

## Abstract

In this presentation we briefly review two approaches to collusion at auction, ring mechanisms and pre-auction bribe offers, that are found in the literature. Included in the ring mechanism discussion are the questions of how a mechanism is decided upon and how a ring could negotiate for a grand coalition, places where bargaining and signaling games like those of pre-auction bribe offers may add understanding. Finally, in the pre-auction bribe offer category, a proposed mechanism currently under development using a double bribe offer is presented along with its first result in the complete information case.

### 1. Motivation

Although the literature on collusion at auction is rich, it remains a small fraction of the overall auction literature and collusion is rarely considered in the mechanism design of an auction. However, the existence of collusion in real world scenarios is well documented, and its effect on seller revenues is potentially great. The problem for collusive bidders becomes how to decide on the split of these earnings between each other.

### 2. Ring mechanisms

The most common approach to modeling a collusive ring's functioning is one of mechanism design. One has to imagine a third party imposing a structure on the ring, where each member of the ring gets to choose whether to participate in the mechanism and, if so, reports his valuation to it. From the set of reported valuations the mechanism determines which ring member gets the item and what side payments are to be made.

Three *ex post* balanced budget examples of ring mechanisms are by McAfee and McMillan [6], Barbar and Forges [1], and Mailath and Zemsky [5]. McAfee and McMillan propose a mechanism that is limited to the homogeneous case. Both Barbar and Forges and Mailath and Zemsky give mechanisms that allow for heterogeneous bidders. Importantly, Mailath and Zemsky introduce a fixed transfer as part of the side-payment scheme.

One of the drawbacks of this mechanism design approach is the question of how the mechanism would be implemented. The general interpretation is that the incentive compatible mechanism reflects an equilibrium in some other mechanism or negotiations. However, of the three mechanisms above, only McAfee and McMillan's one has a known implementation by a simple game (and this one only in the homogeneous case): a pre-auction knockout in the form of a first price auction is held, and the price in the knockout minus the reserve of the seller is divided equally among all losing members of the ring.

Beyond a way to implement a mechanism are two narrower considerations. First, and more difficult, is how does the ring decide between mechanisms, even accepting the mechanisms fully formed. The Mailath and Zemsky mechanism and the Barbar and Forges mechanism have similar properties but would favor different drawn valuations, so the negotiation between the two could have signaling issues. A second consideration is how to adjust the mechanism to make a grand coalition desirable to all members. Here, Mailath and Zemsky's fixed transfer (a property that can be applied to other mechanisms) becomes important, because negotiating between a family of mechanisms differing only in the amount of fixed transfer may allow for grand coalition formation while retaining the incentive compatibility of the mechanism.

### 3. Bribe negotiations

A contrasting approach to the mechanism design one is through looking at simpler games that can model negotiations when one potential bidder approaches another (or several others) and makes a proposal. The analysis focuses on what a bidder signals through an offer and the equilibrium behavior and beliefs that result. Looking at this type of signaling may contribute to understanding of negotiations over ring mechanisms mentioned above. In addition, these offers have the benefit of being a more natural and spontaneous structure on their own.

This bribe-offer approach is rarer, but recently has been done by Eso and Schummer [4] and Chen and Tauman [3]. Eso and Schummer consider a game where one potential bidder, the proponent, makes a single offer of a bribe. The respondent accepts or rejects the bribe and then the game goes to auction with guaranteed enforcement if the bribe was accepted. Chen and Tauman allow for shill bidders and propose an additional bribing structure. Like Eso and Schummer, theirs is a one-time offer.

Somewhat inspired by the wage-buyout negotiation game from Ben-Ner and Jun [2] an interesting variation on the two bidder bribe game that we propose is the double offer: one number as a bribe for bidder 1 to exit the game, the second number for bidder 2 to exit. The respondent can accept one, the other, or neither.

For this game, we model it where two bidders have valuations  $v_1$  and  $v_2$ . Before the auction, there is a bribe game where double offers are made that ends with the offer  $(x_1, x_2)$ , and then they submit their bids  $\hat{v}_1$  and  $\hat{v}_2$ . We consider a second-price auction with a known reserve of  $r$ . Assume there is verification on the bids after auction and that the bribe was conditional to not bidding. Then the bribe payment is

$$P((x_1, x_2), \hat{v}_1, \hat{v}_2) = \begin{cases} (x_1, -x_1) & \text{If } x_1 \text{ accepted} \\ & \text{and } \hat{v}_1 = 0 \\ (-x_2, x_2) & \text{If } x_2 \text{ accepted} \\ & \text{and } \hat{v}_2 = 0 \\ (0, 0) & \text{Otherwise} \end{cases}$$

and the auction result is

$$Q(\hat{v}_1, \hat{v}_2) = \begin{cases} (v_1 - \max\{\hat{v}_2, r\}, 0) & \text{Bidder 1 wins} \\ (0, v_2 - \max\{\hat{v}_1, r\}) & \text{Bidder 2 wins} \end{cases}$$

so the allocation of the surplus is just  $R((x_1, x_2), \hat{v}_1, \hat{v}_2) = P((x_1, x_2), \hat{v}_1, \hat{v}_2) + Q(\hat{v}_1, \hat{v}_2)$ .

Above we said the game "ends with the offer  $(x_1, x_2)$ " because the double offer game could be designed in either one-shot or alternating-offers form. The one-shot form, still undeveloped, may offer an interesting comparison to the results of Eso and Schummer. For the alternating-offers form, we introduce a probability  $(1 - \delta) \in (0, 1]$  that the negotiation will be stopped before the next round. An interpretation for this factor is that they are reaching an agreement in the few moments between their chance to inspect the item and when that item will go up for auction. So,

when a bidder is evaluating the game from a particular node where he thinks he would get an amount  $x$  if he prolongs the game into the next round, he should be willing to accept amounts of  $\delta x$  or greater immediately.

The first case considered is the one of complete information. In this case, one can apply the perfect equilibrium refinement and the bargaining game yields Rubinstein's [7] well-known result. Assuming bidder 1 makes the first offer, if he has the higher valuation the offer is  $(v_1, \frac{\delta}{1+\delta}(v_2 - r))$  and if he has the lower valuation it is  $(\frac{1}{1+\delta}(v_1 - r), 0)$ . The offer is accepted immediately, by bidder 2 accepting  $x_2$  in the first case and  $x_1$  in the second. Furthermore, in a second price auction the agreement is honored. The resulting allocation in the first case is

$$R = \left( v_1 - r - \frac{\delta}{1+\delta}(v_2 - r), \frac{\delta}{1+\delta}(v_2 - r) \right)$$

The more interesting results of course come from relaxing the complete information case and the bidders use the offers as a tool for signaling or discovering information. For instance, consider bidder 1 having a set valuation of 12 and bidder 2 having either a high valuation of 20 or a low one of 5. Bidder 1 could offer amounts where  $x_1$  is acceptable to the high type and  $x_2$  is acceptable to the low type, such as  $(6, 2.5)$ . If bidder 2 is of the high type he accepts the first and if he is the low type he accepts the second, so bidder 1 can effectively separate the two types through his offer.

### 4. Further work

The double bribe offer game can be expanded in two important areas. First, in one-shot form it can be compared to the single bribe offer analyzed by Eso and Schummer. Second, expanding the information scenarios on players' valuations in the extended alternating offers game is the obvious direction, and varying the discount factor or information on it is also possible. In both versions of the game, the ability to give two offers may give the proponent a much more powerful tool in signaling his own valuation and learning his opponent's valuation, much in the way it did in Ben-Ner and Jun's article.

### References

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