solution. Increasing transfer payments, the boundary solution disappears. For example, with $p = 0.15$ welfare of the household supplying unskilled labor is maximized for $\tau_N = 0.06$. Table 4 gives welfare of the unskilled household, J_N, for different values of the labor income tax rate τ_N.

Table 4. Welfare of the household supplying unskilled labor with $p = 0.15$.

τ_N	0	0.06	0.1	0.2	0.3
J_N	-12.179	-12.163	-12.172	-12.262	-12.45

The outcomes of tables 3 and 4 are not too surprising. Since it is in particular skilled labor which benefits from public spending for education, there exists an interior welfare maximizing value of the income tax rate on skilled labor. Unskilled labor profits from human capital formation through the spill-over effect of human capital in the production of the final good which, however, is of course smaller than the direct effect of human capital for skilled labor. Therefore, a boundary solution is obtained with respect to the income tax rate on unskilled labor for the parameter values of the last section. Raising transfer payments makes unskilled labor benefit from human capital to higher degree because a larger share of the tax revenue is transferred to the household with unskilled labor. Therefore, increasing the transfer share generates an interior solution for the income tax rate on unskilled labor, which maximizes welfare of unskilled labor.

5 Conclusion

In this paper we have presented an endogenous growth model with heterogenous agents and human capital, where human capital is the result of public spending for education and of teachers, i.e. human capital, hired by the government. The labor input in the aggregate production function is modelled by a CES function. There exist positive spill-over effects of human capital in the production function which makes unskilled labor benefit from human capital accumulation of skilled labor.

The analysis of fiscal policy in this model has demonstrated that, among other things, there may be over-accumulation of human capital due to too much human capital employed in the educational sector. Further, we could show that a rise of human capital employed in the educational sector raises (reduces) income inequality if the elasticity of substitution between skilled and unskilled labor is relatively small (large). Finally, we demonstrated that growth and welfare maximization may be different goals. So, there may exist a welfare maximizing labor income tax rate even when a higher labor income tax rate always raises the balanced growth rate.

It should be mentioned that all of our results were derived for exogenously given labor supplies. While the assumption of an exogenously given supply of unskilled labor may be justified, this does not necessarily hold for skilled labor. So, it would be interesting to make labor supply an endogenous variable. But given that labor input is modelled by a CES function with skilled and unskilled labor as arguments, the analysis with an endogenous labor supply is far from trivial and results may only be attainable for numerical examples. Such an analysis, however, is left for future research.

A Proof of proposition 1

Since we neglect the economically meaningless BGP with $c^\star = h^\star = 0$ we can consider system (17)-(18) in the rates of growth. Setting $\dot{c}/c = \dot{h}/h = 0$ gives $-\rho + h^\alpha W^{\alpha\sigma/(\sigma-1)}(1-\tau_K)(1-\alpha) = (\epsilon/S)((1-u)L)^\psi h^{(\alpha-1)(1-\psi)} W^{(1-\psi)\alpha\sigma/(\sigma-1)} \Omega^{1-\psi}$. Defining $q = (\epsilon/S)((1-u)L)^\psi h^{(\alpha-1)(1-\psi)} W^{(1-\psi)\alpha\sigma/(\sigma-1)} \Omega^{1-\psi} + \rho - h^\alpha W^{\alpha\sigma/(\sigma-1)}(1-\tau_K)(1-\alpha)$ we can state that a solution h such that $q(\cdot) = 0$ holds gives a BGP for our model. Since $I_E > 0 \leftrightarrow \Omega > 0$, it is easily seen that $\partial q(\cdot)/\partial h < 0$ holds and $\lim_{h\to 0} q(\cdot) = +\infty$ and that $\lim_{h\to\infty} q(\cdot) = -\infty$ implying that there exists a unique $h^\star$ which solves $q(\cdot) = 0$ implying that there is a unique BGP.

To study the dynamics we compute the Jacobian matrix evaluated at the rest point which is given by

$$J = \begin{bmatrix} c^\star & c^\star \alpha (h^\star)^{1-\alpha} W^{\alpha\sigma/(\sigma-1)}((1-\tau_K)(1-\alpha)-1+\Omega) \\ h^\star & a_{22} \end{bmatrix},$$

$a_{22} = \alpha(h^\star)^\alpha W^{\alpha\sigma/(\sigma-1)}(\Omega-1)+(\alpha-1)(1-\psi)(h^\star)^{(\alpha-1)(1-\psi)}W^{(1-\psi)\alpha\sigma/(\sigma-1)}\Omega^{(1-\psi)}(\epsilon/S)((1-u)L)^\psi$.

As concerns the sign of the determinant of this matrix we see that the following holds. $sign(\det J) = sign((\alpha-1)(1-\psi)(h^\star)^{(\alpha-1)(1-\psi)}W^{(1-\psi)\alpha\sigma/(\sigma-1)}\Omega^{(1-\psi)}(\epsilon/S)((1-u)L)^\psi - \alpha(h^\star)^\alpha W^{\alpha\sigma/(\sigma-1)}(1-\tau_K)(1-\alpha))$. It is immediately seen that the sign is strictly negative which is necessary and sufficient for a saddle point.

B Derivation of θ on the BGP

To derive the endogenous value for $\theta = K_L/K$ on the BGP we introduce the new variable $c_L = C_L/K_L$. Differentiating c_L with respect to time gives $\dot{c}_L/c_L = \dot{C}_L/C_L - \dot{K}_L/K_L$. From (10) and (11) we know that the growth rates of consumption of the skilled and unskilled households each equal the growth rate of aggregate consumption for all $t \in [0,\infty)$. Thus, we can write $\dot{c}_L/c_L = \dot{C}/C - \dot{K}_L/K_L$. On the BGP we have $\dot{c}_L/c_L = 0$ implying $\dot{C}/C = \dot{K}_L/K_L$. Solving the latter equality with respect to c_L and using $K_L/K = (K_L/C_L)(C_L/C)(C/K)$ gives equation (21). Since the ratio C_L/C is constant for all $t \in [0,\infty)$ we have $C_L/C = C_L(0)/C(0)$, where $C_L(0)$ and $C(0)$ are endogenous and are obtained from (7) and (14), respectively, showing that they are determined by $K_L(0)$ and $K(0)$.

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