Environmental pollution, the public sector and economic growth

Alfred Greiner*

Abstract

In this paper we present an endogenous growth model with productive public capital and environmental pollution. Emissions result from production and raise the stock of pollution that negatively affects utility of the household. The government levies an income tax and a tax on emissions and uses its revenues for public investment and for abatement of pollution. The paper studies the structure of the model assuming three different scenarios and analyzes how the latter affect the balanced growth rate of the economy. The first scenario posits that abatement is set such that the stock of pollution is constant, in the second scenario pollution declines over time and, in the third scenario, pollution grows at the same positive rate as the economic variables.

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*Department of Business Administration and Economics, Bielefeld University, P.O. Box 100131, 33501 Bielefeld, Germany.

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1 Introduction

One line of research in economics tries to understand the links between economic growth, on the one hand, and environmental pollution, on the other hand. Often, the environment is integrated into economic models by assuming that economic activities go along with emissions, implying that environmental pollution is considered as a negative externality. That line of research goes back to Forster (1973) and was extended by Gruver (1976) or Brock (1977), for example. While the early models incorporating environmental pollution are typically exogenous growth models, more recent approaches model the environment within an endogenous growth framework. Examples for this type of models are the papers by Bovenberg and Smulders (1995) or Gradus and Smulders (1993).[1]

When environmental pollution is modelled as an inevitable by-product of economic activity, such as production for example, the market fails and the government should intervene. The government can try to correct the market failure by giving incentives for private economic agents to invest in abatement that improves the environmental quality or it can undertake abatement by itself. In the economics literature, both approaches can be found. For example, Bovenberg and de Mooij (1997) or Bovenberg and Smulders (1995) assume that private firms engage in abatement whereas Lighthart and van der Ploeg (1994) or Nielson et al. (1995) posit that abatement spending is financed by the government.

As concerns the sources of ongoing growth in the endogenous growth literature, several approaches can be found (for a survey see e.g. Greiner et al., 2005). One line of research assumes that the government can invest in productive public capital which stimulates aggregate productivity. This approach goes back to Arrow and Kurz (1970) who presented models containing that assumption in their book. However, Arrow and Kurz did not present endogenous growth models but limited their analysis to models where growth is an exogenous variable. Futagami et al. (1993) were the first to demonstrate that a productive

[1] For a survey of this type of research see Brock and Taylor (2004).

public capital stock may lead to sustained per-capita growth. As concerns the question of whether public investment in a public capital stock can affect aggregate production possibilities at all, the empirical studies do not reach unambiguous results. Nevertheless, there is sufficient evidence that the government can exert a positive influence on aggregate productivity by investing in a public capital stock.[2]

In this paper, we present a decentralized model of endogenous growth with public capital and environmental pollution where production causes emissions which raise the stock of pollution. Pollution affects utility of the household but does not have immediate effects on production possibilities of the economy. A lot of the models dealing with environmental pollution assume that pollution influences production possibilities either through affecting the accumulation of human capital or by directly entering the production function (see e.g. Bovenberg and de Mooij, 1997, Bovenberg and Smulders, 1995, or Smulders and Gradus, 1996). We pursue a different approach because we want to focus on the role of preferences alone as concerns the dynamics of our model as well as concerns the effects of pollution on long-run growth.

In addition, we distinguish between three different scenarios concerning the evolution of the stock of pollution. In the first scenario, the stock of environmental pollution is constant while economic variables grow at a strictly positive rate in the long-run. The second scenario assumes that the stock of pollution declines, that is the quality of the environment improves, while economic variables grow at a constant rate. In the third scenario, finally, the stock of pollution grows at the same rate as the economic variables in the long-run. In our analysis we focus on the dynamics of our model depending on the scenario under consideration. In addition, we derive long-run growth effects of the different scenarios.

The rest of the paper is organized as follows. In section 2 we present the structure of our model and in section 3 we solve the model and study its dynamics. Section 4 analyzes

[2]For a survey of the literature dealing with that subject see e.g. Romp and de Haan (2005).

how the different scenarios affect economic growth and section 5, finally, concludes the paper.

2 The model economy

In this section we first describe our model economy. We assume a decentralized economy that is composed of three sectors: a household sector, a productive sector and the government. We start with the description of the behavior of the economic agents and with the description of pollution, resulting from production as a negative external effect.

2.1 The household sector

Our economy is represented by one household. The goal of this household is to maximize a discounted stream of utility arising from consumption $C(t)$ over an infinite time horizon subject to its budget constraint. As to the utility function we use the following function:[3]

$$\max_{C(t)} \int_0^\infty e^{-\rho t} U \, dt, \quad \text{where } U = \frac{C^{1-\sigma} X^{-\xi(1-\sigma)}}{1-\sigma}, \tag{1}$$

with $X(t)$ giving the stock of pollution and ρ is the rate of time preference. $1/\sigma > 0$ denotes the inter-temporal elasticity of substitution of consumption between two points in time and $\xi > 0$ gives the (dis)utility of additional pollution. For $\sigma = 1$ the utility function is logarithmic in consumption and pollution.[4]

The budget constraint is given by[5]

$$\dot{K} = (w + rK)(1 - \tau) - C, \tag{2}$$

with $0 < \tau < 1$ the income tax rate. The wage rate is denoted by w. The labor supply L is constant, supplied inelastically, and we normalize $L \equiv 1$. r gives the return to per-capita

[3]For a survey of how to incorporate pollution in the utility function see e.g. Smulders (1995).

[4]In this paper we limit our analysis to the case $\sigma \neq 1$.

[5]In the following we delete the time argument t when no ambiguity arises.

capital K. The budget constraint (2) states that the individual has to decide how much to consume and how much to save, thus increasing consumption possibilities in the future. The depreciation of physical capital is assumed to equal zero.

To derive necessary conditions we formulate the current-value Hamiltonian function as $\mathcal{H}(\cdot) = C^{1-\sigma}X^{-\xi(1-\sigma)}/(1-\sigma) + \lambda(-C + (w + rK)(1-\tau))$, with λ the costate variable. The necessary optimality conditions are given by

$$\lambda = C^{-\sigma}X^{-\xi(1-\sigma)}, \tag{3}$$

$$\dot{\lambda}/\lambda = \rho - r(1-\tau), \tag{4}$$

$$\dot{K} = -C + (w + rK)(1-\tau). \tag{5}$$

Since the Hamiltonian is concave in C and K jointly, the necessary conditions are also sufficient if in addition the transversality condition at infinity $\lim_{t\to\infty} e^{-\rho t}\lambda(t)K(t) \geq 0$ is fulfilled (cf. Seierstad and Sydsaeter, 1987).

2.2 The productive sector and the stock of pollution

The productive sector in our economy is represented by one firm which chooses inputs in order to maximize profits and which behaves competitively. As concerns emissions, $E(t)$, we suppose that it is a by-product of aggregate production, $Y(t)$. In particular, we set $E(t) = \varphi Y(t)$, with $\varphi = constant > 0$.

The stock of pollution X evolves over time according to the following differential equation (cf. Brock and Taylor, 2004),

$$\dot{X} = E - A - \delta_x X,\ 0 < \delta_x, \tag{6}$$

where A gives abatement that reduces pollution and δ_x gives the exponential rate at which the environment dissipates pollution. Further, we posit that $X \geq 0$ holds with $X = 0$ giving the pristine state of the environment.

Emissions are taxed at the rate $\tau_p > 0$ and the firm takes into account that one unit of output causes φ units of emissions for which it has to pay $\tau_p\varphi$ per unit of output. The per-capita production function is written as,

$$Y = K^{\alpha} H^{1-\alpha} L^{1-\alpha} \equiv K^{\alpha} H^{1-\alpha}, \tag{7}$$

with H denoting the stock of productive public capital and $\alpha \in (0,1)$ gives the per-capita capital share. Recall that L is normalized to one so that all economic variables are per-capita quantities.

Assuming competitive markets and taking public capital as given, the first-order conditions for a profit maximum are obtained as,

$$w = (1-\tau_p \varphi)(1-\alpha) K^{\alpha} H^{1-\alpha}, \tag{8}$$

$$r = (1-\tau_p \varphi) \alpha K^{\alpha-1} H^{1-\alpha}. \tag{9}$$

2.3 The government

The government in our economy uses resources for abatement activities, $A \geq 0$, which reduce the stock of pollution. Abatement activities are financed by the tax revenue coming from the tax on emissions, i.e. $A = \eta \tau_p E$, with $0 < \eta$. This means that η determines that part of the emission tax revenue which is used for abatement. Consequently, $1 - \eta$ gives that part of the revenue which is used for public investment in the public capital stock I_p, $I_p \geq 0$. For $\eta > 1$, a certain part of the tax revenue resulting from the taxation of income is used for abatement, in addition to the tax revenue that is gained by taxing emissions.

The government in our economy runs a balanced budget at any moment in time. Thus, the budget constraint of the government is written as,

$$I_p = \tau_p E(1-\eta) + \tau(w + rK). \tag{10}$$

The evolution of public capital is described by,

$$\dot{H} = I_p, \tag{11}$$

where for simplicity we again neglect depreciation.

2.4 Equilibrium conditions and the balanced growth path

Before we define a balanced growth path we define an equilibrium allocation.

Definition 1 *An equilibrium is a sequence of variables* $\{C(t), K(t), H(t), X(t)\}_{t=0}^{\infty}$ *and a sequence of prices* $\{w(t), r(t)\}_{t=0}^{\infty}$ *such that, given prices, the firm maximizes profits, the household solves (1) subject to (2) and the budget constraint of the government (10) is fulfilled.*

Thus, in equilibrium the dynamics of our model are completely described by the following differential equation system:

$$\frac{\dot{C}}{C} = -\frac{\rho}{\sigma} + \frac{\alpha(1-\tau)(1-\varphi\tau_p)}{\sigma}\left(\frac{H}{K}\right)^{1-\alpha} + \xi\left(\frac{1-\sigma}{\sigma}\right)\delta_x - \xi\left(\frac{1-\sigma}{\sigma}\right)\varphi\left(\frac{H}{K}\right)^{1-\alpha}\left(\frac{K}{X}\right)(1-\eta\tau_p), \quad (12)$$

$$\frac{\dot{K}}{K} = -\frac{C}{K} + \left(\frac{H}{K}\right)^{1-\alpha}(1-\tau)(1-\varphi\tau_p),\ K(0) > 0, \quad (13)$$

$$\frac{\dot{H}}{H} = \left(\frac{H}{K}\right)^{-\alpha}\left(\varphi\tau_p\,(1-\eta) + \tau(1-\varphi\tau_p)\right),\ H(0) > 0 \quad (14)$$

$$\frac{\dot{X}}{X} = \varphi\left(\frac{H}{K}\right)^{1-\alpha}\left(\frac{K}{X}\right)(1-\eta\tau_p) - \delta_x,\ X(0) > 0. \quad (15)$$

The initial conditions $K(0)$, $H(0)$ and $X(0)$ are given and fixed and $C(0)$ can be chosen by the economy. It should also be noted that we assume $X(0) > 0$, meaning that the environment initially is polluted.

In the following we will examine our model as to the existence and stability of a balanced growth path (BGP). To do so, we define a BGP as follows.

Definition 2 *A balanced growth path (BGP) is a path such that the economy is in equilibrium and such that consumption, private capital and public capital grow at the same strictly positive constant growth rate, i.e.* $\dot{C}/C = \dot{K}/K = \dot{H}/H = g$, $g > 0$, $g =$ *constant, and either*

(i) $\dot{X} = 0$ *or*

(ii) $\dot{X}/X = -\delta_x$ *or*
(iii) $\dot{X}/X = \dot{C}/C = \dot{K}/K = \dot{H}/H = g$.

This definition shows that on a BGP the growth rates of consumption, of private capital and of public capital are positive and constant over time. As concerns the state of the environment we consider three different scenarios. Scenario (i) describes a situation where the environmental quality is constant over time. Scenario (ii) describes an economy with an improving environment and scenario (iii), finally, gives an economy where the state of the environment deteriorates over time. Note that in scenarios (i) and (ii) the ratio X/K asymptotically converges to zero because pollution, X, is constant or declines in these scenarios, respectively, while K rises. In scenario (iii), X/K is constant because X grows at the same rate as K in that scenario.

It should be pointed out that scenario (i) and scenario (ii) can certainly be considered as reflecting a sustainable BGP because the environment is constant or becomes better in theses scenarios. This does not necessarily hold for scenario (iii) where the environment deteriorates at the same rate as economic variables grow. Nevertheless, if one adopted the definition of sustainability as given by Byrne (1997), scenario (iii) could also be sustainable. This holds because according to that definition, a balanced growth path is said to be sustainable if the growth rate of instantaneous utility is positive, i.e. if $\dot{U}/U > 0$ holds. Depending on the parameters in our model this may hold for scenario (iii) in our economy, too. However, most contributions in the literature define a sustainable BGP as a path on which the quality of the environment is constant. In any case, we also consider scenario (iii) because it may serve as an approximation to real world economies that continuously grow over time and that are characterized by a deterioration of the environmental quality.

To analyze our model further, we first perform a change of variables. Defining $c = C/K$, $h = H/K$ and $x = X/K$, and differentiating these variables with respect to time gives $\dot{c}/c = \dot{C}/C - \dot{K}/K$, $\dot{h}/h = \dot{H}/H - \dot{K}/K$ and $\dot{x}/x = \dot{X}/X - \dot{K}/K$. A rest point of this new system then corresponds to a BGP of our original model economy. The system describing the dynamics around a BGP is given by,

$$
\begin{aligned}
\dot{c} &= c\left(c - \frac{\rho}{\sigma} + \frac{\alpha(1-\varphi\tau_p)(1-\tau)}{\sigma}h^{1-\alpha} - (1-\tau)(1-\varphi\tau_p)h^{1-\alpha}\right) + \\
& \quad c\left(\xi\left(\frac{1-\sigma}{\sigma}\right)\delta_x - \xi\left(\frac{1-\sigma}{\sigma}\right)\varphi\left(\frac{H}{K}\right)^{1-\alpha}(1-\eta\tau_p)x^{-1}\right), \qquad (16) \\
\dot{h} &= h\left(c + h^{-\alpha}\left(\varphi\tau_p\,(1-\eta) + \tau(1-\varphi\tau_p)\right) - (1-\tau)(1-\varphi\tau_p)h^{1-\alpha}\right), \qquad (17) \\
\dot{x} &= x\left(c + \varphi h^{1-\alpha}x^{-1}(1-\eta\tau_p) - \delta_x - (1-\tau)(1-\varphi\tau_p)h^{1-\alpha}\right). \qquad (18)
\end{aligned}
$$

Concerning a rest point of system (16)-(18), it should be noted that we only consider interior solutions with respect to c and h, that is we exclude the economically meaningless stationary point $c^\star = h^\star = 0$.[6]

Before we continue and present results as concerns the dynamics of our economy, we note that scenario (i) is modelled by setting $\eta = (\varphi - \delta_x x h^{\alpha-1})/(\tau_p\varphi)$ which gives $\dot{X} = 0$ for all $t \in [0, \infty)$ making η an endogenous variable. Scenario (ii) is obtained by setting $\eta = 1/\tau_p$ which yields $\dot{X}/X = -\delta_x$ for all $t \in [0, \infty)$.

3 The dynamics of the model

In this section we analyze the dynamics of our model economy. Thus, we study the question of whether a BGP exists, whether it is unique and whether it is stable. As to the uniqueness and stability of a BGP for scenario (i) and for scenario (ii) we can derive the following proposition.

Proposition 1 *Assume that $(\varphi - \tau)/\varphi(1-\tau) < \tau_p < 1/\varphi$ holds. Then, there exists a unique saddle point stable balanced growth path for scenario (i) and for scenario (ii).*

Proof: To prove this proposition we solve $\dot{c}/c = 0$ with respect to c leading to,

$$c \;=\; \rho/\sigma - h^{1-\alpha}\alpha(1-\tau)(1-\tau_p\varphi)/\sigma + (1-\tau)(1-\tau_p\varphi)h^{1-\alpha} +$$

[6] The $\star$ denotes values of the variables on the BGP.

$$\xi(1-\sigma)\varphi h^{1-\alpha}(1-\eta\tau_p)x^{-1}/\sigma - \xi\delta_x(1-\sigma)/\sigma.$$

Inserting this expression in $\dot{h}$ gives for scenario (i),

$$\dot{h} = h(\rho/\sigma + h^{-\alpha}\left(\varphi(\tau_p - 1) + \tau(1-\varphi\tau_p)\right) - (1-\tau)(1-\varphi\tau_p)\alpha h^{1-\alpha}/\sigma),$$

where we used $\eta = (1/\tau_p) - \delta_x x h^{\alpha-1}/(\tau_p\varphi)$ and $x = 0^\star$ because on the BGP K grows faster than X in scenarios (i) and (ii). A $h^\star$ such that $\dot{h} = 0$ gives a BGP for our model. For scenario (ii) we get,

$$\dot{h} = h(\rho/\sigma + h^{-\alpha}\left(\varphi(\tau_p - 1) + \tau(1-\varphi\tau_p)\right) - (1-\tau)(1-\varphi\tau_p)\alpha h^{1-\alpha}/\sigma - \xi\left(1-\sigma\right)\delta_x/\sigma),$$

where we used $\eta = 1/\tau_p$ and $x^\star = 0$. It is easily seen that

$$\lim_{h\to 0}(\dot{h}/h) = +\infty, \text{ for } \tau_p > \frac{\varphi - \tau}{\varphi(1-\tau)}, \text{ and } \lim_{h\to\infty}(\dot{h}/h) = -\infty, \text{ for } \tau_p < 1/\varphi.$$

Further, given the assumptions stated in the proposition, $\partial(\dot{h}/h)/\partial h < 0$ holds so that the existence of a unique BGP is shown.

To show saddle point stability, we compute the Jacobian matrix for scenario (i) and (ii) which is given by,

$$J = \begin{bmatrix} c^\star & \partial\dot{c}/\partial h & 0 \\ h^\star & \partial\dot{h}/\partial h & 0 \\ 0 & 0 & \dot{X}/X - \dot{K}/K \end{bmatrix}$$

where we used $x^\star = 0$. Since X is constant in scenario (i) and declines in scenario (ii), while $\dot{K}/K$ is strictly positive on the BGP, we get the first eigenvalue, μ_1, of this matrix as $\mu_1 = \dot{X}/X - \dot{K}/K < 0$. Further, it is easily seen that, for scenarios (i) and (ii), $c^\star(\partial\dot{h}/\partial h) - h^\star(\partial\dot{c}/\partial h) < 0$ holds so that the second eigenvalue is negative, too, and the third eigenvalue is positive. □

The assumption $(\varphi - \tau)/\varphi(1-\tau) < \tau_p$ is equivalent to $\varphi\tau_p\left(1-\eta\right) + \tau(1-\varphi\tau_p) > 0$ in scenarios (i) and (ii) and is needed for $\dot{H}/H > 0$. It simply states that public investment must be positive for sustained growth which is obvious since productive public spending is the source of ongoing growth in our model. It should be noted that $(\varphi-\tau)/\varphi(1-\tau) < 1/\varphi$ implies $\varphi < 1$. This means that scenarios (i) and (ii) are only feasible if an additional unit of output leads to less than one additional unit of pollution. Loosely speaking, this states that the technology in use must not be too polluting. This is intuitively plausible because it is difficult to have a constant or even improving quality of the environment with ongoing growth if production leads to a strong degradation of the environment. If production is very polluting, i.e. $\varphi \geq 1$, a constant quality of the environment can be achieved only without sustained growth.

Given the assumptions in proposition 1, that proposition demonstrates that there exists a unique saddle point stable BGP for our model economy for scenarios (i) and (ii). It should be mentioned that saddle point stability means that two eigenvalues of the Jacobian matrix are negative and one is positive. This implies that there exists a unique value for initial consumption $C(0)$ such that $c(0) = C(0)/K(0)$ lies on the stable manifold of the rest point of (16)-(18).

From an economic point of view this result tells us that both global and local indeterminacy cannot arise in this economy with a constant or improving environment, i.e. for scenarios (i) and (ii). We here follow Benhabib and Perli (1994) as concerns the definition of local and global determinacy. According to that definition, local determinacy is given if there exist unique values for the variables that are not predetermined but can be chosen at time $t = 0$, such that the economy converges to the BGP in the long-run. This is the case for scenarios (i) and (ii) in our model because there exists a unique value for $c(0)$ such that the economy converges to the BGP.

Global indeterminacy arises if there exists more than one BGP and if the variables that are not predetermined at time $t = 0$ may be chosen to place the economy on the attracting set of either of the BGPs. In this case, the initial choice of the variables does not only affect the transitional growth rates but also the long-run growth rate. If the long-run BGP is unique the economy is said to be globally determinate.

For scenario (iii) the situation is more complex. Now, there may be more than one BGP. Proposition 2 gives the exact outcome.

Proposition 2 *Assume that* $\varphi\tau_p < 1$ *and* $\varphi\tau_p\,(1-\eta)+\tau(1-\varphi\tau_p) > 0$ *hold in scenario (iii). Then, there exists a unique balanced growth path for* $1/\sigma > (\xi-1)/\xi$. *For* $1/\sigma < (\xi-1)/\xi$ *there exist either two BGPs or no BGP.*

Proof: To prove this proposition we set $\dot{h}/h = 0$ giving $c-(1-\tau)(1-\tau_p\varphi) = -h^{-\alpha}(\varphi\tau_p(1-\eta)+\tau(1-\varphi\tau_p))$. Inserting this in $\dot{x}/x = 0$ leads to $\varphi h^{1-\alpha}(1-\eta\tau_p)x^{-1} = \delta_x + h^{-\alpha}(\varphi\tau_p(1-\eta)+\tau(1-\varphi\tau_p))$. Using these relationships we get for $\dot{c}/c$,

$$\dot{c}/c = -\rho/\sigma + (1-\tau)(1-\tau_p\varphi)\alpha h^{1-\alpha}/\sigma - h^{-\alpha}(\varphi\tau_p(1-\eta)+\tau(1-\varphi\tau_p))(1+\xi(1-\sigma)/\sigma).$$

For $1+\xi(1-\sigma)/\sigma > 0 \leftrightarrow 1/\sigma > (\xi-1)/\xi$ we get,

$$\lim_{h\to 0}(\dot{c}/c) = -\infty, \quad \lim_{h\to\infty}(\dot{c}/c) = +\infty \text{ and } \partial(\dot{c}/c)/\partial h > 0,$$

where $\varphi\tau_p < 1$ and $\varphi\tau_p\,(1-\eta)+\tau(1-\varphi\tau_p) > 0$ must hold. This shows that there exists a unique BGP for $1/\sigma > (\xi-1)/\xi$.

If $1+\xi(1-\sigma)/\sigma < 0 \leftrightarrow 1/\sigma < (\xi-1)/\xi$ holds we get,

$$\lim_{h\to 0}(\dot{c}/c) = +\infty \text{ and } \lim_{h\to\infty}(\dot{c}/c) = +\infty \text{ and}$$

$$\frac{\partial(\dot{c}/c)}{\partial h} = \alpha(1-\alpha)h^{-\alpha}(1-\tau)(1-\tau_p\varphi)/\sigma + \alpha h^{-\alpha-1}(\varphi\tau_p(1-\eta)+\tau(1-\varphi\tau_p))(1+\xi\left(\frac{1-\sigma}{\sigma}\right),$$

with $\partial(\dot{c}/c)/\partial h \to -\infty$ (0), for $h \to 0$ (∞). For $h < (>)\, h_1$ the derivative is negative (positive), with $h_1 = -(\varphi\tau_p(1-\eta)+\tau(1-\varphi\tau_p))(1+\xi(1-\sigma)/\sigma)/((1-\alpha)(1-\tau)(1-\tau_p\varphi)/\sigma) > 0$. Further, there exists a unique inflection point of the function $\dot{c}/c$, h_2, given by $h_2 = (1+\alpha)h_1/\alpha > h_1$. This shows that, for $1/\sigma < (\xi-1)/\xi$, there are either two points of intersection of $\dot{c}/c$ with the horizontal axis, and thus two BGPs, or none.[7] □

Proposition 2 shows that the BGP is unique if the inter-temporal elasticity of substitution is larger than one minus the inverse of the parameter determining the (dis)utility

[7] We neglect the case where $\dot{c}/c$ is tangent to the horizontal axis which has Lebesgue measure zero.

of additional pollution, i.e. for $1/\sigma > (\xi - 1)/\xi$. It should also be noted that for a small effect of additional pollution on utility, i.e. for $\xi \leq 1$, uniqueness of the BGP is always given, independent of the inter-temporal elasticity of substitution. Thus, we can state that global indeterminacy can only arise when the effect of pollution on utility is strong. Intuitively, this is plausible because this condition states that the negative external effect of production must be strong so that multiple BGPs can arise.

Besides the parameter giving the (dis)utility resulting from pollution, ξ, the inter-temporal elasticity of substitution of consumption, $1/\sigma$, plays an important role as concerns the question of whether the model is globally indeterminate, which is often the case in endogenous growth models. For our model, we see that a small inter-temporal elasticity of substitution is necessary for multiple BGPs to be feasible. In other papers, a high inter-temporal elasticity of substitution is a necessary condition for multiple BGPs (see for example Benhabib and Perli, 1994). The different outcome in our model, compared to other contributions in the economics literature, is due to the fact that in our model utility does not only depend on consumption but also on the stock of pollution which is a by-product of aggregate production.

Therefore, the outcome stated in propositions 2 makes sense from an economic point of view: Global indeterminacy means that the economy may either converge to the BGP with the high balanced growth rate or to the BGP with the low balanced growth rate in the long-run. Thus, it may either choose a path with higher initial consumption (and lower initial investment) or a path with lower initial consumption (and higher initial investment). In the latter case, the household must be willing to forgo current consumption and shift it into the future. If production does not have negative effects in form of pollution, the household will do that only if it has a high inter-temporal elasticity of substitution of consumption. However, if production does have negative repercussions because it leads to a rise in the stock of pollution, the household is willing to forgo current consumption even with a low inter-temporal elasticity of substitution. In this case, renouncing to consumption today and shifting it into the future yields a higher future marginal utility in scenario (iii) where the environmental quality declines. This holds because marginal (dis)utility of pollution declines with a higher level of consumption for $1/\sigma < 1$ (see also the discussion below following equation (19)). Therefore, the household can reduce (dis)utility of pollution in the future by renouncing to consumption today and shifting it into the future.

As concerns the dynamics around the BGP, this question is more difficult to answer. Proposition 3 gives insight into the local dynamics around a BGP for scenario (iii).

Proposition 3 *Assume that there exists a unique BGP for scenario (iii). Then, the BGP is either saddle point stable or unstable. Assume that there exist two BGPs. Then, the BGP yielding the lower growth rate is either saddle point stable or unstable, while the BGP yielding the higher growth rate is asymptotically stable or unstable.*

Proof: To prove this theorem we compute the Jacobian matrix for the case of scenario (iii) which is obtained as,

$$J = \begin{bmatrix} c^\star & \partial \dot{c}/\partial h & c^\star (x^\star)^{-2} (h^\star)^{1-\alpha} \varphi \xi (1-\sigma)(1-\eta\tau_p)/\sigma \\ h^\star & \partial \dot{h}/\partial h & 0 \\ x^\star & \partial \dot{x}/\partial h & -\varphi (h^\star)^{1-\alpha}(1-\eta\tau_p) \end{bmatrix}$$

The sign of the determinant of the Jacobian is equivalent to the sign of

$$\alpha(1-\alpha)h^{-\alpha}(1-\tau)(1-\tau_p\varphi)/\sigma + \alpha h^{-\alpha-1}(\varphi\tau_p(1-\eta)+\tau(1-\varphi\tau_p))(1+\xi(1-\sigma)/\sigma).$$

If the BGP is unique we have $(1+\xi(1-\sigma)/\sigma) > 0$ implying that $\det J > 0$ holds.

Denoting by μ_j, $j = 1, 2, 3$, the j-th eigenvalue of the Jacobian, we know that $\mu_1 \cdot \mu_2 \cdot \mu_3 = \det J$. Therefore, a positive determinant implies that there are either two negative eigenvalues (or two complex conjugate eigenvalues with negative real parts) and one positive or three positive eigenvalues (or one positive eigenvalue and two complex conjugate eigenvalues with positive real parts).

If there are two BGPs, the BGP associated with the smaller $h^\star$ yields the higher growth rate and vice versa. From the proof of proposition 2 we know that $\dot{c}/c$ first intersects the horizontal axis from above and, then, from below. So, at the first intersection point we have $\partial(\dot{c}/c)/\partial h < 0$ and at the second we have $\partial(\dot{c}/c)/\partial h > 0$. Since

$$\frac{\partial(\dot{c}/c)}{\partial h} = \alpha(1-\alpha)h^{-\alpha}(1-\tau)(1-\tau_p\varphi)/\sigma + \alpha h^{-\alpha-1}(\varphi\tau_p(1-\eta)+\tau(1-\varphi\tau_p))(1+\xi(1-\sigma/\sigma)$$

we know that $\det J < 0$ at the smaller $h^\star$ and $\det J > 0$ at the higher $h^\star$. Thus, the BGP yielding the higher growth rate has either three negative eigenvalues (or one negative eigenvalue and two complex conjugate eigenvalues with negative real parts) or two positive eigenvalues (or two complex conjugate eigenvalues with positive real parts) and one negative. If the determinant is positiv, $\det J > 0$, we have the same situation as for the case of a unique BGP. □

Proposition 2 and proposition 3 demonstrate that for scenario (iii) the dynamics may be more complex, meaning that this scenario may give rise to both global and local indeterminacy. In the next section, we compare the different scenarios as concerns long-run growth.

4 Growth effects of the different scenarios

In this section we first study the question of which scenario generates a higher balanced growth rate. To do so we take scenario (i), where environmental quality is constant over time, as a benchmark. Proposition 4 compares scenario (i) with scenario (ii), that is characterized by an improving environmental quality.

Proposition 4 *For $1/\sigma > (<)\, 1$ scenario (i) is characterized by a lower (higher) balanced growth rate than scenario (ii).*

Proof: To prove this proposition we note from the proof of proposition 1 that a BGP in scenario (i) is given for a $h^\star$ such that

$$\dot{h} = h(\rho/\sigma + h^{-\alpha}\left(\varphi(\tau_p - 1) + \tau(1 - \varphi\tau_p)\right) - (1-\tau)(1-\varphi\tau_p)\alpha h^{1-\alpha}/\sigma) = 0$$

holds.

A BGP in scenario (ii) is given for a $h^\star$ such that

$$\dot{h} = h(\rho/\sigma + h^{-\alpha}\left(\varphi(\tau_p - 1) + \tau(1 - \varphi\tau_p)\right) - (1-\tau)(1-\varphi\tau_p)\alpha h^{1-\alpha}/\sigma) - \xi\delta_x(1-\sigma)/\sigma = 0$$

holds.

This shows that the graph of $\dot{h}$ in scenario (i) is above (below) the graph of $\dot{h}$ in scenario (ii) for $1/\sigma > (<)\, 1$, implying that scenario (i) gives a higher (lower) value $h^\star$. The balanced growth rate is given by (14). Because of $x^\star = 0$, $\eta = 1/\tau_p$ holds for both scenarios (i) and (ii) on the BGP, leading to the result in proposition 4. □

Proposition 4 states that for a relatively high inter-temporal elasticity of substitution, i.e. for $1/\sigma > 1$, the scenario in which environmental quality improves, scenario (ii), yields a higher balanced growth rate than the scenario where the environmental quality is constant, scenario (i). This holds because for an inter-temporal elasticity of substitution larger one, the marginal utility of consumption is the higher the smaller the stock of pollution is, i.e. the better the environmental quality, and vice versa. This is seen by computing the cross derivative of the utility function which is given by,

$$\frac{\partial^2 U}{\partial C \partial X} = -\xi(1-\sigma)C^{-\sigma}X^{-\xi(1-\sigma)-1} > (<)\, 0 \leftrightarrow 1/\sigma < (>)\, 1\,. \qquad (19)$$

Since in scenario (ii) the environmental quality in the future is better than in scenario (i), because the level of X declines in scenario (ii) while it is constant in scenario (i), the household forgoes more consumption today and shifts it into the future in scenario (ii) compared to scenario (i). Therefore, scenario (ii) implies a higher balanced growth rate for $1/\sigma > 1$.

Equation (19) suggests that consumption and a clean environment, i.e. a small value of pollution, are complementary for $1/\sigma > 1$ because, in this case, marginal utility of consumption rises with a decline in the level of pollution. This means that marginal utility of consumption is the higher, the cleaner the environment is. For $1/\sigma < 1$, consumption and pollution can be considered as substitutes because the marginal (dis)utility of additional pollution declines with a rising level of consumption. Thus, one could interpret proposition 4 such that scenario (i) leads to lower growth than scenario (ii) if consumption is complementary to a clean environment. If the household considers consumption as a substitute for the negative effect of pollution, scenario (i) generates a higher balanced growth rate than scenario (ii).

Comparing scenario (i) with scenario (iii) the analysis is more complex. Proposition 5 gives results as concerns the ratio of public capital to private capital on the BGP in these two scenarios as well as concerns the long-run growth rate.

Proposition 5 *A necessary condition for scenario (i) to be associated with a higher $h^\star$ than scenario (iii) is $1/\sigma < 1$. A sufficient condition for scenario (i) to be associated with a lower $h^\star$ than scenario (iii) is $1/\sigma > 1$. Further, $1/\sigma > 1$ is a necessary condition for scenario (i) to generate a higher balanced growth rate than scenario (iii).*

Proof: To prove this proposition we note from the proof of proposition 1 that a $h^\star$ such that

$$\dot{h}/h = \rho/\sigma - (1-\tau)(1-\varphi\tau_p)\alpha h^{1-\alpha}/\sigma + h^{-\alpha}\left(\varphi\tau_p(1-\tau_p^{-1}) + \tau(1-\varphi\tau_p)\right) = 0$$

gives a BGP for scenario (i).

From the proof of proposition 2 we know that a $h^\star$ such that

$$-\dot{c}/c = \rho/\sigma - (1-\tau)(1-\tau_p\varphi)\alpha h^{1-\alpha}/\sigma + h^{-\alpha}\left(\varphi\tau_p(1-\eta) + \tau(1-\varphi\tau_p)\right)(1+\xi(1-\sigma)/\sigma) = 0$$

gives a BGP for scenario (iii).

Further, we know that in scenario (iii), $\eta < 1/\tau_p$ holds on the BGP. Setting $\eta = (1/\tau_p) - \epsilon$, with $\epsilon > 0$, we can write $-\dot{c}/c$ as,

$$
\begin{aligned}
-\dot{c}/c &= \rho/\sigma + h^{-\alpha}\left(\varphi\tau_p(1-\tau_p^{-1}) + \tau(1-\varphi\tau_p)\right)(1+\xi(1-\sigma)/\sigma) + \\
&\quad h^{-\alpha}\varphi\,\tau_p\,\epsilon\,(1+\xi(1-\sigma)/\sigma) - (1-\tau)(1-\tau_p\varphi)\alpha h^{1-\alpha}/\sigma\,.
\end{aligned}
$$

This shows that $1/\sigma > (<)\, 1$ is a sufficient (necessary) condition for the graph of $\dot{h}/h$ to lie below (above) the graph of $-\dot{c}/c$ so that $h^\star$ in scenario (i) is lower (higher) compared to scenario (iii).

To derive growth effects we first note that the balanced growth rate for scenario (i), $g_{(i)}$, and for scenario (iii), $g_{(iii)}$, are obtained from (14) as,

$$
\begin{aligned}
g_{(i)} &= (h^\star_{(i)})^{-\alpha}\left(\varphi\tau_p(1-\tau_p^{-1}) + \tau(1-\varphi\tau_p)\right) \\
g_{(iii)} &= (h^\star_{(iii)})^{-\alpha}\left(\varphi\tau_p(1-\tau_p^{-1}) + \tau(1-\varphi\tau_p)\right) + (h^\star_{(iii)})^{-\alpha}\varphi\,\tau_p\,\epsilon
\end{aligned}
$$

This shows that $h^\star_{(i)} > (<)\, h^\star_{(iii)}$ is sufficient (necessary) for a higher (lower) balanced growth rate in scenario (iii) compared to scenario (i). Thus, proposition 5 is proved. □

This proposition states that a high inter-temporal elasticity of substitution is a necessary condition such that the balanced growth rate in scenario (i), where environmental quality is constant, exceeds the growth rate of scenario (iii), in which the state of the environment degrades over time. The mechanism behind this result is the same as in proposition 4. Thus, only with a large inter-temporal elasticity of substitution, the household may be willing to forgo more consumption today and shift it into the future in scenario (i) compared to scenario (iii), because the marginal utility of consumption rises with a cleaner environment when the inter-temporal elasticity of substitution is larger one.

Proposition 5 also shows that scenario (i) can never lead to a higher balanced growth rate than scenario (iii) if the ratio of public to private capital on the BGP in scenario (iii) is lower than in scenario (i). Thus, for a small inter-temporal elasticity of substitution, i.e. for $1/\sigma < 1$, it is to be expected that scenario (iii) leads to higher long-run growth because $1/\sigma < 1$ is a necessary condition for a lower $h^\star$ in scenario (iii) compared to scenario (i).

An additional result can be derived as concerns the balanced growth rates in scenario (iii) and scenario (i), depending on the number of BGPs in scenario (iii). This holds because the number of BGPs in scenario (iii) depends on the relation between the inter-temporal elasticity of substitution and the parameter giving the (dis)utility of pollution. This is the contents of the following corollary to proposition 5.

Corollary 1 *Assume that there exist two BGPs in scenario (iii). Then, scenario (i) yields a lower balanced growth rate than scenario (iii).*

Proof: To prove this corollary, we recall from proposition 2 that the existence of two BGPs in scenario (iii) implies $1 + \xi(1-\sigma)/\sigma < 0 \leftrightarrow 1/\sigma < (\xi - 1)/\xi$. The proof of proposition 5 shows that the latter inequality is sufficient for $h^\star$ in scenario (i) to be larger than in scenario (iii) so that the long-run growth rate in scenario (iii) is larger than that in scenario (i). □

When there exist two BGPs for scenario (iii) the inter-temporal elasticity of substitution of consumption is smaller than one minus the inverse of the parameter giving the (dis)utility of additional pollution. Then, the balanced growth rate in scenario (iii) goes along with a smaller value of public capital relative to private capital, $h^\star$, compared to scenario (i) so that the growth rate in scenario (i) is smaller than that of scenario (iii). It should be noted that this holds for both BGPs in scenario (iii). Thus, for a sufficiently small inter-temporal elasticity of consumption, both balanced growth rates in the scenario with a declining environmental quality exceed the balanced growth rate of the scenario with a constant environmental quality.

It should also be pointed out that $1/\sigma < 1 - 1/\xi$ is a sufficient condition for the balanced growth rates in scenario (iii) to exceed that of scenario (i). Thus, corollary 1 gives a sufficient condition while the condition formulated in proposition 5, with respect to the inter-temporal elasticity of substitution, is only necessary.

5 Conclusion

In this paper we have presented an endogenous growth model with productive public capital and emissions that raise a stock of environmental pollution, where we assumed that pollution negatively affects utility of the household but does not affect production possibilities directly. In addition, we have considered three different scenarios in our model: first, we studied the scenario where the stock of pollution is constant over time, second, we analyzed the scenario with an improving environmental quality and, finally, we studied the scenario in which environmental pollution grows at the same positive rate as all other endogenous variables.

The analysis has demonstrated that the model is characterized by a unique saddle point stable balanced growth path for the scenarios with a constant or improving environmental quality. However, it also turned out that the latter scenarios are compatible with sustained economic growth only if the production technology in use is not too polluting. The model with the scenario where all endogenous variables grow at the same rate in the long-run may also reveal a more complex dynamic outcome implying that both local and global indeterminacy may arise.

As concerns the balanced growth rate, the analysis of our model has demonstrated that the inter-temporal elasticity of substitution of consumption is decisive as concerns the question of whether a scenario with a cleaner environment goes along with a higher or lower balanced growth rate. Thus, the inter-temporal elasticity of substitution must be high such that the scenarios with a smaller stock of pollution can be associated with a higher balanced growth rate. The reason for this outcome is that the household is more willing to forgo consumption and shift it into the future in scenarios with a cleaner future environment, if the marginal utility of consumption in the future is higher, which is only the case if the inter-temporal elasticity of substitution is large.

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