Economic policy in a growth model with human capital, heterogenous agents and unemployment

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Abstract

In this paper we present an endogenous growth model with human capital, heterogenous agents and unemployment. Two types of households are considered. One household acquires human capital or skills through education while the other household remains low-skilled. Sustained growth is the result of human capital accumulation which is a function of the existing human capital employed in the educational sector and of public spending for teaching materials. Both households are affected by unemployment and, if so, receive unemployment benefits. The government levies an income tax and uses its revenues to pay unemployment benefits, to finance transfers to the low-skilled household and to finance human capital accumulation. The paper studies growth and welfare effects of economic policy and presents a stability analysis of the model.

JEL: E62, H52, O15

Keywords: Human capital, heterogenous agents, unemployment, economic growth

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We thank two referees for competent comments on an earlier version which helped to improve the paper.

1 Introduction

With the publication of the paper by Lucas (1988) the role of human capital has become increasingly popular in building models of economic growth. The paper by Lucas, which is based on the contribution by Uzawa (1965), asserts that the accumulation of human capital is the major source of ongoing growth. Empirical research analyzing the role of human capital indeed seems to find supportive evidence for this view. For example, the survey by Krueger and Lindahl (2001) shows that there is strong evidence that education is positively correlated with income growth at the microeconomic level and the positive correlation seems to be quite robust. However, this does not necessarily hold for the macroeconomic level where the findings are more fragile. For example, human capital is not a robust variable in explaining economic growth according to the study by Sala-i-Martin (1997). But the lack of explanatory power of human capital may be due to measurement errors as pointed out by Krueger and Lindahl who demonstrate that cross-country regressions show a positive and statistically significant correlation with economic growth if measurement errors are taken into account. It should also be pointed out that Levine and Renelt (1992) have demonstrated that human capital, measured by the secondary enrollment rate, is a robust variable in growth regressions, in contrary to the result found in Sala-i-Martin (1997). Because of that, building endogenous growth models with human capital as the engine of sustained growth is certainly justified.

As concerns the formation of human capital, one can find two approaches in the economics literature. On the one hand, there are approaches where human capital formation is only financed by the private sector and, on the other hand, there exist studies where only the public sector spends resources for the formation of human capital. In addition, there also exist contributions where human capital formation is the result of both public and private expenditures. For example, Glomm and Ravikumar (1992) and Blankenau and Simpson (2004) assume that human capital accumulation results from both private and public services. Glomm and Ravikumar present an OLG model with heterogenous agents where human capital accumulation is the result of formal schooling. They demonstrate that public education leads to a faster decline of income inequality whereas private education may lead to higher per-capita incomes. Blankenau and Simpson present an endogenous growth model with both private and public inputs in the process of human capital accumulation. They demonstrate that the response of growth to public education depends on the tax structure, on the level of government spending and on parameters of the production function.

On the other side, Ni and Wang (1994) and Beauchemin (2001) present models where human capital is the result of public spending alone. Ni and Wang assume homogenous agents, as the contribution by Glomm and Ravikumar (1992), and present an OLG model where human capital is the result of public education which is financed by an income tax. Using calibration exercises they derive that the optimal income tax rate is in the range of six to ten percent. Beauchemin presents a political-economic OLG model of growth and human capital accumulation where human capital accumulation is the result of public education. The paper demonstrates that a sufficiently rapid population growth may generate economic stagnation. In Greiner (2008) a growth model with public education and public debt is presented and analyzed. There, the question of how public debt and deficits affect human capital formation and economic growth is analyzed.

An early contribution that studies optimal fiscal policy in an endogenous growth model with human capital and productive public spending is the paper by Corsetti and Roubini (1996). These authors present a general framework where public spending may either enter the production of final goods or the production function of human capital formation. The goal of their paper is to derive optimal tax rates that can replicate the first best optimum. They show that in optimum tax rates are positive so that the externality related rents are taxed away and no public debt is necessary to attain the first-best solution. If there are restrictions as concerns the available tax instruments, the optimal policy may be obtained only if the government borrows or lends in order to smooth distortions over time.

All what these contributions have in common is that they assume full employment on the labor market. However, many European countries experience persistent unemployment in spite of permanently growing GDPs, due to downward labor rigidities. Although wages adjust according to demand on the labor markets, the labor markets are not sufficiently flexible to guarantee full employment. Therefore, allowing for unemployment in an economic growth model seems to be justified. In addition to that most European countries are characterized by relatively high standards of transfer payments and social security that make frictions on the labor market acceptable so that societies remain socially stable.

Therefore, the goal of our contribution is to analyze how fiscal policies, in particular unemployment benefits and social transfers, and labor market rigidities affect growth and welfare in economies that are characterized by features typical for European economies. In addition, we also study stability properties of the model. Thus, we intend to present a general endogenous growth model that captures important features of European economies and to gain insight how fiscal policy works in such economies.

To achieve this goal, we will present an endogenous growth model with human and physical capital where investment in human capital is the result of public spending for education. In addition, we consider an economy with two different types of households. One household supplies skilled labor, due to human capital formation, whereas the other household supplies low-skilled labor but benefits from human capital accumulation through spill-over effects. Households inelastically supply labor on the first and on the second labor market, respectively, and they both save a certain fraction of their income which is subject to the income tax. Both types of households may become unemployed but, if so, they receive unemployment benefits, thus providing income security for households.

The government pays unemployment benefits and pays transfers to the household supplying simple labor, besides financing human capital accumulation. In order to finance its spending, the government levies a distortionary income tax rate. The firm demands two types of labor: skilled labor on the first labor market that is supplied by household one and simple or low-skilled labor on the second labor market, supplied by household two, which receives a lower wage rate. The firm maximizes profits so that the marginal products of labor equal the wage rates, respectively.

The rest of the paper is organized as follows. In the next section, we present the basic structure of our growth model. Section 3 studies growth and welfare effects of fiscal policy along the balanced growth path. Section 4 analyzes stability properties and section 5, finally, concludes.

2 The structure of the growth model

Our economy consists of three sectors: A household sector which receives labor income and income from its saving, a productive sector and the government. First, we describe the productive sector and the wage adjustment process.

2.1 The productive sector and the wage adjustment process

The productive sector is represented by one firm which behaves competitively and which maximizes profits. Production of the firm at time t is given by a Cobb-Douglas production function as

$$Y(t) = AK(t)^{1-\alpha-\beta} \left(h_c(t)L_1^d(t)\right)^{\alpha} \left(\xi h_c(t)L_2^d(t)\right)^{\beta}, \tag{1}$$

where[1] Y gives output, K denotes physical capital and h_c gives per-capita human capital. L_1^d denotes labor demand for skilled labor in the first labor market and L_2^d gives labor demand for simple labor in the second labor market. The parameter $\xi \in (0,1)$ determines the spill-over effect of human capital implying that, due to externalities, low-skilled labor benefits to a certain degree from human capital of skilled labor. The coefficients $\alpha \in (0,1)$ and $\beta \in (0,1)$ give the elasticity of production with respect to skilled and with respect to

[1]From now on we omit the time argument t if no ambiguity arises.

simple labor, respectively, so that $(1-\alpha-\beta)$ is the capital share and A is a technology parameter.

As concerns the wage rate for simple labor we posit that it is a certain fraction $\epsilon \in (0,1)$ of the wage rate for skilled labor, denoted by ω. The assumption that the wage rate for low-skilled labor is a constant fraction of that for high-skilled certainly is a very simplified way of representing the relative wage generation process. But, we think that for the purpose of this paper this can be justified as a first approach.

Static profit maximization, then, gives demand for the two types of labor as

$$L_1^d = c_1\,\alpha\,\omega^{-1/(1-\alpha-\beta)}\,(h_c/K)^{(\alpha+\beta)/(1-\alpha-\beta)} \tag{2}$$

$$L_2^d = c_1\,\beta\,\omega^{-1/(1-\alpha-\beta)}\,(h_c/K)^{(\alpha+\beta)/(1-\alpha-\beta)}/\epsilon, \tag{3}$$

with $c_1 = A^{1/(1-\alpha-\beta)}\alpha^{\alpha/(1-\alpha-\beta)}\beta^{\beta/(1-\alpha-\beta)}(\xi/\epsilon)^{\beta/(1-\alpha-\beta)}$.

Equations (2) and (3) yield $L_1^d/L_2^d = \epsilon\,\alpha/\beta$, showing that the demand for labor in the first labor market relative to demand in the second labor market is determined by the elasticities of production with respect to labor in these two markets and by the wage rate in the second labor market relative to that in the first labor market, ϵ. The higher the elasticity of production with respect to skilled labor and the higher the wage rate in the second labor market relative to the first labor market, the higher is the demand for skilled labor relative to low-skilled labor.

Denoting by r the return to capital, profit maximization yields

$$r = (1-\alpha-\beta)A(h_c/K)^{(\alpha+\beta)/(1-\alpha-\beta)}(\omega/K)^{(-\alpha-\beta)/(1-\alpha-\beta)}(c_1\alpha)^{\alpha}(c_1\beta/\epsilon)^{\beta}, \tag{4}$$

with L_1^d and L_2^d substituted by the expressions given in (2) and (3).

We should like to point out that the use of a Cobb-Douglas production function implies that the elasticity of substitution between capital and labor and between the two types of labor is each equal to one. This implies that the two types of labor can be substituted by each other to a certain degree.

As concerns the evolution of the wage rate, we assume that it depends negatively on the unemployment rate on the two labor markets and positively on the growth rate of physical capital. The reason for this assumption is that the change in the wage rate will be the smaller the higher the unemployment rate in an economy, as described by a Phillips curve relationship.[2] In addition, in a growing economy wage demands of unions will be the higher the larger the growth rate of the economy. Further, the capital stock positively affects labor productivity so that a higher growth rate of capital implies a higher growth rate of labor productivity. It should also be noted that our assumption as to the wage formation implies that, along a balanced growth path, the wage rate grows at the same rate as capital and GDP.

With this assumption, the growth rate of the wage to capital ratio, $x \equiv \omega/K$, can be described by

$$\frac{\dot{x}}{x} = \beta_{L1}\left(\frac{L_1^d - L_1^n}{(1-u)L_1}\right) + \beta_{L2}\left(\frac{L_2^d - L_2^n}{L_2}\right), \tag{5}$$

with $(1-u)L_1$ and L_2 labor supply on the first and on second labor market, respectively, and L_i^n, $i = 1, 2$, the natural levels of employment in the two markets. This means that the wage rate grows at the same rate as the capital stock if skilled and simple labor demand is equal to its natural level, respectively. It must also be mentioned that uL_1, $u \in (0, 1)$, of the skilled labor is hired by the government as teachers in the educational sector, that is described in detail in the next subsection, so that $(1-u)L_1$ of skilled labor is available in the final goods sector. The parameters $\beta_{L1} > 0$ and $\beta_{L2} > 0$ determine the speed of adjustment and reflect labor market rigidities. The less flexible labor markets are, the smaller will be the parameters, implying that changes in labor demand affect the growth of the wage rate only to a minor degree. Further, since we allow for substitution between the two types of labor we suppose that demand on both labor markets influences the wage adjustment process.

[2] An extensive discussion of the role of the Phillips curve in dynamic macroeconomics can be found e.g. in Flaschel et al. (1997). As regards the Phillips curve see also Blanchard and Katz (1999).

We should also point out that there is one good in our economy that can be either consumed or invested. Consequently all variables are real including the return to capital and the wage rate. Next, we describe the government sector and human capital accumulation.

2.2 Human capital formation and the government

Human capital in our economy is produced in the schooling sector where an exogenously given number of students is educated. As mentioned above, the government hires the fraction u of the skilled labor force as teachers. Additionally, the government uses public resources for education in the schooling sector, like expenditures for books and other teaching material, which is an input in the process of human capital formation, too. Thus, the input in the schooling sector is composed of teachers and of schooling expenditures and we assume decreasing returns to scale to each input but constant returns to both inputs. The evolution of per capita human capital, then, is a function of teachers per student and of expenditures per student.

It should be noted that human capital, which is embodied in students, becomes available to the whole active skilled labor force in the economy, once students become employees. The reason for this assumption is to be seen in spill-over effects of knowledge, which leads to a diffusion of knowledge among the labor force. At first sight, this seems to be a strong assumption. But if one takes into account that in reality newly hired employees interact with existing staff and both learn from each other, this assumption becomes comprehensible.

As concerns the production function for human capital formation we assume a Cobb-Douglas specification. The differential equation describing the change in human per capita capital can be written as

$$\dot{h}_c = \kappa (u h_c L)^{\psi} (I_E)^{1-\psi}/S - \delta_h h_c, \tag{6}$$

with I_E public resources used in the schooling sector, $\kappa > 0$ a technology parameter and $0 < \psi < 1$ is the elasticity of human capital formation with respect to teachers. The parameter δ_h gives depreciation taking into account that a certain fraction of human capital gets lost, for example due to unemployment. Finally, the variable S gives the number of students in the economy.

The government in our economy receives tax revenues from capital and labor income taxation it then uses for the remuneration of the teachers, for public spending in the schooling sector, for transfer payments to low-skilled labor, T_P, and for unemployment benefits, U_p. The budget of the government is balanced at each point in time. Thus, the period budget constraint of the government is given by

$$T = I_E + \omega u L + T_p + U_b, \tag{7}$$

with T denoting tax revenue. As concerns transfer payments, T_p, we assume that this variable makes a certain part of the tax revenue, i.e. $T_p = \phi T$, with $0 < \phi < 1$.

2.3 The household sector

The household sector is composed of two types of households. The first household supplies skilled labor, which is employed either in the production of the final good or in the educational sector, while the second household supplies low-skilled labor. We assume that both households behave as immortal families corresponding to finite-lived individuals who are connected via intergenerational transfers that are based on altruism. Thus, although individuals have finite lives each family is considered as a dynasty where the decision maker behaves as if he had an infinite time horizon (cf. Barro and Sala-i-Martin, 1995, chapter 2.1).

The overall number of skilled people is composed of a stock of students, S, and of a stock of employees, L, who constitute the active labor force and produce goods or are hired as teachers. At each point of time a certain number of students, which is determined exogenously, enters the stock of students and a certain number of students becomes employees. We assume that the number of students becoming employees just equals the number of new students so that the overall stock of students is constant. Further, the number of students becoming employees equals the number of employees leaving the active labor force, so that the active labor force and, thus, the total stock of skilled labor is constant, too, just like the number of low-skilled labor.

The household sector maximizes the discounted streams of utility arising from per-capita consumption, C_i, $i = 1, 2$, over an infinite time horizon subject to their budget constraints, taking factor prices as given. The utility function of both households is assumed to be logarithmic, $U(C_i) = \ln C_i$, $i = 1, 2$, and the households supply labor inelastically. Both households may become unemployed but, if so, receive unemployment benefits from the government that make a certain percentage of the market wage rate.

The maximization problem of the household in the first labor market, then, can be written as

$$\max_{C_1} \int_0^\infty e^{-\rho t} \ln C_1 dt, \tag{8}$$

subject to

$$(1-\tau)\left(\omega L_1^d + u\omega L_1 + rK_1 + \lambda\omega(L_1 - L_1^d - uL_1)\right) = \dot{K}_1 + \delta K_1 + C_1. \tag{9}$$

The parameters $\rho > 0$, $\tau \in (0,1)$ and $\delta \in (0,1)$ are the subjective discount rate, the income tax rate and the depreciation rate of capital, respectively, and $K_1 > 0$ and $C_1 > 0$ give the capital stock owned by the household in the first labor market and its level of consumption. $\lambda \in (0,1)$ gives that part of the market wage that is paid by the government as unemployment benefit and we assume that the total income, including unemployment benefits, is subject to the income tax.

To solve this problem we formulate the current-value Hamiltonian which is written as

$$H_1 = \ln C_1 + \gamma_1((1-\tau)\left(\omega L_1^d + u\omega L_1 + rK_1 + \lambda\omega(L_1 - L_1^d - uL_1)\right) - \delta K_1 - C_1), \tag{10}$$

with γ_1 the shadow-price of capital for household 1. Necessary optimality conditions are given by

$$C_1^{-1} = \gamma_1 \tag{11}$$

$$\dot{\gamma}_1 = (\rho + \delta)\gamma_1 - \gamma_1(1 - \tau)\, r \tag{12}$$

If the transversality condition $\lim_{t\to\infty} e^{-\rho t} K_1/C_1 = 0$ holds, which is fulfilled for a time path on which capital grows at the same rate as consumption, the necessary conditions are also sufficient.

The maximization problem of the household in the second labor market is given by

$$\max_{C_2} \int_0^\infty e^{-\rho t} \ln C_2 dt, \tag{13}$$

subject to

$$(1 - \tau) \left(\epsilon\, \omega L_2 + r K_2 + \lambda\, \epsilon\, \omega (L_2 - L_2^d)\right) + T_p = \dot{K}_2 + \delta K_2 + C_2. \tag{14}$$

The capital stock owned by household two is denoted by $K_2 > 0$ and $C_2 > 0$ is its consumption. The household in the second labor market also saves but we assume that it disposes of a smaller capital stock than the household in the first labor market, i.e. $K_2 < K_1$. Further, it receives transfer payments from the government, T_p, in addition to its market income.

Again, we formulate the current-value Hamiltonian which is

$$H_2 = \ln C_2 + \gamma_2((1 - \tau) \left(\epsilon\, \omega L_2^d + r K_2 + \lambda\, \epsilon\, \omega (L_2 - L_2^d)\right) + T_p - \delta K_2 - C_2), \tag{15}$$

with γ_2 the shadow-price of capital for household 2. Necessary optimality conditions are obtained as

$$C_2^{-1} = \gamma_2 \tag{16}$$

$$\dot{\gamma}_2 = (\rho + \delta)\gamma_2 - \gamma_2(1 - \tau)\, r \tag{17}$$

These conditions are again sufficient if the transversality condition $\lim_{t\to\infty} e^{-\rho t} K_2/C_2 = 0$ is fulfilled.

The growth rates of consumption of the households are obtained from (11)-(12) and (16)-(17) as

$$\frac{\dot{C}_i}{C_i} = -\rho + (1-\tau)r, \; i = 1, 2. \tag{18}$$

Using $C_1 + C_2 = C$, the growth rate of aggregate consumption is given by

$$\frac{\dot{C}}{C} = \frac{\dot{C}_1}{C_1}\frac{C_1}{C} + \frac{\dot{C}_2}{C_2}\frac{C_2}{C} = \left(-\rho + (1-\tau)\, r\right)\left(\frac{C_1}{C} + \frac{C_2}{C}\right), \tag{19}$$

with $C_1/C + C_2/C = 1$.

3 The balanced growth path

An equilibrium allocation is defined as an allocation such that the firm maximizes profits implying that factor prices equal their marginal products (equations (2), (3) and (4)), the households solve (8) subject to (9) and (13) subject to (14), respectively, the wage rate relative to capital evolves according to (5) and the budget constraint of the government (7) is fulfilled and the limiting transversality conditions hold.

The economy-wide resource constraint in this economy is obtained by combining the budget constraint of private households, equations (9) and (14), with the budget constraint of the government (7) as

$$\frac{\dot{K}}{K} = \frac{Y}{K} - \frac{C}{K} - \frac{I_E}{K} - \delta, \tag{20}$$

where I_E is given by

$$I_E = T(1-\phi) - \omega u L_1 - \lambda\omega(L_1(1-u) - L_1^d + \epsilon(L_2 - L_2^d)). \tag{21}$$

Aggregate consumption evolves according to equation (19) with r given by (4) so that the growth rate of aggregate consumption can be written as

$$\frac{\dot{C}}{C} = (1-\tau)\,(1-\alpha-\beta)\,\left(\frac{Y}{K}\right) - (\rho+\delta), \tag{22}$$

with Y/K given by

$$\frac{Y}{K} = A\, h^{(\alpha+\beta)/(1-\alpha-\beta)} x^{(-\alpha-\beta)/(1-\alpha-\beta)} (c_1\alpha)^{\alpha} (c_1\beta/\epsilon)^{\beta}, \tag{23}$$

where we defined $h \equiv h_c/K$.

Human capital, finally, grows according to

$$\frac{\dot{h}_c}{h_c} = (\kappa/S)(u\,L_1)^{\psi}\left(\frac{I_E}{h_c}\right)^{1-\psi} - \delta_h. \tag{24}$$

Thus, the economy is completely described by equations (5), (20), (22) and (24) plus the limiting transversality conditions of the households and initial conditions with respect to the capital stocks.

A balanced growth path (BGP) is defined as a path on which all endogenous variables grow at the same constant rate, i.e. $\dot{K}/K = \dot{C}/C = \dot{h}_c/h_c = \dot{\omega}/\omega = g > 0$ holds, with $g = constant$. To analyze our economy around a BGP we define the new variable $c \equiv C/K$ and we use $h = h_c/K$. Differentiating these variables with respect to time and using $x = \omega/K$ from (5), a three dimensional system of differential equations results, given by $\dot{c}/c = \dot{C}/C - \dot{K}/K$, $\dot{h}/h = \dot{h}_c/h_c - \dot{K}/K$ and $\dot{x}/x$ which can be written as follows,

$$\begin{aligned}
\dot{c} &= c\left((1-\tau)(1-\alpha-\beta)\left(\frac{Y}{K}\right) - \rho - u\omega L_1 - \lambda\omega(L_1(1-u) - L_1^d + (L_2 - L_2^d)\epsilon)\right) + \\
&\quad c\left(c + (1-\phi)\left(\frac{T}{K}\right) - \left(\frac{Y}{K}\right)\right) && (25)\\
\dot{h} &= h\left((\kappa/S)(u\,L_1)^{\psi}\left(\frac{I_E}{h_c}\right)^{1-\psi} - \delta_h - u\omega L_1 - \lambda\omega(L_1(1-u) - L_1^d + (L_2 - L_2^d)\epsilon)\right) + \\
&\quad h\left(\delta + c + (1-\phi)\left(\frac{T}{K}\right) - \left(\frac{Y}{K}\right)\right) && (26)\\
\dot{x} &= x\left(h^{(\alpha+\beta)/(1-\alpha-\beta)}x^{(-\alpha-\beta)/(1-\alpha-\beta)}\,c_1\,\left(\alpha\beta_{L1}/(L(1-u)) + \beta\beta_{L2}/(\epsilon L_2)\right)\right) + \\
&\quad x\left(\rho + \delta - (1-\tau)(1-\alpha-\beta)\left(\frac{Y}{K}\right) - \beta_{L1}L_1^n/(L(1-u)) - \beta_{L2}L_2^n/(\epsilon L_2)\right) && (27)
\end{aligned}$$

with L_1^d, L_2^d, I_E and Y/K given by (2), (3), (21) and (23), respectively. The expression for T/K is $T/K = \tau(1-\alpha-\beta)(Y/K) + \tau h^{(\alpha+\beta)/(1-\alpha-\beta)}x^{(-\alpha-\beta)/(1-\alpha-\beta)}(1-\lambda)c_1(\alpha+\beta) + \tau x(uL_1 + \lambda L_1(1-u) + \lambda\epsilon L_2)$. It should be noted that the tax revenue T grows at the same rate as capital K on the BGP, because the return to capital r is constant and the wages grow at the same rate as capital. Thus, T/K is constant along the BGP.

A solution of $\dot{c} = \dot{h} = \dot{x} = 0$ with respect to h, c, x gives a BGP for our model and the corresponding ratios $h^\star, c^\star, x^\star$ on the BGP.[3] Proposition 1 gives results as concerns existence of a BGP.

Proposition 1 *Assume that $I_E/h_c > (\kappa(uL)^\psi/\delta_h)^{1/(\psi-1)}$ holds in equilibrium. Then, there exists a unique balanced growth path for the model economy.*

Proof: See appendix.

This proposition shows that the balanced growth path for this economy is unique provided that public investment in the educational sector is large enough. The fact that educational investment has to be positive and sufficiently large for sustained growth is not too surprising because human capital formation is the source of ongoing growth in this model. It should also be pointed out that along the BGP total employment equals its natural level which is due to the formulation of the Phillips curve in equation (5). But, since we use a Cobb-Douglas production function that allows substitution between the two types of labor, labor demand for one type of labor can be below its natural level while the other type of labor exceeds its natural level of employment.

4 Comparative statics of the balanced growth path

In this section we want to study how the balanced growth rate reacts to changes in parameters. In particular, we are interested in the question of how labor market rigidities, modelled in our approach by the parameters $\beta_{L,i}$, $i = 1, 2$, affect the balanced growth rate as well as how transfer payments and unemployment benefits influence growth. Thus, we first analyze variations in the adjustment speed. Proposition 2 gives the result.

Proposition 2 *A rise in the adjustment speed for skilled labor raises (leaves unchanged, reduces) the balanced growth rate if and only if $L_1^n/L_2^n > (=, <)\, \alpha\, \epsilon/\beta$. A rise in the ad-*

[3]The $\star$ denotes BGP values and we exclude the economically meaningless BGP $h^\star = c^\star = x^\star = 0$.

justment speed for low-skilled labor raises (leaves unchanged, reduces) the balanced growth rate if and only if $L_2^n/L_1^n < (=, >)\ \beta/(\alpha\,\epsilon)$.

Proof: See appendix.

To interpret that proposition we first note that variations in the speed of adjustment do not affect the ratio of human capital to physical capital on the BGP. Thus, it is the relation of labor demand relative to its natural level that determines whether a rise in the adjustment speed raises the balanced growth rate. For example, if labor demand for skilled labor is smaller than its natural level, a higher speed of adjustment for skilled labor will raise the growth rate. The reason for that outcome is that a more flexible labor market reduces unemployment, thus, raising the balanced growth rate. This interpretation becomes obvious when it is recalled that $\alpha\epsilon/\beta = L_1^d/L_2^d$.

Of course, the same holds for low-skilled labor. If low-skilled labor is employed below its natural level, a higher speed of adjustment for low-skilled labor will increase the balanced growth rate because it implies a smaller rate of unemployment. Thus, a more flexible labor market for that type of labor that is in excess supply, i.e. where demand is below its natural level, leads to a higher balanced growth rate.

In the next two propositions, we analyze how fiscal policy affects the balanced growth rate.[4] The next proposition deals with growth effects of raising transfer payments.

Proposition 3 *A shift of resources to transfer payments reduces the balanced growth rate.*

Proof: See appendix.

Proposition 3 shows that more transfer payments imply a decline in the balanced growth rate. The economic mechanism behind that result is obvious. Due to the budget constraint of the government a rise in transfers leads to a decline in public spending for

[4] We do not study effects of varying the income tax rate. Since the government uses its tax revenues to finance productive investment in education, there will exist a growth maximizing income tax rate as in growth models with public infrastructure (see e.g. Greiner and Hanusch, 1998). The same holds for human capital employed in the educational sector.

education. As a consequence, the balanced growth rate declines because investment in human capital is reduced.

When analyzing growth effects of increasing unemployment benefits, the outcome changes. Proposition 4 gives the result.

Proposition 4 *A rise in unemployment benefits λ reduces (leaves unchanged) the balanced growth rate if and only if* $c_1(\alpha + \beta) < (=) L_1(1-u) + \epsilon L_2$.

Proof: See appendix.

In order to interpret that proposition we note that the condition in the proposition can be rewritten as $(L_1^d - L_1(1-u)) + \epsilon(L_2^d - L_2) < (=) 0$. This shows that higher unemployment benefits reduce the balanced growth rate if labor demand is smaller than labor supply. The economic mechanism behind that result is the same as in proposition 3. More unemployment benefits reduce public investment in education and, as a consequence, the ratio of human to physical capital on the BGP and, thus, economic growth. Only in case of full employment variations in unemployment benefits do not affect the balanced growth rate which is obvious since, in that case, unemployment payments of the government are equal to zero anyhow.

In the next section we analyze how fiscal policy affects welfare on the BGP.

5 Welfare effects of fiscal policy

We analyze welfare effects of fiscal policy by comparing welfare on the balanced growth path for different fiscal policy parameters. Further, we have to introduce a social welfare function. In this paper, we use a Bernoulli-Nash function that gives social welfare W and that is written as

$$W = W_1^{\mu_1} \cdot W_2^{\mu_2}, \tag{28}$$

with W_1 and W_2 welfare of household 1 and household 2, respectively, and $\mu_i \geq 0$, $i = 1, 2$, weights given to the welfare of the households, where we set $\mu_1 = \mu_2 = 1$ so that welfare of the two households receive the same weight.

Along the BGP, individual welfare is given by

$$W_i = \int_0^{\infty} e^{-\rho t} \ln C_i(t) = \frac{g}{\rho^2} + \frac{\ln C_i(0)}{\rho},\ i = 1, 2, \tag{29}$$

with g denoting the balanced growth rate. It should be noted that along the BGP, consumption and the capital stocks of the two households grow at the same rate as the economy. Hence, we have $\dot{K}_i/K_i = \dot{C}_i/C_i = g = (1-\tau)r - (\rho+\delta)$. This gives initial consumption of the two households as $C_1 = \rho K_1 + (1-\tau)\omega(uL_1 + L_1^d + \lambda(L_1(1-u) - L_1^d))$ and $C_2 = \rho K_2 + (1-\tau)\epsilon\omega(L_2^d + \lambda(L_2 - L_2^d)) + T_p$. Of course, we have $C_1 + C_2 = C$ and $K_1 + K_2 = K$. Along the BGP, the distribution of capital between the two households is constant because all capital stocks grow at the same rate. Therefore, on the BGP the initial value of consumption of the two households depends on the initial distribution of the capital stocks, on the initial value of the aggregate capital stock and on the wage rate relative to capital on the BGP.

In order to study how fiscal policy affects welfare in this economy we resort to a numerical example. As regards the parameter values we use the following values as benchmark, $A = 1$, $\alpha = 0.4$ and $\beta = 0.2$. Those values for α and β imply an elasticity of output with respect to skilled labor and with respect to low-skilled labor of 40 and 20 percent, respectively, and the elasticity with respect to physical capital is 40 percent. The parameter determining spill-overs of human capital in the production is set to $\xi = 0.15$. Labor supply is set to 0.1 for both types of labor, i.e. $L_1 = L_2 = 0.1$, and low-skilled labor gets 50 percent of the wage rate of skilled labor, $\epsilon = 0.5$. The natural rate of unemployment is assumed to be 2.5 percent for skilled labor and 7.5 for low-skilled so that we set $\bar{L}_1 = 0.0975$ and $\bar{L}_2 = 0.0925$. It should be noted that this implies that low-skilled labor is more likely to become unemployed than skilled labor.

The adjustment speed for both types of labor is set to 1 percent, $\beta_{L1} = \beta_{L2} = 0.01$. Further, we assume that the number of students relative to total labor supply is 10 percent, implying $S = 0.02$ and 7 percent of skilled labor supply is employed in the education sector, $u = 0.07$. The elasticity of human capital formation with respect to educational investment is 50 percent, i.e. $\psi = 0.5$, and we set $\kappa = 0.14$. The depreciation rates of physical and human capital are 5 percent, $\delta = \delta_h = 0.05$, and the rate of time preference is $\rho = 0.05$. Finally, the fiscal parameters are set to $\tau = 0.15$, $\phi = 0.1$ and $\lambda = 0.7$, giving an income tax rate of 15 percent and implying that 10 percent of the tax revenue is paid as transfers and unemployed receive 70 percent of the wage rate.

With these parameter values we first compute welfare for different values of transfers paid to the relatively poor household. Table 1 shows the outcome for different values of the parameter ϕ, where we set $K_1/K = 0.75$ implying that the first household owns 75 percent of the capital stock in this economy[5] and we set $K(0) = 100$.

	$\phi = 0.05$	$\phi = 0.1$	$\phi = 0.15$	$\phi = 0.2$
W	3364.29	3287.2	3202.69	3110.46
W_1	71.3246	70.2074	69.039	67.8124
W_2	47.1688	46.8212	46.3895	45.8686
$C_1(0)$	20.1651	19.894	19.6114	19.3158
$C_2(0)$	6.0265	6.1787	6.3195	6.4478
g	2.8%	2.6%	2.4%	2.1%

Table 1: Welfare, initial consumption and the growth rate for different values of ϕ.

Table 1 shows that a rise in transfer payments reduces both the balanced growth rate and welfare in this economy. The increase in transfer raises initial consumption of the relatively poorer household, however, this increase is not sufficiently large to compensate the decline in the growth rate. As a consequence, welfare of the poorer household declines although transfers are increased. The level of initial consumption of the relatively rich

[5]The ratio K_1/K is not decisive. For example, setting $K_1/K = 0.9$ gives the same qualitative results.

household declines as transfers to the poor rises and, of course, its welfare, too.

In table 2 we report the results of varying the unemployment benefits.

	$\lambda = 0.6$	$\lambda = 0.7$	$\lambda = 0.8$	$\lambda = 0.9$
W	3284.96	3287.2	3289.45	3291.65
W_1	70.2433	70.2074	70.172	70.1363
W_2	46.7656	46.8212	46.877	46.9322
$C_1(0)$	19.917	19.894	19.8711	19.8481
$C_2(0)$	6.1576	6.1787	6.1998	6.2209
g	2.6%	2.6%	2.6%	2.6%

Table 2: Welfare, initial consumption and the growth rate for different values of λ.

Table 2 shows that welfare in the economy rises as unemployment benefits become higher. It can also be seen that the relatively rich household is worse off with higher unemployment benefits. Thus, both its initial consumption and its welfare decline as λ rises. The poor household, however, has both higher initial consumption and higher welfare, the larger unemployment benefits are. Further, the increase in welfare of the poorer household outweighs the decline in welfare of the richer household so that total welfare rises.

The economic mechanism behind that outcome is that the quantitative decline of the balanced growth rate is only small. Hence, the balanced growth rate is reduced but the decline is less than 0.1%. This is due to the fact both households get unemployment benefits and pay taxes on their total income. Therefore, the decline in the tax revenue and, thus, the reduction of public investment in education, is smaller compared to the case when transfer payments are increased that are not subject to the income tax. Consequently, the negative effect of a smaller balanced growth rate is negligible and this measure raises welfare in the economy.

Up to now, we have studied growth and welfare effects along the BGP. In the next section, we analyze stability properties of our model.

6 Stability of the balanced growth path

In order to find how fiscal parameters affect stability we resort to the numerical example from the last section and compute the eigenvalues of the Jacobian matrix. As in the last section we study the effects of varying transfer payments and unemployment benefits. Table 3 shows the signs of the eigenvalues of the Jacobian matrix for different values of transfer payments, ϕ.

	$\phi = 0.05$	$\phi = 0.1$	$\phi = 0.2$	$\phi = 0.35$
eigenvalues	$+; -a \pm bi$	$+; -a \pm bi$	$+; -a \pm bi$	$+; +a \pm bi$

Table 3: Eigenvalues of the Jacobian for different values of ϕ $(a, b > 0, i = \sqrt{-1})$.

Table 3 demonstrates that more transfer payments, i.e. a higher value for ϕ, tends to make the economy unstable. Hence, this table suggests that higher spending for education and less for unproductive purpose do not only raise economic growth but also tend to stabilize the economy. Thus, for values of ϕ larger than about 35% percent all eigenvalues are positive or have positive real parts. It should also be mentioned that the economy is characterized by transitory oscillations until it reaches the BGP in the long-run since the eigenvalues are complex conjugate.

However, the question of for which parameter values the economy becomes unstable also depends on the flexibility of the wage adjustment process, that is on the value of β_{L1} and β_{L2}. This is illustrated in table 4, where we set $\beta_{L1} = \beta_{L2} = 0.1$ which is ten times as high as in the benchmark case.

	$\phi = 0.05$	$\phi = 0.25$	$\phi = 0.55$	$\phi = 0.75$
eigenvalues	$-; -; +$	$-; -; +$	$+; -a \pm bi$	$+; +a \pm bi$

Table 4: Eigenvalues of the Jacobian for different ϕ, $\beta_{L1} = \beta_{L2} = 0.1$ $(a, b > 0, i = \sqrt{-1})$.

With higher values for the coefficients β_{L1}, and β_{L2} the economy is stable even for relatively large transfer payments. Thus, only for values of ϕ larger than about 75 percent the economy is now unstable. But it must also be pointed out that the balanced growth rate becomes negative if ϕ becomes larger than 55 percent. In that case, educational investment is not sufficiently high to compensate the decline in human capital, due to oblivion, and the economy is characterized by a declining GDP on the BGP.

An additional result is that the economy is not characterized by transitory oscillations if transfer payments are sufficiently small, ϕ smaller than about 25 percent. Hence, fiscal policy may also be decisive as to whether the economy shows transitory cycles on the transition path or whether the adjustment is monotonic. In the next two tables we analyze effects of varying the amount of unemployment benefits.

Qualitatively, the results are the same as in the case of varying transfer payments. Hence, for small values of λ, determining the amount of unemployment benefits, the economy is stable with two negative real roots. If the parameter λ is increased, the eigenvalues become complex with two negative real roots. That means that the economy is stable but it shows transitory oscillations until it reaches the BGP in the long-run. If unemployment benefits are further increased the economy again becomes unstable.[6] Table 5 illustrates this case.

[6]In tables 3, 4 and 5, the dynamic system (25)-(27) undergoes a Hopf bifurcation leading to unstable limit cycles for $\phi = 0.3119$, $\phi = 0.7409$ and for $\lambda = 0.9093$, respectively.

	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 0.9$	$\lambda = 0.95$
eigenvalues	$+; -a \pm bi$	$+; -a \pm bi$	$+; -a \pm bi$	$+; +a \pm bi$

Table 5: Eigenvalues of the Jacobian for different values of λ $(a, b > 0, i = \sqrt{-1})$.

Finally, table 6 shows that the economy is always stable for $\beta_{L1} = \beta_{L2} = 0.1$. Again, for small values of λ all eigenvalues are real with two being negative, for larger values of λ two eigenvalues become complex conjugate with negative real parts. Hence, the economy may produce transitory oscillations, but it always converges to the BGP in the long-run.

	$\lambda = 0.5$	$\lambda = 0.75$	$\lambda = 0.8$	$\lambda = 1$
eigenvalues	$-; +; -$	$-; -; +$	$+; -a \pm bi$	$+; -a \pm bi$

Table 6: Eigenvalues of the Jacobian for different λ, $\beta_{L1} = \beta_{L2} = 0.1$ $(a, b > 0, i = \sqrt{-1})$.

7 Conclusion

In this paper we have presented an endogenous growth model with human capital and unemployment. Human capital is the result of public spending for education and the government sector also finances transfers to the poorer household and unemployment benefits.

The analysis of our model has demonstrated that more flexible labor markets, flexible in the sense that changes in labor demand imply a stronger reaction in the wage rate, go along with a higher balanced growth rate if labor demand is smaller than the natural level of employment. Further, higher transfer payments reduce the balanced growth. The same holds for higher unemployment benefits unless labor demand equals labor supply.

The reason for the latter two results is that, due to the budget constraint of the government, higher transfers and unemployment spending reduce productive investment in the educational sector.

As concerns welfare, numerical examples have shown that higher transfer payments reduce welfare in the economy due to the large decline in the growth rate so that welfare of both rich as well as poor households declines. However, this does not necessarily hold for unemployment benefits. The reason for that outcome is that variations in unemployment benefits affect economic growth to a lesser extent than variations in transfer payments. In that case, a rise in unemployment benefits reduces welfare of the rich household but raises that of the poor so that overall welfare can rise.

Hence, the general conclusion that can be drawn from our analysis is that social spending programs that benefit poorer households can raise welfare provided that the growth rate does not decline too strongly, even if these policy measures reduce welfare of relatively rich households.

Finally, a study of the stability properties has demonstrated that fiscal policy affects stability properties of the model economy as well. We found that higher unproductive public spending for transfers and for unemployment benefits can make the economy unstable. But, the more flexible labor markets are, flexible in the sense that the wage rate adjusts quickly to variations in labor demand, the more social spending is feasible without endangering stability.

Appendix

Proof of proposition 1

To prove this proposition we note that $\dot{c}/c = 0$ gives $c + \delta - uL_1\omega - \lambda\omega(L_1(1-u) - L_1^d + \epsilon(L_2 - L_2^d)) + (1-\phi)T/K - Y/K = (\rho + \delta) - (1-\tau)(1-\alpha-\beta)(Y/K)$.

Using this, we can rewrite $\dot{h}/h$ as

$$\dot{h}/h = (\kappa/S)(u\,L_1)^{\psi}\left(\frac{I_E}{h_c}\right)^{1-\psi} - \delta_h + \rho + \delta - (1-\tau)(1-\alpha-\beta)\left(\frac{Y}{K}\right)$$

with

$$\frac{Y}{K} = A\,h^{(\alpha+\beta)/(1-\alpha-\beta)}x^{(-\alpha-\beta)/(1-\alpha-\beta)}(c_1\alpha)^{\alpha}(c_1\beta/\epsilon)^{\beta}$$

From (21) we get for I_E/h_c: $I_E/h_c = h^{-1+(\alpha+\beta)/(1-\alpha-\beta)}x^{(-\alpha-\beta)/(1-\alpha-\beta)}((1-\phi)\tau(1-\alpha-\beta)(Y/K) + (\lambda + (1-\phi)\tau(1-\lambda))c_1(\alpha+\beta)) + h^{-1}x((1-\phi)\tau(uL_1 + \lambda L_1(1-u) + \lambda\epsilon L_2) - uL_1 - \lambda(L_1(1-u) + \epsilon L_2))$.

Further, from $\dot{x}/x = 0$ we get

$$x = h^{\alpha+\beta}\left(\frac{c_1(\alpha\beta_{L1}/(L(1-u)) + \beta\beta_{L2}/(\epsilon L_2))}{\beta_{L1}L_1^n/(L(1-u)) + \beta_{L2}L_2^n/L_2}\right)^{1-\alpha-\beta}$$

Replacing x in Y/K and in I_E/h_c leads to

$$\frac{\dot{h}}{h} \equiv q(h,\cdot) = (\kappa/S)(u\,L_1)^{\psi}h^{-(1-\alpha-\beta)(1-\psi)}\,C_1^{1-\psi} + (\rho+\delta-\delta_h) - h^{\alpha+\beta}\,C_2,$$

with $C_i > 0$ $i = 1,2$, constants, given by $C_1 = \tau(1-\phi)(1-\alpha-\beta)A(c_1\alpha)^{\alpha}(c_1\beta/\epsilon)^{\beta} + (1-\lambda)c_1(\alpha+\beta) + uL + \lambda L_1(1-u) + \lambda\epsilon L_2) + \lambda c_1(\alpha+\beta) - uL - \lambda(L_1(1-u) + \epsilon L_2)$ and $C_2 = (1-\tau)(1-\alpha-\beta)A(c_1\alpha)^{\alpha}(c_1\beta/\epsilon)^{\beta}$.

Thus $\dot{h}/h \to +\infty$, for $h \to 0$, $\dot{h}/h \to -\infty$, for $h \to \infty$ and $\partial(\dot{h}/h)/\partial h < 0$ such that there exists a unique $h^\star$ with $q(h^\star,\cdot) = 0$. □

Proof of proposition 2

To prove that proposition, we note that the balanced growth rate is given by equation (22). Using (23) and $x = x(h,\cdot)$ obtained from solving $\dot{x}/x = 0$ in the proof of proposition 1, we get

$$\frac{\dot{C}}{C} = -(\rho+\delta) + (1-\tau)(1-\alpha-\beta)A(c_1\alpha)^{\alpha}(c_1\beta/\epsilon)^{\beta}h^{\alpha+\beta}\cdot \left(\frac{\beta_{L1}L_1^n/(L(1-u)) + \beta_{L2}L_2^n/L_2}{c_1(\alpha\beta_{L1}/(L(1-u)) + \beta\beta_{L2}/(\epsilon L_2))}\right)^{\alpha+\beta} \quad (A.1)$$

From the proof of proposition 1 it is easily seen that h on the BGP is invariant with respect to variations in $\beta_{L,i}$, $i = 1, 2$. Using that and differentiating $(A.1)$ with respect to $\beta_{L,i}$, $i = 1, 2$, it is easily seen that we get the result in proposition 2. □

Proof of proposition 3

From $(A.1)$ one realizes that transfer payments do not directly affect the balanced growth rate but only through variations in h. The effect of varying ϕ on the value of h along the BGP is obtained by implicitly differentiating $q(h, \cdot)$ from proposition 1. It is easily seen that $\partial q(h, \cdot)/\partial\phi < 0$ and $-\partial q(h, \cdot)/\partial h > 0$ so that a rise in ϕ implies a decrease in h on the BGP and, therefore, a lower balanced growth path. □

Proof of proposition 4

Unemployment benefits λ do not directly affect the balanced growth rate $(A.1)$. Implicitly differentiating $q(h, \cdot)$ from proposition 1 gives $-\partial q(h, \cdot)/\partial h > 0$ and $\partial q(h, \cdot)/\partial\lambda < (=)\, 0$ for $c_1\,(\alpha + \beta) < (=)\, L_1(1 - u) + \epsilon L_2$. □

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