Relative and Individual Regulation: An Investigation of Investment Incentives under a Cost-Plus Approach *

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Abstract

In this paper we analyze the effects of a modified Yardstick competition on firms' cost-reduction efforts. Departing from the existing literature, we use a relative cost-plus approach: firms are regulated on the basis of other firms' performances, but they are granted a mark-up and not a lump-sum transfer in order to be compensated for their investments. We show that the trade-off between encouraging cost reduction and minimizing prices under individual cost-plus regulation disappears under relative cost-plus regulation but that the latter is not always implementable. Indeed, in some econonomic environments, firms wil prefer to stay out of the market instead of being regulated under this regime, whatever the value of the mark-up granted by the regulator. Furthermore, we extend our model by including technical spillovers and we investigate their effects on firms' cost reduction efforts and on the efficiency of the whole industry. Finally, we allow for quality-enhancing investments and study the interaction between them and cost reduction investments.

Keywords: Yardstick Competition, Cost Reduction, Cost-Plus Regulation.

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1 Introduction

Yardstick competition, see SHLEIFER[1985], is a well-established and applied method to regulate *local monopolies*. Apart from the reimbursing of hospitals, see DRANOVE[1987], Yardstick competition is also used in the electricity industry, see JOSKOW/SCHMALENSEE[1986], and in the telecommunication sector, see FCC[1997]. Considering firms of the same industry, the idea of Shleifer's approach is the regulation of these firms using constructed benchmarks. These are based on costs of all other firms in the considered industry, but they are independent of the costs of the firm the benchmark is created for. Due to this independence, firms are expected to have a high incentive to reduce and reveal their costs.

This paper investigates the effects of a combination of Yardstick competition and *cost-plus* regulation on firms' cost reduction incentives in a one-shot game.[1] More precisely, we analyze the cost reduction efforts of symmetric firms when the regulated price is equal to the average of the marginal costs of the other firms plus a relative mark-up. Doing so, we depart from the traditional Yardstick competition since we do not allow for transfers on the part of the regulator. Our approach is motivated by the empirical observation that in many industries regulators set prices using some regulatory rule (mainly a price-cap or a cost-plus rule) without making any transfer to the regulated firms. Even though it is clear that making side payments leads to less deadweight loss than setting prices higher than marginal costs, there are some rationales for not using transfers. First, collecting public funds is costly and, second, making transfers may increase the likelihood of regulatory capture since the regulator cannot be directly monitored by consumers (through prices)

We start by assuming that our relative mark-up parameter is exogenously given. We do so because real-world regulators' decisions do not always result in a socially desirable value of the mark-up since they may be influenced by interest groups such as lobbies.[2] Then, it is interesting to know how the investment incentives and prices are affected by a mark-up which may not be set optimally. We determine the Nash equilibria of our modified Yardstick game for any value of the mark-up parameter and compare them with the cost reduction effort of a firm under individual cost regulation.[3] From this point of view, our paper is close to DALEN[1996], who

[1] This combination meets the requirement of JASKOW[2006], who postulates that regulation has to be seen as a bundle of different instruments.

[2] For on overview about lobbyism activities in the European Union, see SVENDSEN[2002].

[3] Our approach abstracts from collusive behavior under Yardstick competition. For an analysis of incentives to collude, see TANGERÅS[2002], and LAFFONT/MARTIMORT[2000].

compares individual regulation to Yardstick competition. He considers an assymetric information framework and investigates firms' informational rents (granted through transfers) and their incentives for conducting industry-specific and firm-specific investments. DALEN[1996] finds that the optimal regime for encouraging investments depends crucially on their nature: Yardstick competition leads to more firm-specific investments and individual regulation entails more industry-specific investments. Note that, in this paper, the advantages of Yardstick competition are strongly related to the reduction of the regulator's informational problem. In contrast to DALEN[1996], we assume a complete information framework, which implies that we abstract from a big advantage of Yardstick competition.[4]

We show that relative cost-plus regulation is not always implementable. Indeed, for some values of the model parameters, the break-even constraint of the regulated firms is not met for any value of the mark-up parameter. This result hinges on the fact that we do not allow for transfers and points out a weakness of the relative mark-up as a regulatory instrument. But whenever our modified Yardstick competition is implementable, it dominates the individual cost-plus regulation from static and dynamic perspectives since it leads to higher investment (lower underinvestment) and lower prices. This is due to the fact that relative regulation creates an artificial competition between firms in which cost reduction efforts become strategic complements. One of our main findings is that the cost reduction effort under our modified Yardstick competition is decreasing in the mark-up, which is contrary to the case of individual cost regulation, where the cost reduction is increasing in the mark-up. This difference between the two regimes explains why the tension between encouraging cost reductions and minimizing prices that characterizes individual cost-plus regulation is absent under its relative counterpart. Hence, under relative cost-plus regulation, the regulator does not have to allow for high prices in order to stimulate cost-reducing investments. We also show that granting a too high mark-up allows firms, under some general conditions, to replicate the unregulated monopoly outcome. Even though we focus on the cost reduction efforts, we do not disregard the effects of our two regulation regimes on prices. In the case of regulation with transfers, prices are equal to marginal costs and consequently they are negatively related to cost reduction which makes it possible to derive the comparative statics on prices from those on cost reductions. In our setting (without transfers), prices are affected by the mark-up parameter in a *direct* way and not only indirectly through cost reductions, which makes the price analysis not just a corollary of the investment analysis. Note also that

[4] Since Yardstick competition reduces regulators' informational problem, we disregard a welfare enhancing effect; see AURIOL[2000], TANGERÅS[2002, 2003].

Then, we endogenize the mark-up parameter in a complete information framework. We determine the socially optimal mark-up using a standard welfare function and we compute the investment-maximizing mark-up. It turns out that under relative cost-plus regulation, both criteria lead to the same mark-up choice, which is not the case under individual regulation.

Finally, we extend our model in two directions. First, we include (small) technical spillovers in our framework and investigate their effects on the cost reduction efforts of the regulated firms and on the efficiency of the whole industry. Second, we allow firms to undertake quality-improving investments. Following BUEHLER/SCHMUTZLER/BENZ[2005] we assume that these investments have a demand-enhancing effect. Different from TANGERÅS[2002], we consider the case where prices are regulated on cost only, not on quality. This captures the fact that regulatory authorities are rather incapable to consider all possibilities of quality enhancement. We study the interplay between quality investments and cost reduction efforts in such a setting.

The remainder of this paper is organized as follows. Our model of cost-plus regulation is introduced and solved in section 2. In this section we determine equilibrium investments and prices under three regimes: non-regulated monopoly, individual cost-plus regulation and relative cost-plus regulation. In section 3, we compare the outcomes of the two latter regimes. Section 4 is devoted to the endogenization of the mark-up parameter and to welfare analysis. In section 5, we extend our model by allowing for technical spillovers and quality-enhancing investments. Section 6 concludes.

2 The model

We consider an industry of N symmetric firms. These firms are assumed to be *local monopolists* on their market, e.g. electricity network providers, railway system operators, hospitals etc. The demand for each firm i is given by

$$D(p_i) = a - p_i, \tag{1}$$

where a denotes the market size and p_i the price for the product of firm i. We abstract from fixed costs and assume that the initial marginal cost of each local monopolist is given by $c < a$. Furthermore, firms can spend an effort, denoted u, which reduces their marginal costs linearly. Formally, the constant marginal cost of firm i after undertaking an investment u_i is given by:

$$c_i = c - u_i. \tag{2}$$

where $u_i \in [0, c]$. Let us denote by $C_i(u_i)$ the cost of a cost reduction effort u_i. We choose the following convex quadratic expression for this cost:

$$C_i(u_i) = \frac{\gamma}{2} u_i^2.$$

We first examine the investment efforts and the prices under two benchmark regimes, namely non-regulated monopoly and individual cost-plus regulation. Then we proceed with the analysis of relative cost-plus regulation and show the advantages and drawbacks of this kind of regulation.

2.1 Investment under non-regulated monopoly

In the case of an unregulated monopoly the net profit function of each firm is given by:

$$\pi^M(p, u) = (a - p)(p - c + u) - \frac{1}{2}\gamma u^2.$$

Once the cost reduction u is implemented, the monopolist sets the (monopoly) price:

$$p^M(u) = \frac{1}{2}(c - u + a),$$

which results in the following profit:

$$\pi^M(p^M(u), u) = \frac{1}{4}(a - c + u)^2 - \frac{1}{2}\gamma u^2.$$

Let us suppose first suppose that $\gamma > \frac{1}{2}$. Under this assumption, the profit function $\pi^M(p^M(u), u)$ is concave with respect to the effort. Using the first-order condition:

$$\frac{d\pi^M(p^M(u), u)}{du} = 0,$$

it is straightforward to see that the unconstrained maximum is reached at $u = \frac{a-c}{2\gamma-1}$. Taking into account the restriction $u \in [0, c]$, we get the monopoly cost reduction effort:

$$u^M = \begin{cases} \frac{a-c}{2\gamma-1} & if \quad a \leq 2\gamma c \\ c & if \quad a > 2\gamma c. \end{cases}$$

Thus, the monopoly price is given by:

$$p^M = \begin{cases} \frac{(a+c)\gamma-a}{2\gamma-1} & if \quad a \leq 2\gamma c \\ \frac{a}{2} & if \quad a > 2\gamma c. \end{cases}$$

and the net monopoly profit is:

$$\pi^M = \begin{cases} \frac{\gamma(a-c)^2}{2(2\gamma-1)} & if \quad a \leq 2\gamma c \\ \frac{a^2}{4} - \frac{1}{2}\gamma c^2 & if \quad a > 2\gamma c. \end{cases}$$

Note that if $\gamma \leq \frac{1}{2}$, the function $u \longrightarrow \pi^M(p^M(u), u)$ is increasing over the interval $[0, c]$ and hence reaches its maximum value at $u = c$. Since the combination of the inequalities $\gamma \leq \frac{1}{2}$ and $a > c$ leads to $a > 2\gamma c$, the above expressions of the monopoly cost reduction effort, price and profits remain true even when $\gamma \leq \frac{1}{2}$.

2.2 Individual cost-plus regulation

Under *individual cost-plus regulation,* firms are regulated on the basis of their own production costs. More precisely, we assume that the maximum allowed price equals the production cost of the regulated firm, where a mark-up on this cost is guaranteed:[5]

$$p^{IR} = \mu(c - u), \tag{3}$$

where $\mu \geq 1$ denotes the (relative) mark-up parameter, c the initial cost level, and u the cost reduction effort of the firm.

The maximization program of an individually regulated firm (under cost-plus regulation) is:

$$\max_{u \in [0,c], p \in [0,a]} \pi(p, u) = (p - c + u)(a - p) - \frac{1}{2}\gamma u^2 \tag{4}$$

subject to the regulatory constraint:

$$p \leq \mu(c - u).$$

Before giving the formal solution of this maximization program, let us analyze the marginal effect of cost-reduction on the firm's profit when the regulatory constraint is binding. In this case the profit function is given by

$$\pi(u) = (\mu - 1)(c - u)(a - \mu(c - u)) - \frac{1}{2}\gamma u^2.$$

It is then clear that a cost-reduction has a negative price effect which is decreasing in the mark-up parameter μ, and a positive demand effect which is increasing in the mark-up parameter

[5] Such a cost based regulation method can be found in the regulation of the german electricity industry for example.

μ. This entails that a higher mark-up will have a (weakly) positive effect on the investment incentives, since it will decrease the negative price effect and increase the positive demand effect.

The following proposition summarizes a firm's behavior under individual cost-plus regulation.

Proposition 1 *Denote $\mu^M = \frac{(a+c)\gamma - a}{2\gamma c - a}$. Under individual cost-plus regulation, the optimal investment and pricing strategy is as follows:*

1. *If $\mu \leq \frac{a}{2c}$, then the regulated firm does not invest in cost reduction and the pricing regulatory constraint is binding: $u^{IR}(\mu) = 0$ and $p^{IR}(\mu) = \mu c$.*
2. *If $\frac{a}{2c} < \mu \leq \mu^M$, then the regulated firm undertakes a cost reduction $u^{IR}(\mu) = \frac{(2\mu c - a)(\mu - 1)}{2\mu(\mu-1)+\gamma}$ and the pricing regulatory constraint is binding : $p^{IR}(\mu) = \frac{a\mu(\mu-1)+\mu\gamma c}{2\mu(\mu-1)+\gamma}$.*
3. *If $\mu > \mu^M$, then the optimal investment and pricing strategy depends on the model parameters in the following way:*
   - If $a < 2\gamma c$, then the regulatory constraint is not binding and the regulated firm is able to implement the monopoly outcome: $u^{IR}(\mu) = u^M$ and $p^{IR}(\mu) = p^M$.
   - If $a \geq 2\gamma c$, then the regulated firm undertakes a cost reduction $u^{IR}(\mu) = \frac{(2\mu c - a)(\mu - 1)}{2\mu(\mu-1)+\gamma}$ and the pricing regulatory constraint is binding : $p^{IR}(\mu) = \frac{a\mu(\mu-1)+\mu\gamma c}{2\mu(\mu-1)+\gamma}$.

Proof: See appendix.

We will focus in this discussion on the cases $a < \min(2c, 2\gamma c)$ (sufficiently small market) and $a > \max(2c, 2\gamma c)$ (sufficiently large market).

Suppose that $a < \min(2c, 2\gamma c)$. In this case, the inequality $\mu \leq \frac{a}{2c}$ is never satisfied and consequently the effort is strictly positive independent of the value of the mark-up parameter μ. This means that the positive demand effect always outweighs the negative price effect if the market is sufficiently small. Furthermore, the investment effort is decreasing in the market size and is increasing in the mark-up parameter μ whenever $\mu \leq \mu^M$, while the price is increasing in the mark-up paramater μ (and of course in the market size) whenever $\mu \leq \mu^M$. Note that μ^M can be interpreted in the case of small markets as the monopoly replicating mark-up since for any $\mu \geq \mu^M$ the firms act as if they were not regulated.

Suppose now that $a > \max(2c, 2\gamma c)$. In this case, the firm does not invest in cost reduction unless $\mu > \frac{a}{2c}$, which means that the positive demand effect outweighs the price effect if and only if the mark-up is sufficiently high. Note that whenever the firm invests, its investment is increasing in the mark-up and the price is increasing in the mark-up. The monopoly outcome $\left(u^M, p^M\right)$ is never reached in this case but is the limit as the mark-up parameter goes to infinity.

2.3 Relative cost-plus regulation

The fundamental idea of yardstick competition is the construction of a benchmark, on which firms are regulated.[6] This benchmark is generally constructed in such a way that the cost reduction of firm i does not *directly* affect the price firm i is regulated on. Hence, without strategic interaction, firms should fully benefit from cost reduction. In our model, we assume, as in SHLEIFER[1985], that the benchmark for firm i is based on the marginal costs of all other $N-1$ firms in the following way:

$$\bar{c}_i = \frac{1}{N-1} \sum_{j \neq i} c_j. \tag{5}$$

However, in contrast to Shleifer we adopt a cost-plus approach instead of allowing for lump-sum transfers. In other words, the regulator sets a *maximum allowed price* for every firm i, which is based on $\bar{c}_i$ and guarantees additionally a mark-up captured by a parameter $\mu \geq 1$. More precisely, the regulated price of firm i is given by

$$p_i^R = \mu \bar{c}_i, \tag{6}$$

with $\mu \geq 1$.

The timing of the game is as follows. In the first stage, the N firms are informed that they are going to be regulated by a relative cost-plus rule (like the one presented above) with a mark-up parameter μ and each firm has to decide whether to stay in its market or leave it. If a firm decides to stay in its market, it commits itself to respect the regulatory constraints and to produce a quantity at least equal to the demand corresponding to the regulated price (hence, quantity is not a strategic variable in this game). Needless to say, firms leave their markets if they expect to get negative profits. In the second stage, firms undertake investments and the corresponding cost reductions immediately come into effect.

We determine now the game's perfect Nash equilibria using backward induction.

[6] See SHLEIFER[1985].

2.3.1 Investment and pricing stage under Yardstick competition

We begin by solving the last stage of the game, namely the investment and pricing stage, assuming that the N firms have accepted the contract proposed by the regulator. The maximization program of firm i, given the cost reduction efforts of the other firms, is given by:

$$\max_{u_i \in [0,c], p_i \in [0,a]} \pi_i(p_i, u_i) = (p_i - c + u_i)(a - p_i) - \frac{1}{2}\gamma u_i^2, \tag{7}$$

subject to the regulatory constraint

$$p_i \leq \mu \bar{c}_i = \mu \left(c - \bar{u}_i \right),$$

where $\bar{u}_i = \frac{1}{N-1} \sum\limits_{j \neq i} u_j$.

The following proposition summarizes the outcomes of the investment and pricing stage.

Proposition 2 *The outcomes of the investment and pricing stage depend on the model parameters in the following way:*

1. *If the markets are sufficiently small relative to the investment cost ($a \leq \gamma c$), then the regulated firms reduce their cost by $u^* (\mu) = \frac{a - \mu c}{\gamma - \mu}$ and get the regulated price $p^* (\mu) = \mu \frac{c\gamma - a}{\gamma - \mu}$, if the mark-up granted by the regulator is not too high $\left(\mu \leq \mu^M\right)$, and reduce their cost by $u^M = \frac{a-c}{2\gamma - 1}$ and set the monopoly price $p^M = \frac{(a+c)\gamma - a}{2\gamma - 1}$ otherwise.*
2. *If the markets are medium-sized relative to the investment cost ($\gamma c < a \leq 2\gamma c$), then the regulated firms invest $u^* = c$ and get the regulated price $p^* = 0$, if the mark-up granted by the regulator is not too high $\left(\mu \leq \mu^M\right)$, and reduce their cost by $u^M = \frac{a-c}{2\gamma-1}$ and set the monopoly price $p^M = \frac{(a+c)\gamma - a}{2\gamma - 1}$ otherwise.*
3. *If the markets are sufficiently large relative to the investment cost ($a > \gamma c$), then the firms invest $u^* = c$ and get the regulated price $p^* = 0$ independent of the value of the mark-up granted by the regulator.*

Proof: See appendix.

We focus first on the case $a \leq \gamma c$. Note that under this assumption, the cost reduction effort undertaken by the regulated firms is positively affected by the size of their markets, which is different from the case of individual regulation. More surprisingly, it is negatively affected by

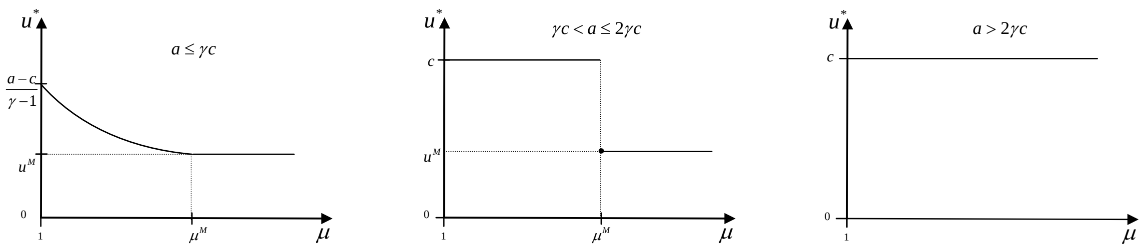

Figure 1: Optimal cost-reduction effort as a function in μ.

the mark-up parameter μ. These two results are due to the effects of the market size and the mark-up on demand: for given cost reduction efforts of its "competitors", the marginal benefit of cost reduction by firm i is equal to the demand $a - p_i^R = a - \mu(c - \bar{u}_i)$, which is clearly increasing in the market size a and decreasing in μ. As for the equilibrium price, it is straightforward to show that it is increasing in the mark-up parameter μ when the regulatory constraint is binding. It is also worth noting that the cost reduction effort and the price are independent of the number of regulated firms, which makes Yardstick competition quite different from usual forms of competition.

Let us turn now to the case $c\gamma < a \leq 2c\gamma$. Due to the positive effect of the market size on the investment effort, a higher market size (relative to the previous case) may make it optimal for the firms to choose the maximum investment (the one that leads to a cost equal to zero). This happens whenever the mark-up parameter is not too high. If the latter condition does not hold, then two other Nash equilibria may emerge, as seen in the proof of Proposition 2, and the pareto-dominant Nash equilibrium is the monopoly-replicating equilibrium.

Finally, when the market is sufficiently large relative to the investment cost ($a > 2c\gamma$) the optimal strategy of each firm, given independently of the other firms' strategies, is to choose the maximum investment level. The prisoner's dilemma outcome is unavoidable in this case whatever the mark-up granted by the regulator.

Turning to the effect of the mark-up parameter on investment efforts and prices, we easily derive from the previous proposition the following result (illustrated in figure (1)):

Corollary 1 *Conditional on the firms' participation, the cost reduction effort of the firms is non-increasing* *in the mark-up and - as a result- the price is non-decreasing in the mark-up.*

Note that under our modified Yardstick competition, the effect of granting a higher mark-up on consumers' surplus is unambiguously negative, since it leads to a higher price. The next subsection shows that even if the regulator is solely concerned by consumers' surplus, it cannot set a too small mark-up because firms may then prefer to leave the market.

2.3.2 Participation stage

Before solving this stage, note that the profits of the regulated firms do not depend on the number of firms, if they decide to stay in their markets . This implies that we cannot have an asymmetric Nash equilibrium in this stage. In other words, either all firms decide to stay in their markets or they all leave.

Let us first assume that $a \leq \gamma c$. In this case, the net profit function of the regulated firms is given by:

$$\pi(\mu) = \begin{cases} \frac{\gamma(a-c\mu)(a+c\mu+2c\gamma(\mu-1)-2a\mu)}{2(\mu-\gamma)^2} & if \quad \mu \leq \mu^M \\ \pi^M & if \quad \mu > \mu^M. \end{cases}$$

It is straightforward to show that $\pi(\mu)$ is strictly increasing over the interval $\left[1, \mu^M\right]$ and that

$$\pi(\mu) = 0 \Longleftrightarrow \mu = \mu_0 = \frac{2c\gamma - a}{2c\gamma - 2a + c}.$$

Hence, under the condition $a \leq \gamma c$, the net profit of the regulated firms is negative if the mark-up parameter is below some threshold, namely $\mu_0 = \frac{2c\gamma-a}{2c\gamma-2a+c}$, and is positive if the mark-up parameter is greater than this threshold. The profit function is easily derived under the two other cases $\gamma c < a \leq 2\gamma c$ and $a > 2\gamma c$. The results under the three scenarios are illustrated in figure (2).

The following proposition summarizes the outcomes of the participation stage.

Proposition 3 *The acceptance of the regulation contract by firms depends on the model parameters and the relative mark-up in the following way:*

1. *If the markets are sufficiently small relative to the investment cost ($a \leq \gamma c$), then the firms will accept the contract proposed by the regulator, if and only if the mark-up parameter is greater than the threshold $\mu_0 = \frac{2c\gamma-a}{2c\gamma-2a+c}$, which is decreasing in the investment cost parameter γ and increasing in the market size a.*

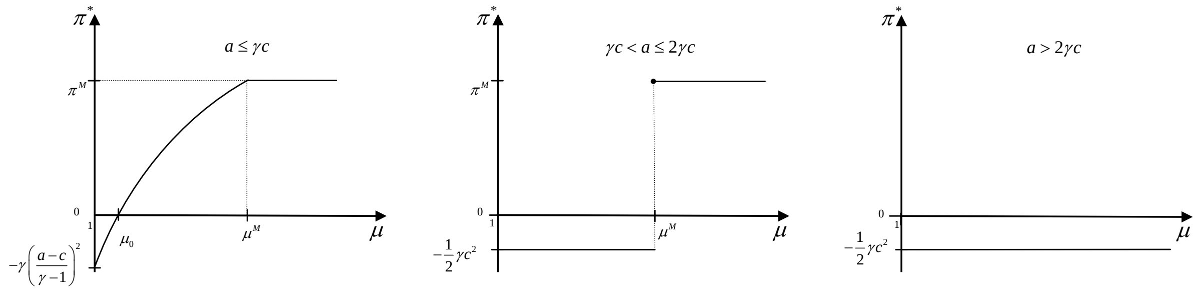

Figure 2: Profits as a function of μ.

2. *If the markets are medium-sized relative to the investment cost*($\gamma c < a \leq 2\gamma c$)*, then the firms will accept the contract proposed by the regulator, if and only if the mark-up parameter* μ *is at least equal to the monopoly-replicating mark-up* μ^M*:* $\mu \geq \mu^M$*.*

3. *If the markets are sufficiently large relative to the investment cost* ($a > 2\gamma c$)*, then the firms will prefer to leave the market independent of the value of the mark-up parameter.*

This proposition shows that it is not always possible to implement our modified Yardstick competition in the sense that under some circumstances firms will prefer to leave the market instead of being regulated this way. One of the main advantages of relative regulation is that it pushes firms to over-invest relative to what they would do, if they were not regulated or if they were individually regulated, but this advantage may turn out to be a serious drawback in some environments. More precisely, in relatively large markets, the artificial dynamic competition created by relative cost-plus regulation leads to a high level of investment which does not allow firms to recoup their costs. Anticipating this, firms will prefer not to be active on their markets. Note that this problem of firms'participation is a major difference with the traditional Yardstick competition. Indeed, when transfers are allowed, the regulator can always make firms accept the regulatory contract.

2.3.3 Overinvestment or underinvestment?

We know that some forms of competition in investments may lead to overinvestment relative to the socially optimal level (for instance, patent races). It is interesting to know whether our modified Yardstick competition has this feature or not.

We focus on the case of sufficiently small markets ($a \leq \gamma c$) and we assume that $\mu_0 \leq \mu \leq \mu^M$. The remaing cases are not relevant since under them, either we get the monopoly outcome or it is impossible to implement our modified Yardstick competition.

Let us assume that a social planner chooses the investment level that maximizes the social welfare function defined as the sum of the industry profits and the consumers' surplus:

$$\max_{(u_1,u_2,...,u_N)\in[0,c]^N} W(\mu, u_1, u_2, ..., u_N) = \sum_{i=1}^{N} [\pi(\mu, u_i, \bar{u}_i) + CS(\mu, u_i)]$$

where:

$$\pi(\mu, u_i, \bar{u}_i) = (\mu (c - \bar{u}_i) - c + u_i)(a - \mu (c - \bar{u}_i)) - \frac{1}{2}\gamma u_i^2$$

and

$$CS(\mu, \bar{u}_i) = \frac{1}{2} (a - \mu c + \mu \bar{u}_i)^2$$

subject to the break-even constraints:

$$\pi(\mu, u_i, \bar{u}_i) \geq 0 \text{ for all } i \in \{1, ..., N\}.$$

The following proposition gives the comparison between the equilibrium investment level under relative cost-plus regulation and the socially optimal level of investment.

Proposition 4 *Suppose that $a \leq \gamma c$. For any $\mu \in [\mu_0, \mu^M]$, the unique (and symmetric) socially optimal investment level $u^{**}(\mu)$ is higher than the equilibrium investment level $u^*(\mu)$ under relative cost-plus regulation.*

Proof: See appendix.

This proposition shows that there is always underinvestment under relative cost-plus regulation. The risk of overinvestment associated with some forms of competition is then absent in the case of our modified Yardstick competition. This is compatible with another feature of Yardstick competition, namely the fact that the outcomes do not depend on the number of firms involved.

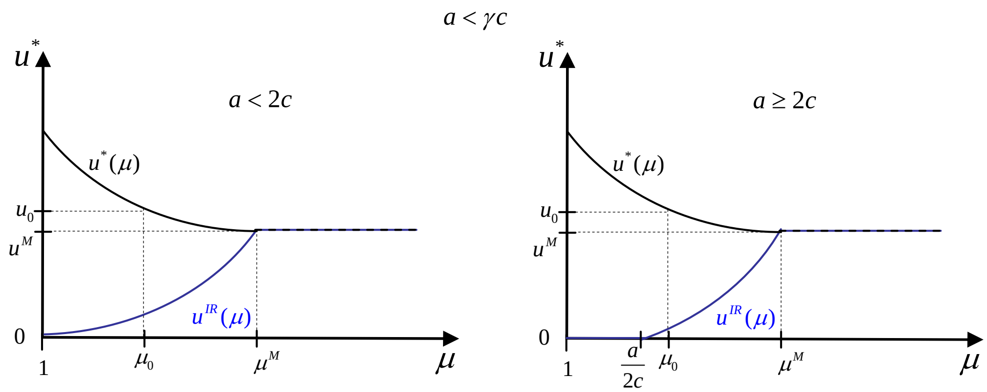

Figure 3: Optimal cost reduction effort under individual and relative regulation

3 Individual vs relative cost-plus regulation

Let us compare the outcomes under individual and relative cost-plus regulation. Under individual regulation, there is a trade-off between encouraging investment and minimizing prices: in order to make firms invest more, the (relative) mark-up has to increase, which leads to a higher price. This tension disappears under relative cost-plus regulation: since the cost reduction effort is decreasing in the mark-up, a smaller mark-up leads to higher investment and lower price. The comparison of these two regimes is given by the following proposition, which we easily derive from the three previous propositions.

Proposition 5 *Individual Vs relative cost-plus regulation (for a given mark-up)*

1. *If the markets are sufficiently small relative to the investment cost* $(a \leq \gamma c)$ *then:*

   a- If the mark-up is sufficiently small $(\mu < \mu_0)$*, firms leave the markets, if they are regulated under relative cost-plus regime, whereas they stay in the market and invest an amount increasing in the mark-up, if they are regulated on an individual basis.*

   b- If the mark-up is intermediate $\left(\mu_0 \leq \mu < \mu^M\right)$*, firms invest more and get a lower price under relative cost-plus regulation than under individual regulation.*

   c- If the mark-up parameter is high enough $\left(\mu \geq \mu^M\right)$*, firms are indifferent between the two regimes: in both cases they are able to implement the monopoly outcome.*

2. *If the markets are medium-sized relative to the investment cost* $(\gamma c < a \leq 2\gamma c)$ *then:*

a- If the mark-up is smaller than the monopoly replication mark-up $\left(\mu < \mu^M\right)$*, firms leave the markets, if they are regulated relative cost-plus regulation, whereas they stay in the market and invest an amount increasing in the mark-up if they are regulated on an individual basis.*

b- If the mark-up parameter is high enough $\left(\mu \geq \mu^M\right)$*, firms are indifferent between the two regimes: In both cases they are able to implement the monopoly outcome.*

3. *- If the markets are sufficiently large relative to the investment cost* $(a \geq 2\gamma c)$*, then firms leave the markets, if they are regulated under our modified Yardstick competition, whereas they stay in the market and invest an amount, which is increasing in the mark-up, whenever* $\mu < \mu^M$ *and replicate the monopoly outcome if* $\mu \geq \mu^M$*.*

The case of sufficiently small markets relative to the investment cost $(a \leq \gamma c)$ - which is in some way the most interesting one- is illustrated in figures (3) and (4) . Figure (3) suggests that in this case, we have a stronger result than the one presented proposition (5): For any values μ_1 and μ_2 in $\left[\mu_0, \mu^M\right]$ the cost-reduction effort of the firms under relative cost-plus regulation is higher under relative cost-plus regulation with a mark-up μ_1 than under individual regulation with a mark-up μ_2. Summing-up all the results of this section, we get the following corollary.

Corollary 2 *For a given mark-up parameter, relative cost-plus regulation (weakly) dominates individual cost-plus regulation from a static and a dynamic point of view, whenever it does not lead to firms leaving their markets.*

This proposition suggests that the only advantage of individual cost-plus regulation relative to our modified yardstick competition is that firms always get nonnegative profits and hence have no incentive to leave the market.

4 Endogenization of the mark-up

So far, we have considered the mark-up μ as an exogenous parameter. Hereafter we assume that it is set by a regulator under complete information: The regulator can observe the marginal product costs of the firms and their investment costs. This is a strong assumption. However, since this paper does not focus on the regulator's decision but on firm's investments given a regulatory environment, we prefer to abstract from the informational asymmetry that may exist between the firms and the regulator.

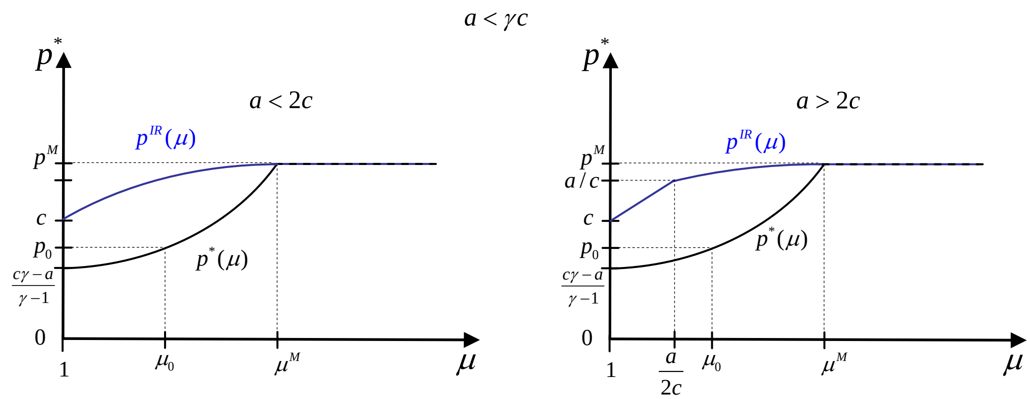

Figure 4: Prices under individual and relative regulation

We suppose that $a < \gamma c$ and that firms are regulated using our modified Yardstick competition. We assume that the regulator seeks to maximize (with respect to the mark-up parameter μ) the social welfare defined as a the sum of the industry profits and the consumers' surplus:

$$W^Y(\mu) = \sum_{i=1}^{N} [\pi_i^*(\mu) + CS_i^*(\mu)] = N(\pi^*(\mu) + CS^*(\mu)) \tag{8}$$

where:

$$\pi^*(\mu) = (\mu - 1)(c - u^*(\mu))(a - \mu(c - u^*(\mu)) - \frac{1}{2}\gamma u^*(\mu)^2$$

and

$$CS^*(\mu) = \frac{1}{2}(a - \mu c + \mu u^*(\mu))^2$$

subject to the (common) break-even constraint:

$$\pi^*(\mu) \geq 0 \,.$$

The solution of this program is given by the following proposition.

Proposition 6 *Maximizing the social welfare function defined by expression (8), a regulator chooses to set the mark-up parameter to the minimum value guaranteeing that firms do not leave their markets:* $\mu = \mu_0 = \frac{2c\gamma - a}{2c\gamma - 2a + c}$, *which happens to be also the investment-maximizing mark-up under the firms' participation constraint.*

Proof: See appendix

Note that these results are quite predictable, since, as we pointed out before, there is no trade-off between the static and dynamic objectives of the regulator under relative cost-plus regulation.

The regulator is able to control the intensity of the artificial competition between firms through the choice of the mark-up parameter. Indeed, the latter can be seen as an inverse measure of this form of competition. Proposition 7 can then be interpreted in the following way: the regulator will set the intensity of the artificial competition to the maximum value that does not lead to firms leaving their markets.

In the previous section we compared the outcomes of individual regulation and relative regulation from static and dynamic perspectives, for *a given (same) value of the mark-up paramater* (see Proposition 5 and its corollary). In the subsequent analysis, we aim at comparing the maximized welfare under individual regulation $\max_{\mu\in[1,\mu^M]} W^{IR}(\mu)$ to the maximized welfare under our modified Yardstick competition $\max_{\mu\in[\mu_0,\mu^M]} W^{Y}(\mu) = W^{Y}(\mu_0)$. Note that the fact that relative regulation dominates individual regulation from a static and dynamic perspective for any given $\mu \in \left[\mu_0, \mu^M\right]$ does not imply that $\max_{\mu\in[1,\mu^M]} W^{IR}(\mu) \leq \max_{\mu\in[\mu_0,\mu^M]} W^{Y}(\mu)$ since the welfare-maximizing value of the mark-up under individual regulation may well be in the interval $]1, \mu_0[$. In other words, it is unclear *a priori* whether the break-even constraint which obliges the regulator to set a sufficiently high mark-up will or will not affect the dominance result we obtained in the previous sections. The following proposition gives an answer to this question.

Proposition 7 *Suppose that $a \leq \gamma c \leq 2c$. If under each regulation regime, the mark-up parameter is set by the regulator in a way that maximizes the social welfare, then relative cost-plus regulation leads to a higher social welfare than individual regulation, that is:*

$$\max_{\mu\in[1,\mu^M]} W^{IR}(\mu) \leq \max_{\mu\in[\mu_0,\mu^M]} W^{Y}(\mu)$$

Proof: See appendix.

The fact that the regulator has to grant a sufficiently high mark-up to make firms accept the regulatory contract under our modified Yardstick competition restricts her choice of the mark-up parameter under this regime. The previous proposition suggests that even with this restriction, the regulator can improve welfare with respect to optimal individual regulation.

5 Extensions under relative regulation

5.1 Spillovers

Assume that firms are regulated using our modified Yardstick competition. We suppose in this subsection that each firm can benefit from the cost-reducing investment of the other firms through spillovers. We capture these spillovers by a parameter $\beta \in [0,1]$, such that any cost reduction u by one of the firms allows all the other firms to reduce costlessly their marginal costs by βu. We assume hereafter that $a < \gamma c$ and that $\mu \in \left[\mu_0, \mu^M\right[$. We know from our previous analysis that in this case the regulatory pricing constraint is binding under yardstick competition (without spillovers). It is easy to see that this remains true, when we focus on sufficiently small spillovers. Thus, for sufficiently small values of β, the profit function of firm i is given by:

$$\pi_i(u_1, u_{2,...}, u_N) = \left(p_i - c + u_i + \beta \sum_{j \neq i} u_j\right)(a - p_i) - \frac{1}{2}\gamma u_i^2$$

where

$$\begin{aligned}
p_i &= \mu\left(c - \frac{1}{N-1}\sum_{j\neq i}\left(u_j + \beta\sum_{k\neq j} u_k\right)\right) \\
&= \mu\left(c - \bar{u}_i - \frac{\beta}{N-1}\sum_{j\neq i}\sum_{k\neq j} u_k\right) \\
&= \mu\left(c - \bar{u}_i - \frac{\beta}{N-1}\sum_{j\neq i}\left(\sum_{k\neq i} u_k + u_i - u_j\right)\right) \\
&= \mu\left(c - \bar{u}_i - \frac{\beta}{N-1}\left((N-1)\,u_i - (N-1)\,\bar{u}_i + (N-1)^2\,\bar{u}_i\right)\right) \\
&= \mu\left(c - (1 + \beta\,(N-2)\,\bar{u}_i - \beta u_i)\right)
\end{aligned}$$

The first-order condition $\frac{\partial \pi_i}{\partial u_i} = 0$ gives the reaction function of firm i :

$$u_i = \frac{1}{\gamma - 2\mu\beta + 2\mu^2\beta^2}\left[a - \mu c + \mu\beta\,(2\mu c - a - c) + \bar{u}_i\left[\mu + \beta\left(\mu\,(N-2) - 2\mu^2\right) + \beta^2\left(\mu\,(N-1) - 2\mu^2\,(N-2)\right)\right]\right],$$

which allows us to derive the unique Nash equilibrium:

$$u^*(\beta) = \frac{a - \mu c + \mu\beta\,(2\mu c - a - c)}{\gamma - \mu + \mu\beta(2-N) + \mu(2\mu-1)\beta^2(N-1)}.$$

What can we say about the effect of very small spillovers on the cost reduction effort $u^*(\beta)$ of each firm and on the overall cost reduction $(1 + (N-1)\beta)\, u^*(\beta)$ realized by each one of them? In other words, what are the signs of $\frac{du^*(\beta)}{d\beta}|_{\beta=0}$ and $\frac{d((1+(N-1)\beta)u^*(\beta))}{d\beta}|_{\beta=0}$?

It is straightforward to show that:

$$\frac{du^*(\beta)}{d\beta}|_{\beta=0} = \frac{N\mu\,(a-\mu c) + \mu(2\mu c - a - a\mu) + \gamma\left(2\mu^2 c - 2\mu c - \mu a + a\right)}{(\gamma-\mu)^2},$$

which can be written as:

$$\frac{du^*(\beta)}{d\beta}|_{\beta=0} = \frac{(N-2)\,\mu\,(a-\mu c) + (\mu-1)\left[\mu\,(2\gamma c - a) - a\gamma\right]}{(\gamma-\mu)^2}. \tag{9}$$

We assumed that $a < \gamma c$ and $\mu < \mu^M$. Under these assumptions, we know that $\mu^M < \frac{a}{c}$, which leads to $\mu < \frac{a}{c}$. Then, the first term of the numerator $(N-2)\,\mu\,(a-\mu c)$ is an increasing linear function of the number of regulated firms N (and is nonnegative for any $N \geq 2$). This ensures the existence of a threshold $\tilde{N}\,(a,c,\gamma,\mu) \geq 2$ such that for any $N \geq 2$:

$$\frac{du^*(\beta)}{d\beta}|_{\beta=0} \geq 0 \Longleftrightarrow N \geq \tilde{N}\,(a,c,\gamma,\mu)$$

In order to determine whether $\tilde{N}\,(a,c,\gamma,\mu) = 2$ or $\tilde{N}\,(a,c,\gamma,\mu) > 2$, we need to investigate the second term of the numerator of expression (9). Using the fact that $\mu > 1$, it is clear that this term is nonnegative, if and only if $\mu\,(2\gamma c - a) - a\gamma \geq 0$. Since $a < \gamma c < 2\gamma c$, the latter condition can be written as:

$$\mu \geq \frac{a\gamma}{2\gamma c - a}$$

Note that $\tilde{\mu} := \frac{a\gamma}{2\gamma c - a} < \mu^M$. This entails that the latter condition is fulfilled for any $\mu \in \left[\mu_0, \mu^M\right[$, if $\tilde{\mu} \leq \mu_0$, and only for $\mu \in \left[\tilde{\mu}, \mu^M\right[$ if $\tilde{\mu} > \mu_0$. Then, for any $\mu \in \left[\mu_0, \mu^M\right[$, we have:

$$\tilde{N}\,(a,c,\gamma,\mu) = 2 \Longleftrightarrow \mu \geq \max(\tilde{\mu}, \mu_0).$$

These results lead to the following proposition which gives some sufficient conditions under which small spillovers have a positive effect on the investment effort.

Proposition 8 *The marginal effect of small spillovers on the investment effort under relative cost-plus regulation is positive, if at least one of the following conditions holds:*

- *The mark-up parameter is sufficiently large*
- *The number of regulated firms is sufficiently large.*

DALEN[1998] finds that spillovers reduce firms' investment incentives, but since he examines the other extreme case, namely perfect spillovers, his result cannot be compared to ours.

Let us now turn to the marginal effect of spillovers on the overall cost reduction realized by each firm, that it is $(1+(N-1)\beta)\,u^*(\beta)$. It is straightforward to show that:

$$\begin{aligned} \frac{d\left((1+(N-1)\beta)\,u^*(\beta)\right)}{d\beta}|_{\beta=0} &= \frac{du^*(\beta)}{d\beta}|_{\beta=0} + (N-1)u^*(0) \qquad (10) \\ &= \frac{(N-2)\,\gamma\,(a-\mu c) + (2\gamma c - a + c)\,\mu^2 - \mu\,(a\gamma + 3\gamma c) + 2a\gamma}{(\gamma-\mu)^2}. \end{aligned}$$

Since the latter expression is a positively-sloped linear function of N, we can claim that there exists a threshold $\hat{N}\,(a,c,\gamma,\mu) \geq 2$ such that for any $N \geq 2$:

$$\frac{d\left((1+(N-1)\beta)\,u^*(\beta)\right)}{d\beta}|_{\beta=0} \geq 0 \Longleftrightarrow N \geq \hat{N}\,(a,c,\gamma,\mu)$$

It is obvious that $\tilde{N}\,(a,c,\gamma,\mu) \geq \hat{N}\,(a,c,\gamma,\mu)$: If spillovers have a positive effect on the cost reduction efforts, they have *a fortiori* a positive effect on the overall cost reduction of each firm.

Beside the fact that first term of the numerator of (10) is nonnegative (for reasons previously cited), note that the second term is always positive, if $c \geq 1$. Indeed, using the fact that $\mu^2 > \mu$ and $2\gamma c - a + c > 0$ we have:

$$(2\gamma c - a + c)\,\mu^2 - \mu\,(a\gamma + 3\gamma c) + 2a\gamma > \mu\,(-\gamma c - a\gamma - a + c) + 2a\gamma.$$

Using now the fact that $\mu < \frac{a}{c}$ and $-\gamma c - a\gamma - a + c < 0$, we get:

$$(2\gamma c - a + c)\,\mu^2 - \mu\,(a\gamma + 3\gamma c) + 2a\gamma > \frac{a}{c}\,(-\gamma c - a\gamma - a + c) + 2a\gamma = a\,(1+\gamma)\,(1-\frac{1}{c}),$$

which is the positive, if $c \geq 1$. Under this assumption, this entails $\hat{N}\,(a,c,\gamma,\mu) = 2$.

These findings can be summed up in the following way:

Proposition 9 *The marginal effect of small spillovers on the efficiency of the whole industry under relative cost-plus regulation is positive, if at least one the following conditions holds:*

- *The mark-up parameter is sufficiently large*
- *The number of regulated firms is sufficiently large*
- *The marginal production cost incurred by firms, if they do not invest, is sufficiently high.*

5.2 Quality investment

We assume in this subsection that the firms can undertake, simultaneously with their cost reduction, a quality-enhancing investment that shifts the demand upwards. More precisely, by incurring a cost $\frac{1}{2}\lambda\theta^2$, the regulated firms can increase the demand from $D(p) = a - p$ to $D(p,\theta) = a + \theta - p$. This means that from the firms' point of view, a quality-enhancing investment "enlarges" the market they operate in: The market size increases from a to $a+\theta$. This implies that, for a given $\theta \geq 0$, the analysis of the case without quality investments remains true, whenever we take into account the fact the new market size is $a + \theta$. It is then important to look at how the relevant thresholds are affected by the market size. For instance, it is easy to show that under the assumption $a < \gamma c$, the threshold $\mu^M(a) = \frac{(a+c)\gamma - a}{2\gamma c - a}$ is increasing in a. This implies that for any given $\theta \geq 0$, it holds that $\mu^M(a) \leq \mu^M(a+\theta)$.

The maximization program of firm i, given the cost reduction efforts of the other firms, is the following:

$$\max_{u_i \in [0,c], p_i \in [0,a], \theta_i \in [0,+\infty[} \pi_i(p_i, u_i, \theta_i) = (p_i - c + u_i)(a + \theta_i - p_i) - \frac{1}{2}\gamma u_i^2 - \frac{1}{2}\lambda\theta_i^2$$

under the regulatory constraint

$$p_i \leq \mu\left(c - \bar{u}_i\right).$$

We focus on the values of the mark-up parameter in the range $\left[\mu_0, \mu^M(a)\right[$ and we assume hereafter that $c > 1$ and $\lambda > 1$. The last assumption ensures that there exists a threshold $\bar{\theta}$ (depending only on a) such that the net profit $\pi_i(p_i, u_i, \theta_i)$ is smaller than $\pi_i(p_i, u_i, 0)$ for any $\theta_i \geq \bar{\theta}$, independent of the values of $\mu \in \left[\mu_0, \mu^M(a)\right[, p_i \in [c, a], u_i \in [0, c]$, $\bar{u}_i \in [0, c]$. This implies that maximizing the function $\theta_i \longrightarrow \pi_i(p_i, u_i, \theta_i)$ over the interval $\left[0, \bar{\theta}\right]$ is equivalent to maximizing it over the interval $[0, +\infty[$. Consequently, by assuming that $a + \bar{\theta} \leq \gamma c$, we are sure that we can conduct the same analysis as in the case without quality investment (under the condition $a \leq \gamma c$) without risking to discard a potential maximizing value of θ_i. Since $\mu^M(a) \leq \mu^M(a+\theta)$, the assumption $\mu \in \left[\mu_0, \mu^M(a)\right[$ entails that for any given $\theta \geq 0$, we have: $\mu \leq \mu^M(a+\theta)$. Then we derive from the analysis of the case without quality investment that the regulatory constraint is binding and that the solution of the program $\max_{u_i \in [0,c], p_i \in [0,a]} \pi_i(p_i, u_i, \theta_i)$ is given by:

$$u_i\left(\theta_i, \bar{u}_i\right) = \frac{1}{\gamma}(a + \theta_i - \mu c + \mu \bar{u}_i)$$

$$p_i\left(\bar{u}_i\right) = \mu\left(c - \bar{u}_i\right).$$

At this point we need to maximize $\pi_i(p_i(\theta_i, \bar{u}_i), u_i(\theta_i, \bar{u}_i), \theta_i)$ with respect to θ_i. Using the first-order condition, we get:

$$\theta_i(\bar{u}_i) = \frac{\gamma c(\mu - 1) + a - \mu c - (\gamma - 1)\mu \bar{u}_i}{\lambda\gamma - 1},$$

which leads to the investment reaction function of firm i :

$$u_i(\bar{u}_i) = u_i(\theta_i(\bar{u}_i), \bar{u}_i) = \frac{a\lambda - c - \mu c(\lambda - 1) + \mu(\lambda - 1)\bar{u}_i}{\lambda\gamma - 1}.$$

Solving these N equations, it is straightforward to show that the unique Nash equilibrium is symmetric and characterized by:

$$\hat{u}(\mu, \gamma, \lambda) = \frac{\lambda(a - \mu c) + c(\mu - 1)}{\lambda(\gamma - \mu) + \mu - 1}$$
$$\hat{\theta}(\mu, \gamma, \lambda) = \frac{(\mu - 1)(\gamma c - a)}{\lambda(\gamma - \mu) + \mu - 1}$$
$$\hat{p}(\mu, \gamma, \lambda) = \mu\frac{\lambda(\gamma c - a) + c - 1}{\lambda(\gamma - \mu) + \mu - 1}.$$

How are the equilibrium cost-reduction, quality and price affected by the mark-up parameter μ and by the cost parameters γ and λ? The answer is given by the following proposition, which is simply derived from the expressions of $\hat{u}(\mu, \gamma, \lambda)$, $\hat{\theta}(\mu, \gamma, \lambda)$ and $\hat{p}(\mu, \gamma, \lambda)$.

Proposition 10 *Consider firms which are regulated under a relative cost-plus regime. If these firms can invest in cost-reduction and quality, then under some general conditions (stated in the analysis above):*

1. *Firms' investment in cost reduction $\hat{u}(\mu, \gamma, \lambda)$ is decreasing in the mark-up parameter μ, and in the investment cost parameters γ and λ.*

2. *Firms' investment in quality $\hat{\theta}(\mu, \gamma, \lambda)$ is increasing in the mark-up parameter μ and the cost-reducing investment cost parameter γ and is decreasing in the quality investment cost parameter λ.*

3. *The equilibrium price $\hat{p}(\mu, \gamma, \lambda)$ is increasing in the the mark-up parameter μ and in the investment cost parameters γ and λ.*

Proposition 10 shows that the cost-reduction investment (respectively the price) is decreasing (respectively increasing) in the mark-up like in the case without quality investment. Hence the interpretation of the mark-up parameter as an inverse measure of the "artificial competition" resulting from relative regulation still holds. However, the mark-up value affects positively the investment in quality. This is partly due to the fact that the firms are regulated only with respect to their costs. Under this assumption, the marginal benefit of enlarging the demand through higher quality is clearly increasing in the mark-up.

The previous proposition also states that the more costly the investment in quality the lower the investment in cost reduction. The logic behind this result is as follows : a more costly investment in quality induces less quality enhancement which entails a lower market size, which implies a lower investment in cost reduction. Furthermore, we showed that the investment effort in quality is increasing in the cost of cost-reducing investments. This can be interpreted in the following way: when the investment in cost reduction becomes costlier, the regulated firms have an incentive to shift some of their investment from cost reduction to quality improvement. Finally, note that the previous analysis can be applied to any demand-enhancing investment and not only to quality investments.

6 Conclusion

This paper has investigated investment incentives and prices under individual and relative cost-plus regulation. The main difference with the existing literature is that the only regulatory instrument available to the regulator is the relative mark-up granted to firms. In particular, transfers are not possible. We show that this feature of our modified Yardstick competition has two important implications. First, relative cost-plus regulation is not implementable in some economic environments. In other words, even very high relative mark-ups may be insufficient to compensate firms for their production and investment costs when they are relatively regulated. This is for instance the case when markets are very large. Second, optimal relative cost-plus regulation leads to a second best outcome: there is always underinvestment in cost reduction relative to the socially optimal level. This contrasts with Schleifer's Yardstick competition (with transfers) where the regulator is able to implement the first best outcome.

One of the main messages of this paper is about the consequences of granting (too) high relative mark-ups to regulated firms. In the case of individual regulation, the conventional wisdom that a higher mark-up leads to higher investment is true and may serve as a justification for granting high mark-ups, particularly if the regulator is mainly concerned with encouraging cost reduction investments. This justification cannot be used when firms are regulated in a relative way since in this case, high mark-ups entail not only high prices but weak cost reductions as well. This is due to the fact that the fundamuntal tension between static and dynamic efficiency under individual cost-plus regulation disappears under its relative couterpart, making it always optimal to grant the smallest mark-up that allows firms to recoup their costs. It is also woth noting that in both regimes, granting high relative mark-ups may allow firms to behave like unregulated monopolists.

Comparing individual and relative regulation, we find that the latter dominates the former from a static and a dynamic point of view when the same relative mark-up is used. If the mark-ups are different, it is shown that the investment under relative regulation is still higher than the investment under individual regulation. Considering welfare, we compare the two regimes when the mark-up is set optimally by the regulator. It appears that even if the regulator has to grant a sufficiently high mark-up to make sure that firms do not leave the market under relative regulation, she can still reach higher welfare than under optimal individual regulation.

The fact that the equilibrium cost reduction under our modified Yardstick competition is decreasing in the mark-up parameter (whenever the latter regime is implementable) allows us to interpret this parameter as an inverse measure of the ”artificial competition” usually associated to relative regulation. Hence the regulator can control the degree of competition in the industry. through the relative mark-up she grants to firms, but not through the number of regulated firms. Indeed, we show that, at least from a theoretical perspective, the outcome of our modified Yardstick regulation does not depend on the number of firms.

Finally, our model is extended by allowing for technical spillovers and quality improvements. It is shown that small spillovers can have a positive effect on the cost reduction investment under some conditions. Considering quality investments, we provide some insights of the interplay between quality-enhancement and cost-reduction investments under relative regulation. We show for example that an increase of the mark-up provide firms with an incentive to shift some of their investment effort from cost-reduction to quality improvement.

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Appendix

Proof of proposition 1:

The function $\pi(\cdot, u)$ is a concave quadratic function. The point where it reaches its unconstrained maximum is given by the first-order condition:

$$\frac{\partial \pi}{\partial p} = 0 \Longrightarrow p = \frac{1}{2}(c - u + a)$$

Taking into account the regulatory constraint $p \leq \mu(c-u)$, the function $\pi(\cdot, u)$ reaches its maximum at:

$$p = \min\left(\mu(c-u), \frac{1}{2}(c-u+a)\right).$$

The comparison of $\mu(c-u)$ and $\frac{1}{2}(c-u+a)$ leads to the following result:

$$p = \begin{cases} \frac{1}{2}(c-u+a) & if \quad u \leq c - \frac{a}{2\mu-1} \\ \mu(c-u) & if \quad u > c - \frac{a}{2\mu-1} \end{cases}$$

which implies:

$$\pi(p(u), u) = \begin{cases} \frac{1}{4}(a-c+u)^2 - \frac{1}{2}\gamma u^2 & if \quad u \leq c - \frac{a}{2\mu-1} \\ (\mu-1)(c-u)(a-\mu c+\mu u) - \frac{1}{2}\gamma u^2 & if \quad u > c - \frac{a}{2\mu-1} \end{cases}. \tag{11}$$

Note that $c - \frac{a}{2\mu-1}$ may be negative and hence an irrelevant threshold. This is the case, whenever $\mu < \frac{a+c}{2c}$.

$$\frac{a+c}{2c} < \mu^M \Longleftrightarrow a < 2\gamma c. \tag{12}$$

Let us assume hereafter that $\gamma > \frac{1}{2}$ (we will get back to the case $\gamma \leq \frac{1}{2}$ at the end of the following analysis). Under this assumption the unconstrained maximum of $u \longrightarrow \frac{1}{2}(c-u+a)$ is reached at $u^M = \frac{a-c}{2\gamma-1} < c$ and the unconstrained maximum of the concave function $u \longrightarrow (\mu-1)(c-u)(a-\mu(c-u)) - \frac{1}{2}\gamma u^2$ is reached at:

$$\tilde{u}(\mu) = \frac{(2\mu c - a)(\mu - 1)}{2\mu(\mu-1)+\gamma} < c.$$

Note that:

$$\tilde{u}(\mu) \geq 0 \Longleftrightarrow \mu > \frac{a}{2c}.$$

Note also that if $a \leq 2c$, then the latter condition is fulfilled for any $\mu \geq 1$. This is why we distinguish the following two cases:

Case 1: $a \leq 2c$

a- Let us first examine the case $\mu \geq \frac{a+c}{2c}$. Under this condition, $c - \frac{a}{2\mu-1}$ is a relevant threshold in the sense that $c - \frac{a}{2\mu-1} \in [0, c]$. It follows that, in order to determine the maximum of $u \longrightarrow \pi(p(u), u)$ over the interval $[0, c]$, we need to compare the threshold $c - \frac{a}{2\mu-1}$ to both u^M and $\tilde{u}(\mu)$. We get the following results:

$$c - \frac{a}{2\mu - 1} < \tilde{u}(\mu) \Longleftrightarrow (2\gamma c - a)\,\mu < \gamma(a + c) - a \tag{13}$$

$$c - \frac{a}{2\mu - 1} < u^M \Longleftrightarrow (2\gamma c - a)\,\mu < \gamma(a + c) - a. \tag{14}$$

At this point, we need to distinguish two cases:

a.1. If $a < 2\gamma c$, then the (common) inequality on the left-hand side of (13) and (14) can be written as:

$$\mu < \frac{\gamma(a + c) - a}{2\gamma c - a} = \mu^M. \tag{15}$$

The comparison of $\frac{a+c}{2c}$ and μ^M leads to :

$$\frac{a + c}{2c} < \mu^M \Longleftrightarrow a < 2\gamma c.$$

Then, we need to distinguish two subcases:

- If $\frac{a+c}{2c} \leq \mu < \mu^M$, then the solution of $\max\limits_{u\in[0,c]} \pi(p(u), u)$ is $u = \tilde{u}(\mu)$. It follows that the solution of the maximization program (4) is given by:

$$u^{IR}(\mu) = \tilde{u}(\mu) \text{ and } p^{IR}(\mu) = \mu(c - \tilde{u}(\mu)) = \frac{a\mu(\mu - 1) + \mu\gamma c}{2\mu(\mu - 1) + \gamma}$$

- If $\mu > \mu^M$, then the solution of $\max\limits_{u\in[0,c]} \pi(p(u), u)$ is $u = u^M$. It follows that the solution of the maximization program (4) is given by:

$$u^{IR}(\mu) = u^M \text{ and } p^{IR}(\mu) = p^M$$

a.2. If $a \geq 2\gamma c$, then the inequality on the left hand side of (13) and (14) can be written as:

$$\mu > \frac{\gamma(a + c) - a}{2\gamma c - a}$$

It is easy to check that under condition $a > 2\gamma c$, we have $\frac{\gamma(a+c)-a}{2\gamma c-a} < 1$. This implies that the previous condition is always satisfied (since $\mu > 1$). Then the solution of $\max\limits_{u\in[0,c]} \pi(p(u), u)$ is

$u = \tilde{u}(\mu)$ and the solution of the maximization program (4) is:

$$u^{IR}(\mu) = \tilde{u}(\mu) \text{ and } p^{IR}(\mu) = \frac{a\mu(\mu-1) + \mu\gamma c}{2\mu(\mu-1)+\gamma}.$$

b- Let us now examine the case $\mu < \frac{a+c}{2c}$. Under this condition $c - \frac{a}{2\mu-1}$ is no more a relevant threshold in the definition of $\pi(p(u), u)$. In other words, the expression of $\pi(p(u), u)$ for any $u \in [0, c]$ is simply given by:

$$\pi(p(u), u) = (\mu-1)(c-u)(a-\mu c+\mu u)) - \frac{1}{2}\gamma u^2.$$

It follows that the solution of $\max_{u\in[0,c]} \pi(p(u), u)$ is $u = \tilde{u}(\mu)$ and hence the solution of the maximization program (4) is given by:

$$u^{IR}(\mu) = \tilde{u}(\mu) \text{ and } p^{IR}(\mu) = \frac{a\mu(\mu-1) + \mu\gamma c}{2\mu(\mu-1)+\gamma}.$$

Case 2: $a > 2c$

a- Let us first examine the case $\mu \leq \frac{a}{2c}$. This condition implies that $\mu < \frac{a+c}{2c}$, which entails that for any $u \in [0, c]$ $\pi(p(u), u)$ is given by:

$$\pi(p(u), u) = (\mu-1)(c-u)(a-\mu c+\mu u)) - \frac{1}{2}\gamma u^2 \tag{16}$$

The unconstrained maximum of this concave quadratic function is reached at $\tilde{u}(\mu)$, which is negative under the condition $\mu \leq \frac{a}{2c}$. It follows that its maximum over $[0, c]$ is reached at $u = 0$. Then the solution of the maximization program (4) is:

$$u^{IR}(\mu) = 0 \text{ and } p^{IR}(\mu) = \mu c.$$

b- Let us now turn to the case $\mu > \frac{a}{2c}$. This case has to be divided in two subcases:

b$_1$. $\frac{a}{2c} < \mu < \frac{a+c}{2c}$: Under this condition, the expression of $\pi(p(u), u)$ is given by (11) and $\tilde{u}(\mu) \in]0, c[$. As a consequence the solution of the maximization program (4) is:

$$u^{IR}(\mu) = \tilde{u}(\mu) \text{ and } p^{IR}(\mu) = \frac{a\mu(\mu-1) + \mu\gamma c}{2\mu(\mu-1)+\gamma}$$

b$_2$. $\mu \geq \frac{a+c}{2c}$: Under this condition the results are the same as in the case $a \leq 2c$.

We have analyzed all cases exhaustively under the assumption $\gamma > \frac{1}{2}$. How is it changed when $\gamma \leq \frac{1}{2}$? Under this assumption, the maximum over the interval $\left[0, c - \frac{a}{2\mu-1}\right]$ of the net profit when the regulatory constraint is not binding, namely $u \longrightarrow \frac{1}{4}(a-c+u)^2 - \frac{1}{2}\gamma u^2$, is reached at $u = c - \frac{a}{2\mu-1}$. Moreover the condition $a < 2\gamma c$ cannot hold, since $a > c$. Except for this case, the results of the previous analysis remain true. QED.

Proof of proposition 2:

Let us first maximize $\pi_i(p_i, u_i)$ with respect to u_i. The function $\pi_i(p_i, \cdot)$ is a concave, quadratic function. The point where it reaches its maximum over the interval $[0, c]$ can be simply derived from its unconstrained maximum given by the first order condition:

$$\frac{\partial \pi_i}{\partial u_i} = 0 \Longrightarrow u_i = \frac{1}{\gamma}(a - p_i).$$

Hence, we can state that for any $p_i \in [0, a]$, the function $\pi_i(p_i, \cdot)$ reaches its maximum over $[0, c]$ at:

$$u_i(p_i) = \min\left(c, \frac{1}{\gamma}(a - p_i)\right). \tag{17}$$

Given this result the maximization program of firm i reduces to:

$$\max_{p_i \in [0,a]} \pi_i(p_i, u_i(p_i)) \tag{18}$$

under the constraint:

$$p_i \leq \mu \bar{c}_i = \mu\left(c - \bar{u}_i\right).$$

We need to distinguish two cases:

Case 1: $a \leq c\gamma$

In this case, $\frac{1}{\gamma}(a - p_i) \leq c$ for any $p_i \in [0, a]$, then from (17) we get: $u_i(p_i) = \frac{1}{\gamma}(a - p_i)$, which entails that:

$$\pi_i(p_i, u_i(p_i)) = (a - p_i)\left[p_i\left(1 - \frac{1}{2\gamma}\right) + \frac{a}{2\gamma} - c\right].$$

The unconstrained maximum of this concave function of p_i is given by the first order condition $\frac{\partial \pi_i}{\partial p_i} = 0$ which can be written as:

$$p_i = \frac{(a+c)\gamma - a}{2\gamma - 1} = p^M.$$

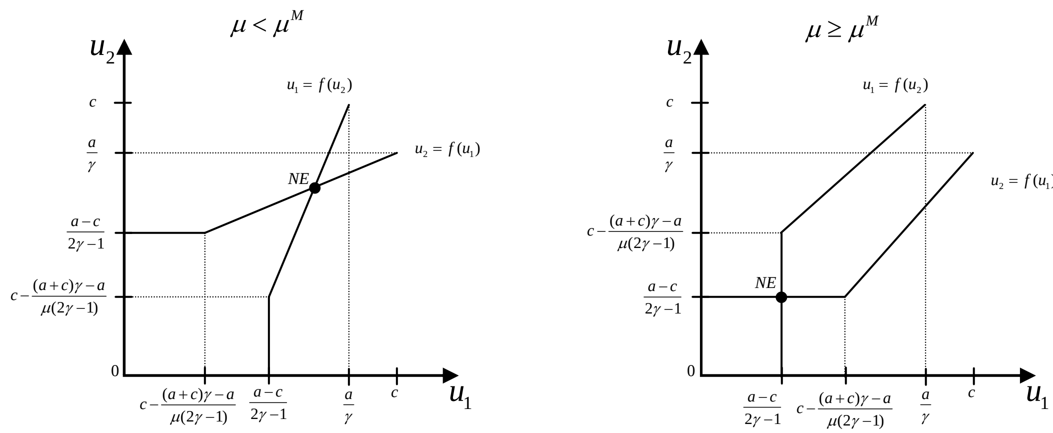

Figure 5: Reaction curves for $(N = 2)$ of case 1.

Taking into account the regulatory constraint, the solution of the maximization of our modified Yardstick program (7) is given by:

$$p_i = \begin{cases} p^M & if \quad \mu\left(c-\bar{u}_i\right) > \frac{(a+c)\gamma-a}{2\gamma-1} \\ \mu\left(c-\bar{u}_i\right) & if \quad \mu\left(c-\bar{u}_i\right) \leq \frac{(a+c)\gamma-a}{2\gamma-1}, \end{cases}$$

which can be rewritten as:

$$p_i = \begin{cases} p^M & if \quad \bar{u}_i < c - \frac{(a+c)\gamma-a}{\mu(2\gamma-1)} \\ \mu\left(c-\bar{u}_i\right) & if \quad \bar{u}_i \geq c - \frac{(a+c)\gamma-a}{\mu(2\gamma-1)}. \end{cases}$$

Since $u_i(p_i) = \frac{1}{\gamma}(a - p_i)$, the optimal cost reduction effort of firm i, given the efforts of the other firms, is as follows:

$$u_i = \begin{cases} u^M & if \quad \bar{u}_i < c - \frac{(a+c)\gamma-a}{\mu(2\gamma-1)} \\ \frac{1}{\gamma}\left(a-\mu c+\mu\bar{u}_i\right) & if \quad \bar{u}_i \geq c - \frac{(a+c)\gamma-a}{\mu(2\gamma-1)}. \end{cases}$$

Note that the efforts act as strategic complements. This is part of the dynamic artificial competition usually associated with Yardstick regulation. Given the reaction functions of the N firms, we can solve for the Nash equilibria of the investment and pricing subgame. The threshold

$$\mu^M = \frac{(a+c)\gamma - a}{2\gamma c - a},$$

turns to be important. More precisely, some tedious computations lead to the following results :

- If $\mu \leq \mu^M$, then the unique Nash equilibrium investment, which is unsurprisingly symmetric, is given by:

$$u_1^* = u_2^* = ... = u_N^* = u^*(\mu) = \frac{a - \mu c}{\gamma - \mu},$$
$$p_1^* = p_2^* = ... = p_N^* = p^*(\mu) = \mu \frac{c\gamma - a}{\gamma - \mu}.$$

- If $\mu > \mu^M$, then the unique Nash equilibrium is given by:

$$u_1^* = u_2^* = ... = u_N^* = u^M = \frac{a - c}{2\gamma - 1}$$

$$p_1^* = p_2^* = ... = p_N^* = p^M = \frac{(a + c)\gamma - a}{2\gamma - 1}.$$

An illustration of these results is given for $N = 2$ in figure (5). Note that in the first case, the regulatory pricing constraint is binding, while it is not in the second case. This implies that granting an excessive mark-up (i.e. $\mu > \mu^M$) to regulated firms has no regulatory effect.

Case 2: $a > c\gamma$

In this case, the optimal cost reduction effort of firm i for a given price p_i is such that:

$$u_i(p_i) = \begin{cases} c & if \quad p_i \leq a - c\gamma \\ \frac{1}{\gamma}(a - p_i) & if \quad p_i > a - c\gamma. \end{cases}$$

Firm i's maximized profit function with respect to investment for a given price p_i is then:

$$\pi_i(p_i, u_i(p_i)) = \begin{cases} \frac{1}{2\gamma}(a - c)^2 & if \quad p_i \leq a - c\gamma \\ (a - p_i)\left[p_i\left(1 - \frac{1}{2\gamma}\right) + \frac{a}{2\gamma} - c\right] & if \quad p_i > a - c\gamma. \end{cases}$$

In order to maximize this function with respect to p_i, we first have to compare $a - c\gamma$ to $p^M = \frac{(a+c)\gamma - a}{2\gamma - 1}$. This comparison leads to the definition of a new threshold for the market size parameter a. More precisely, we need to distinguish two subcases:

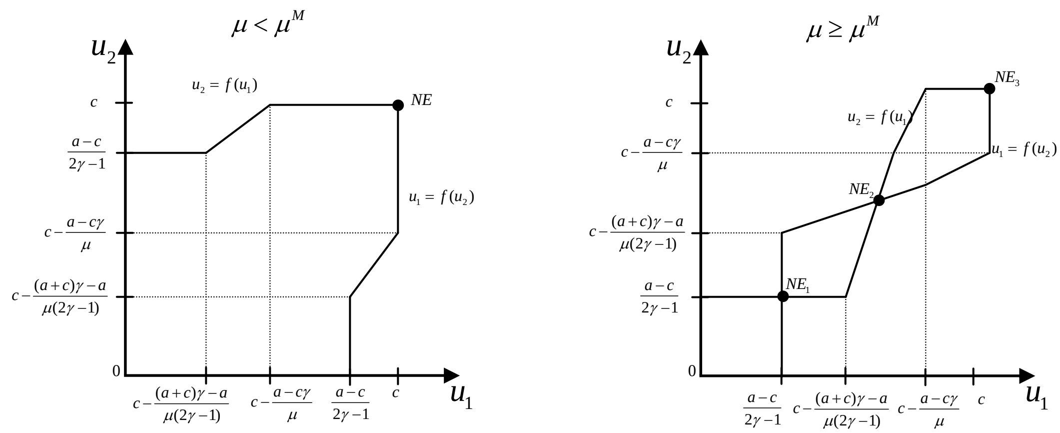

Figure 6: Reaction curves for $(N = 2)$ of subcase 2.1.

Subcase 2.1: $c\gamma < a \leq 2c\gamma$

Under this assumption, we find that $a - c\gamma < p^M$, which implies that the unconstrained maximum of the function $p_i \longrightarrow \pi_i(p_i, u_i(p_i))$ is reached at $p_i = p^M$. Since

$$a - c\gamma < \mu\left(c - \bar{u}_i\right) \Longleftrightarrow \bar{u}_i < c - \frac{a - c\gamma}{\mu}$$

and

$$p^M < \mu\left(c - \bar{u}_i\right) \Longleftrightarrow \bar{u}_i < c - \frac{(a + c)\,\gamma - a}{\mu\,(2\gamma - 1)},$$

the optimal effort of firm i, given the efforts of the other firms, is now:

$$u_i = \begin{cases} u^M & if & \bar{u}_i < c - \frac{(a+c)\gamma - a}{\mu(2\gamma-1)} \\ \frac{1}{\gamma}\left(a - \mu c + \mu \bar{u}_i\right) & if & c - \frac{(a+c)\gamma - a}{\mu(2\gamma-1)} \leq \bar{u}_i \leq c - \frac{a - c\gamma}{\mu} \\ c & if & \bar{u}_i > c - \frac{a - c\gamma}{\mu} \end{cases}$$

The Nash equilibria of the investment and pricing subgame in this subcase are as follows:

- If $\mu \leq \mu^M$, then the unique Nash equilibrium (see figure (6)) is:

$$u_1^* = u_2^* = ... = u_N^* = c$$

$$p_1^* = p_2^* = ... = p_N^* = 0$$

- If $\mu > \mu^M$, then there exist 3 Nash equilibria (see figure (6)) and all of them are symmetric. The set of these equilibria is

$$\left\{ \left(u^M, p^M\right); \left(\frac{a-\mu c}{\gamma-\mu}, \mu\frac{c\gamma-a}{\gamma-\mu}\right); (c,0) \right\}.$$

We use pareto-dominance as a criterion to select the Nash equilibrium that will emerge when there are multiple equilibria. It is clear that the first equilibrium pareto-dominates the two others.

Subcase 2.2: $a > 2c\gamma$

This subcase is simpler to analyze than the latter, because under this condition $p^M \leq a - c\gamma$, which implies that the function $p_i \longrightarrow \pi_i(p_i, u_i(p_i))$ is constant over $[0, a-c\gamma]$ and decreasing over $[a-c\gamma, a]$. Under these circumstances, the optimal effort of firm i is independent of the efforts of the other firms (it is a dominant strategy) and is given by $u_i = c$. Then there exists a unique Nash equilibrium given by:

$$u_1^* = u_2^* = ... = u_N^* = c$$

$$p_1^* = p_2^* = ... = p_N^* = 0.$$

QED.

Proof of proposition 4:

Some tedious computations lead to:

$$\frac{\partial W(\mu, u_1, u_2, ..., u_N)}{\partial u_i} = a + \left(\mu^2 - 2\mu\right)c - \left(\gamma + \frac{\mu^2}{N-1}\right)u_i + \left(2\mu - \mu^2.\frac{N-2}{N-1}\right)\frac{1}{N-1}\sum_{j\neq i} u_j.$$

So the first-order conditions:

$$\frac{\partial W(\mu, u_1, u_2, ..., u_N)}{\partial u_i} = 0 \text{ for all } i \in \{1, 2, ..., N\} \tag{19}$$

are a set of N linear equations in $(u_1, u_2, ..., u_N)$. We need to show that these linear equations are independent. Consider the symmetric square matrix $A_N = (\alpha_{i,j}(N))_{1\leq i,j \leq n}$ defined by

$$\alpha_{i,j}(N) = \begin{cases} -\left(\gamma + \frac{\mu^2}{N-1}\right) & if \quad i = j \\ \left(2\mu - \mu^2.\frac{N-2}{N-1}\right)\frac{1}{N-1} & if \quad i \neq j. \end{cases}$$

The linear equations (19) are independent if and only if the determinant of the matrix A_N is different from 0. We need the following lemma for the subsequent analysis:

Lemma: Let $k \in N^*$ and $a, b \in R$. Consider the square matrix $D_k = (d_{i,j})_{1 \le i,j \le k}$ defined by:

$$d_{i,j} = \begin{cases} a & if \quad i = j \\ b & if \quad i \neq j. \end{cases}$$

then

$$\det D_k = (a - b)^{k-1} (a + (k-1)\, b)$$

Proof: Available upon request (recursive proof).

Applying this lemma to A_N, it is straightforward to show that:

$$\det A_N = \left(-\gamma - \frac{2\mu}{N-1} - \frac{\mu^2}{(N-1)^2} \right)^{N-1} \left(-\gamma + 2\mu - \mu^2 \right).$$

Since $1 < \mu \le \mu^M < \frac{a}{c} \le \gamma$ then $-\gamma + 2\mu - \mu^2 < -\gamma + \mu < 0$. Moreover, the first term between brackets is different from 0, so that $\det A_N \neq 0$. We can then conclude that the first-order conditions (19) have a unique solution. It is easy to check that this solution is symmetric and given by:

$$\hat{u}_1(\mu) = \hat{u}_2(\mu) = ... = \hat{u}_N(\mu) = \hat{u}(\mu) = \frac{a + \mu c\, (\mu - 2)}{\gamma + \mu\, (\mu - 2)}$$

Note that $\hat{u}(\mu) \in [0, c]$ (this is simply derived from $a \le \gamma c$).

Furthermore, A_N is the Hessian matrix of the N-dimensional quadratic function g_μ:$(u_1, u_2, ..., u_N) \longrightarrow W(\mu, u_1, u_2, ..., u_N)$. Let us now show that A_N is negative definite which will allow us to state that function g_μ has a unique global maximum (reached at $(\hat{u}(\mu), \hat{u}(\mu), ..., \hat{u}(\mu))$) and has the shape of an N-dimensional elliptic paraboloid. We know that, in order to demonstrate that A_N is negative definite, it is sufficient to show that for all $l \in \{1, 2, ..., N\}$, the matrix $A_{N,l}$, defined as the matrix resulting from taking the first l rows and first l columns of A_N, satisfies the following condition:

$$(-1)^l \det A_{N,l} > 0.$$

Since the matrices $A_{N,l}$ are also of the kind of D_k, we can apply the previous lemma to compute their determinant. We get that:

$$\begin{aligned} (-1)^l \det A_{N,l} &= (-1)^l \left(-\gamma - \frac{2\mu}{N-1} - \frac{\mu^2}{(N-1)^2} \right)^{l-1} \left(-\gamma - \frac{\mu^2}{N-1} + (l-1) \left(2\mu - \mu^2 . \frac{N-2}{N-1} \right) \frac{1}{N-1} \right) \\ &= -\left(\gamma + \frac{2\mu}{N-1} + \frac{\mu^2}{(N-1)^2} \right)^{l-1} \left(-\gamma - \frac{\mu^2}{N-1} + (l-1) \left(2\mu - \mu^2 . \frac{N-2}{N-1} \right) \frac{1}{N-1} \right) \end{aligned}$$

At this point, we need to dsitinguish two cases. If $2\mu - \mu^2.\frac{N-2}{N-1} \leq 0$ then the second term between brackets is negative and consequently $(-1)^l \det A_{N,l} > 0$. If $2\mu - \mu^2.\frac{N-2}{N-1} > 0$ then it follows from $l < N$ that the second term between brackets satisfies:

$$-\gamma - \frac{\mu^2}{N-1} + (l-1)\left(2\mu - \mu^2.\frac{N-2}{N-1}\right)\frac{1}{N-1} < -\gamma - \frac{\mu^2}{N-1} + (N-1)\left(2\mu - \mu^2.\frac{N-2}{N-1}\right)\frac{1}{N-1} = -\gamma + 2\mu - \mu^2$$

Since we have already shown that $-\gamma + 2\mu - \mu^2 < 0$, we can state that in this case too, we have: $(-1)^l \det A_{N,l} > 0$. QED.

Since our aim is not to give the expression of the socially optimal investment effort but rather to compare it to the equilibrium effort under relative regulation, we will not determine under what circumstances the break-even constraints are binding (or not). Consider the domain

$$D = \left\{(u_1, u_2, ..., u_N) \in [0, c]^N \mid \pi(\mu, u_i, \bar{u}_i) > 0 \text{ for all } i \in \{1, ..., N\}\right\}$$

Given the shape of the function g_μ:$(u_1, u_2, ..., u_N) \longrightarrow W(\mu, u_1, u_2, ..., u_N)$, there are two possible cases:

Case 1: The unconstrained global maximum is reached within D, that is $\pi(\mu, \hat{u}(\mu), \hat{u}(\mu)) > 0$: in this case, the unique socially optimal investment effort is symmetric and given by $u^{**}(\mu) = \hat{u}(\mu)$ (the break-even constraints are not binding)

Case 2: The unconstrained global maximum is reached outside D,that is $\pi(\mu, \hat{u}(\mu), \hat{u}(\mu)) \leq 0$: in this case, **all** the break-even constraints are binding and the unique socially optimal investment effort is symmetric and given by the solution in u of $\pi(\mu, u, u) = 0$, which we denote by $\breve{u}(\mu) : u^{**}(\mu) = \breve{u}(\mu)$.

Let us show that in both cases, $u^*(\mu) \leq u^{**}(\mu)$. First, note that in case 1, $u^{**}(\mu)$ can be rewritten as $u^{**}(\mu) = \frac{a-\mu c+\mu c(\mu-1)}{\gamma-u+\mu(\mu-1)}$ which can be easily compared to $u^*(\mu) = \frac{a-\mu c}{\gamma-u}$. Indeed, it is straightforward to show that the function $h : X \longrightarrow \frac{a-\mu c+cX}{\gamma-u+X}$ is increasing in X, which makes $h(0) < h(\mu(\mu-1))$, that is $u^*(\mu) < u^{**}(\mu)$. Second, note that in case 2, the function $u \longrightarrow \pi(\mu, u, u)$ is a concave quadratic function such that $\pi(\mu, 0, 0) \geq 0$. Then it is easy to see graphically (or analytically) that $\pi(\mu, u, u) \geq 0 \iff u \leq \breve{u}(\mu)$. Since we are under the conditions $a \leq \gamma c$ and $\mu_0 \leq \mu \leq \mu^M$, we know from Proposition 3 that the firms will accept the relative regulation contract, which means that: $\pi(\mu, u^*(\mu), u^*(\mu)) \geq 0$. This implies that $u^*(\mu) \leq \breve{u}(\mu) = u^{**}(\mu)$ .Note that the inequality is strict unless $\mu = \mu_0$. QED.

Proof of proposition 6:

Some tedious computations lead to :

$$\frac{d^2W^Y(\mu)}{d\mu^2} = N \cdot \frac{-\gamma}{(\mu-\gamma)^4} R(\mu).$$

where

$$R(\mu) = 2(\gamma c - a)^2 \mu + 6\gamma ca - 2ac\gamma^2 - 3c^2\gamma^2 - 3a^2 + c^2\gamma^3 + \gamma a^2$$

Since $R(\mu)$ is increasing in μ, then for any $\mu \geq 1 : R(\mu) \geq R(1)$. It is easy to show that $R(1) = (\gamma - 1)(\gamma c - a)^2$ which is strictly positive since $\gamma > \frac{a}{c} > 1$. It follows that $R(\mu) > 0$ and consequently

$$\frac{d^2W^Y(\mu)}{d\mu^2} < 0 \text{ for any } \mu \geq 1.$$

It is also straightforward to show that:

$$\frac{dW^Y(\mu)}{d\mu}|_{\mu=1} = 0.$$

We can then conclude that $\frac{dW^Y(\mu)}{d\mu} < 0$ for all $\mu > 1$ which means that $W^Y(\mu)$ is decreasing in μ over $]1, +\infty[$. This implies that the maximization of $W^Y(\mu)$ subject to the constraint $\pi^*(\mu) \geq 0$ which is equivalent, as shown previously, to $\mu \geq \mu_0$ leads to the choice $\mu = \mu_0$. QED.

It remains to show that μ_0 is the investment-maximizing mark-up value under the constraint $\mu \geq \mu_0$. This is easily derived from the fact that the equilibrium investment effort $u^*(\mu)$ is decreasing in μ.

Proof of proposition 7:

First, it is easy to see that $W^Y(\mu_0) = N\left(\frac{2\gamma(a-c)}{2\gamma-1}\right)^2$. Since we need to consider explicitly the dependance of $W^{IR}(\mu)$ and $\left(\frac{2\gamma(a-c)}{2\gamma-1}\right)^2$ upon the variable a, we will denote $W^{IR}(\mu, a)$ instead of $W^{IR}(\mu)$ and $W_0^Y(a)$ instead of $W^Y(\mu_0)$.

We showed that under the condition $a \leq \gamma c$, we have $\mu^M \leq \frac{a}{c}$ and consequently $\mu^M \leq \gamma$. Instead of fixing $a \in]c, \gamma c]$ and making μ vary in $\left[1, \mu^M\right]$, let us fix μ in $[1, \gamma]$ and make a vary in the interval $[\mu c, \gamma c]$. We aim to show that for all $a \in [\mu c, \gamma c]$

$$W^{IR}(\mu, a) \leq W_0^Y(a).$$

Let us define sufficient conditions such that the previous inequality holds. Note first that, given the expression of $u^{IR}(\mu)$, the function $a \longrightarrow W^{IR}(\mu, a)$ is clearly quadratic. Note also that $a \rightarrow W_0^Y(a)$ is quadratic and increasing over $[\mu c, \gamma c]$. Then, for the previous inequality to be satisfied for any $a \in [\mu c, \gamma c]$, it is sufficient that the three following conditions hold:

1. $W^{IR}(\mu, \mu c) \leq W_0^Y(\mu c)$
2. $W^{IR}(\mu, \gamma c) \leq W_0^Y(\gamma c)$

3- $\frac{\partial W^{IR}(\mu,a)}{\partial a}|_{a=\mu c} \leq \frac{dW_0^Y(a)}{da}|_{a=\mu c}$

Some routine computations lead to the fact that $\frac{W^{IR}(\mu,\mu c)}{W_0^Y(\mu c)}, \frac{W^{IR}(\mu,\gamma c)}{W_0^Y(\gamma c)}, \frac{\frac{\partial W^{IR}(\mu,a)}{\partial a}|_{a=\mu c}}{\frac{dW_0^Y(a)}{da}|_{a=\mu c}}$ do not depend on the parameter c. They only depend on the variables $\mu \in [1, \gamma]$ and $\gamma \in \left[\frac{a}{c}, 2\right]$ (since we supposed that $a \leq \gamma c \leq 2c$). It is then sufficient to show that the three previous functions take values greater than 1 for any $(\mu, \gamma) \in [1, 2]^2$. Ploting these functions with Maple, we find that they all satisfy this condition. QED