The competitive firm and the role of information about uncertain factor prices

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Abstract

This paper studies the role of information about uncertain input prices for a competitive firm. The production decision has to be taken when the price of an input factor is perceived as random. However, a signal is observable in advance, conveying some information about the future factor price. Transparency is linked to the informational content of this signal. The impact of a higher level of transparency is analyzed from an ex ante perspective, i.e. before the information signal is observed. The change of factor use, production amount and cost are determined by comparing the strength of the curvatures of total and marginal product. By contrast, ex ante profit always increases.

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1 Introduction

Starting with the seminal work of Sandmo (1971) much has been written on the theory of the competitive firm under price uncertainty. These studies replace the classical assumption that the firm knows the price at the time when the production decision is made by the assumption that only the probability distribution is known and that the firm is averse to risk. Despite this generalization, the decision problem still seems to be somewhat particular in nature. But Milgrom (1994) shows that any comparative static conclusion about investment under uncertainty in Sandmo's model also holds for a larger class of expected return function. This class contains for example the standard portfolio problem. A number of studies not only incorporate uncertainty in the firm's decision problem but also discuss the effects of increasing uncertainty. This is operationalized using concepts like a mean preserving spread. Consequently, the stochastic nature is a purely exogenous structural element in these models. Causes of the change in the level of uncertainty are not modeled explicitly.

More recently, Broll and Eckwert (2006) study a firm in a framework where price uncertainty due to a random exchange rate is to some extent caused by imperfect information. Risk is therefore in part endogenous and may be reduced by the revelation of more precise information. This does not reduce uncertainty per se, as done in the models mentioned above, but affects the updated beliefs after the information has become available. In addition to the production decision, the firm is allowed to trade forward delivery contracts on a futures market. But in contrast to Feder et al. (1980) the setup also captures the interaction of the available information with the terms of contracting on the forward exchange market. They show that the effect of better information on the output decision depends on the curvature of the inverse marginal cost function.

The focus of this paper is the role of more reliable information about random input instead of output prices on the production decision and performance of a competitive firm in such a framework. If the input factor is imported from abroad, the randomness of its price may be caused by exchange rate uncertainty. In addition it may have various other reasons. The price of some agricultural products, for example, fluctuates considerably due to random weather conditions.

A single competitive firm is considered, which produces a homogeneous good from a single input factor and sells it at a given price. At the time the production decision has to be taken the price of the input factor is random. Yet, in advance of the production decision an information signal can be observed. This signal contains some information about the future factor price, which is used to update the prior belief. The factor market's transparency is associated with the reliability of this signal in the sense of Blackwell (1951, 1953). The effects of higher transparency on the ex ante expected volumes of factor use, production, cost and profit are studied. This distinguishes the analysis from others that focus on the interim perspective and consider an increase in risk or the (monotone) dependence on the observed signal (Batra and Ullah 1974, Coes 1977, Milgrom 1994, Athey 2000).

The chosen framework allows to state the results in terms of conditions on the firm's technology, i.e. production function. It turns out that more transparency leads on average to more (less) factor use, if the marginal product is convex (concave). This can be linked to the trend of the strength of the the law of diminishing marginal returns. Ex ante expected production may increase or decrease depending on the relation of the curvatures of total and marginal product. This relation compares the strength of two (potentially opposing) effects. The former is the loss in average production caused by a spread in optimal factor inputs due to diminishing marginal returns. The latter one is caused by the change in average factor use, which may be positive or negative as just mentioned. In a number of cases average production may decrease although average factor use increases. A similar kind of result is derived for the cost from ex ante perspective. Here the magnitude of more or less average factor use is interacting with an efficiency gain by better decisions due to more informative signal observations. Under certain conditions it occurs that the efficiency gain is big enough so that average expenditure for the factor decreases although the demand increases on average. By contrast, ex ante profit always increases if more reliable information about the future factor price is available, independent of the specific type of production function. This means that the efficiency effect always dominates the effect caused by diminishing marginal returns.

The welfare implications of higher transparency are not discussed in detail in this work. For a risk averse firm with a random return function Athey (2000) gives necessary and sufficient conditions for the distribution of an information signal to lead to higher welfare (ex ante expected utility). But in contrast to her setup, more reliable information leads in the presented one not only to better decisions but on the other hand also destroys risk sharing opportunities. Eckwert and Zilcha (2003) show that the overall effect on welfare therefore depends on the decision maker's level of risk aversion. For small levels of risk aversion welfare increases with more transparency as the former effect is dominating and for high levels welfare is decreasing because the latter one is bigger. This result is also valid for the presented model.

The remainder of this paper is organized as follows: In Section 2 the model is presented and the used notion of transparency is introduced. Section 3 presents the results on the effects of more transparency about the future factor price on factor use, production, cost and profit. Finally, Section 4 provides a brief conclusion. All proofs are relegated to a separate appendix.

2 The Model

Consider a competitive risk-averse firm which extends over two periods in time ($t = 1, 2$). In period $t = 2$ it produces a homogeneous good by the use of an input factor and sells it. The good will serve as numeraire throughout the analysis. The firm's technology is represented by the production function $f : \mathbb{R}_+ \to \mathbb{R}_+, x \mapsto f(x)$, where x denotes the used factor amount. It is assumed that f is strictly increasing, strictly concave and thrice continuously differentiable. At date $t = 1$, when the firm's production decision has to be taken, the price of the input factor is uncertain. It is represented by a real-valued random variable $\tilde{z}$ that assumes values in $\Omega = [\underline{z}, \bar{z}]$, where $0 < \underline{z} < \bar{z} < \infty$. From this perspective profit in $t = 2$ is therefore uncertain. However, prior to making its decision, the firm observes an information signal y, which is correlated with the realization of the future factor price z. Thus, the relevant expectation for $\tilde{z}$ is the updated posterior belief. In addition, the firm has access to a futures market which allows it to hedge the factor price risk. The futures market opens at period $t = 1$ after the information signal y has been observed. A futures contract pays one unit of the factor at period $t = 2$. Hence, the payoff is worth z. Let $h \in \mathbb{R}$ denote the forward commitment of the firm, i.e. h denotes the number of futures contracts purchased (or sold, if negative). It is assumed that the terms of forward contracting are unbiased, which implies that the futures market clears at price

$$z(y) := \mathrm{E}\left[\tilde{z}|y\right], \tag{1}$$

i.e. the conditional mean of a contract's payoff. Both the payoff and the purchase price of the contract fall due in the second period. The timing of events is outlined in Figure 1.

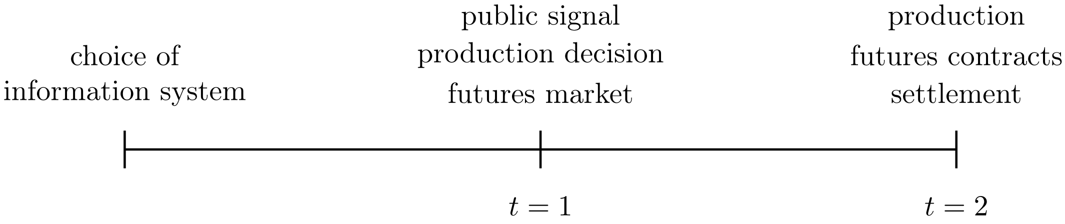

Figure 1: The timing of events.

2.1 Production Decision

In period $t = 1$ the firm has to make its production decision after observing the signal y but before the realization of the factor price. It buys h contracts on the futures market to hedge the risk and its random profit is given by

$$\tilde{\Pi} = f(x) - \tilde{z}x + h(\tilde{z} - z(y)). \tag{2}$$

Let $U : \mathbb{R}_+ \to \mathbb{R}, \Pi \mapsto U(\Pi)$ be the firm's von Neumann-Morgenstern utility function and assume that U is strictly increasing, strictly concave and twice continuously differentiable. The firm maximizes expected utility by solving

$$\max_{h,x} \mathrm{E}[U(\tilde{\Pi})|y] \tag{3}$$

for optimal futures commitment and factor use. The necessary first-order conditions, which are also sufficient, are

$$\mathrm{E}[U'(\tilde{\Pi})(f'(x) - \tilde{z})|y] = 0, \tag{4}$$

$$\mathrm{E}[U'(\tilde{\Pi})(\tilde{z} - z(y))|y] = 0. \tag{5}$$

The solutions to (4) and (5) are given by

$$f'(x^*) = z(y), \tag{6}$$

$$h^* = x^*. \tag{7}$$

The firm fully hedges the factor price risk on the futures market, buying a number of futures contracts equal to the factor amount used for production.[1] Notice that

[1] The use of (5) to derive complete hedging is not really necessary. Note that if the futures price is actuarial fair, then the firm's expected profit is $f(x) - z(y)x$ for any h. Since setting $h = x$ allows it to reach $f(x) - z(y)x$ with certainty, the assumption of risk aversion directly implies that this is the optimal level.

the optimal factor amount, which is implicitly determined by (6), depends on the observed signal y only via the conditional expectation $z(y)$.[2] Therefore it will be denoted by $x(z(y))$ in the following. Substituting (7) into (2), profit in the second period is therefore given by

$$\Pi(z(y)) = f(x(z(y))) - z(y)x(z(y)), \tag{8}$$

which also depends only on the conditional expected factor price. To assure that the optimal factor use $x(z(y))$ is in fact positive for every signal realization y, assume that $f'(0) > \bar{z}$. Since f' is strictly decreasing, it follows from (6) that $x(z(y))$ is decreasing in $z(y)$.[3] This means that the smallest factor amount is chosen when the factor price is expected to be the highest possible, $\bar{z}$, and the highest amount will be chosen if the price is expected to be the lowest possible, $\underline{z}$. The optimal factor amount is therefore bounded from below by $\underline{x}$ and from above by $\bar{x}$, i.e. $x(z(y)) \in [\underline{x}, \bar{x}] \subset \mathbb{R}_{++}$ for all $z(y) \in [\underline{z}, \bar{z}]$, where the two constants $\underline{x}$ and $\bar{x}$ are implicitly defined by

$$f'(\underline{x}) = \bar{z}, \tag{9}$$

$$f'(\bar{x}) = \underline{z}. \tag{10}$$

Next the generation of the signal by an information system will be explained and the notion of informativeness introduced.

2.2 Information Systems

Before the first period an information system is chosen which generates the public information signal. This information system, denoted by (G, Y), specifies for each state of nature $z \in \Omega$ a conditional probability function over the set of signals Y, where Y is a subset of some Euclidian space. Let $g : Y \times \Omega \to \mathbb{R}_+, (y, z) \mapsto g(y|z)$ be the conditional probability density for the realization of signal y given factor price z. The firm knows the information system (G, Y) by which the signal is

[2] This resembles the result of Feder, Just and Schmitz (1980). They show that in a similar framework with output price uncertainty the production decision also depends only on the futures price. Moreover, the firm fully hedges the risk if the subjective price expectation equals the futures price.

[3] This leads directly to a monotone comparative static result: A higher signal induces a lower factor choice if and only if it implies a higher conditional expected value. A necessary condition for the latter property is that the conditional signal distributions satisfy the monotone likelihood ratio property (Milgrom 1981).

generated. On the basis of observing y it can therefore update its prior belief $\pi : \Omega \to \mathbb{R}_+, z \mapsto \pi(z)$ using Bayes' rule. The density $\mu : Y \to \mathbb{R}_+, y \mapsto \mu(y)$ denotes the density of the prior signal occurrence distribution and is given by

$$\mu(y) = \int_{\Omega} g(y|z)\pi(z)dz \quad \forall\, y \in Y. \tag{11}$$

It is assumed that $\mu(y) > 0$ for all $y \in Y$ since any signal with zero probability density could obviously be deleted from the information system. The density function of the updated posterior distribution $\nu : \Omega \times Y \to \mathbb{R}_+, (z, y) \mapsto \nu(z|y)$ is then well-defined by

$$\nu(z|y) = \frac{g(y|z)\pi(z)}{\mu(y)} \quad \forall\, z \in \Omega\ \forall\, y \in Y. \tag{12}$$

Hence, the conditional expectation of the future factor price takes the form

$$z(y) = \mathrm{E}[\tilde{z}|y] = \int_{\Omega} z\, \nu(z|y)dz \quad \forall\, y \in Y. \tag{13}$$

Blackwell (1951, 1953) defines a criterion that compares the informativeness of different information systems. He formalizes the following intuitive idea: if the transmission of a signal generated by an information system is garbled by a stochastic transformation uncorrelated with the true state of nature it loses informativeness. Therefore he regards an information system as more informative than an alternative one if the latter can be generated from the former by adding some random transmission error.

Definition 1 *Let (G, Y) and $(\bar{G}, \bar{Y})$ be two information systems. (G, Y) is said to be more informative than $(\bar{G}, \bar{Y})$, denoted $(G, Y) \succ^i (\bar{G}, \bar{Y})$, if there exists an integrable function $\gamma : \bar{Y} \times Y \to \mathbb{R}_+, (\bar{y}, y) \mapsto \gamma(\bar{y}, y)$ such that the two following conditions hold:*

$$\forall\, y \in Y : \quad \int_{\bar{Y}} \gamma(\bar{y}, y)d\bar{y} = 1, \tag{14}$$

$$\forall\, \bar{y} \in \bar{Y}\ \forall\, z \in \Omega : \quad \bar{g}(\bar{y}|z) = \int_{Y} \gamma(\bar{y}, y)g(y|z)dy. \tag{15}$$

In the definition above the transformation of signals from Y into signals in $\bar{Y}$ is represented by the term $\gamma(\bar{y}, y)$ in (15). It can be interpreted as the conditional probability density of receiving signal $\bar{y}$ when actually signal y was sent.

The following characterization of Blackwell's informativeness criterion facilitates the analysis. A proof can be found in Hermelingmeier (2008).

Lemma 1 *If the information system* (G, Y) *is more informative than the information system* $(\bar{G}, \bar{Y})$*, the distribution of* $z(\tilde{y})$ *is a mean-preserving spread of the distribution of* $z(\tilde{\bar{y}})$*.*

Lemma 1 states that the conditional expected factor prices are more dispersed in the sense of Rothschild and Stiglitz (1970) if the information system is more informative. This reflects more sensitivity of the posterior belief with respect to changes in the observed signal as it carries more information.

3 Transparency, Production and Profit

Factor price transparency is modeled by the informational content of the public signal generated in the first period: the factor price transparency is said to be higher if more precise information about the future factor price z can be inferred from observing the signal realization y. That means that the economic environment is characterized as more transparent if risk is reduced through better information rather than through less economic uncertainty per se.

Definition 2 *Let* (G, Y) *and* $(\bar{G}, \bar{Y})$ *be two information systems. The factor price transparency is said to be higher under* (G, Y) *than under* $(\bar{G}, \bar{Y})$ *if* $(G, Y) \succ^{i} (\bar{G}, \bar{Y})$*.*

Now the question is tackled, how factor use, production, cost, and profit are affected as the factor price transparency becomes higher. For this purpose consider an information system (G, Y), generating the signal realization y in the first period. This determines the optimal factor use $x(z(y))$, production amount $f(x(z(y)))$ and profit $\Pi(z(y))$. Moreover, actual cost in optimum is given by

$$c(z(y)) := z(y)x(z(y)). \tag{16}$$

At the time when the information system determining the level of transparency is chosen however, i.e. from ex ante perspective, the signal realization in the first period is random. Therefore the impact of a more transparent factor price on factor use, production, cost and profit is analyzed in a natural way in terms of the ex ante expectation before the signal has been observed. The ex ante average factor use is then defined by the expression

$$X((G, Y)) := E_y[x(z(y))] = \int\limits_{Y} x(z(y))\mu(y)dy. \tag{17}$$

The analog expressions for ex ante production, cost and profit are

$$F((G,Y)) := E_y[f(x(z(y)))], \tag{18}$$

$$C((G,Y)) := E_y[c(z(y))], \tag{19}$$

$$P((G,Y)) := E_y[\Pi(z(y))]. \tag{20}$$

Now, assume the factor price transparency becomes higher, i.e. a more informative information system in the sense of Definition 1 is chosen. The analysis starts with a statement about the average factor use. Depending on the curvature of the marginal product function, it may increase or decline with more transparency.

Proposition 1 *When the factor price transparency becomes higher, the average factor use decreases (increases), if the marginal product function f' is concave (convex) on $[\underline{x}, \bar{x}]$.*

Obviously, this implies that the average optimal factor amount is independent of the chosen information system if the marginal product is linear on that interval. To understand the critical role of the curvature of f' in Proposition 1 consider the determination of the optimal factor amount by (6) again. It says that the firm chooses the factor amount $x(z(y))$ so as to equate marginal cost $z(y)$ and marginal revenue $f'(x(z(y)))$ of the used factor amount. According to Lemma 1, the conditional price expectations, i.e. expected marginal costs, get more dispersed when the factor price becomes more transparent. The ex ante expectation $\mathrm{E}_y[z(y)]$, however, is equal to the unconditional expectation due to the law of total probability, independently of the chosen information system. This means that, in total, the increases and decreases of conditional factor prices are of equal magnitude. The resulting changes of the associated optimal factor amounts, however, depend on how fast marginal returns are diminishing or, more formally, the curvature of f'. From (6) follows that, if f' is concave, increased expected prices have a bigger impact on the optimal factor amount by decreasing it than decreased prices by increasing it. If f' is convex, the impact of decreased expected prices is bigger than that of increased ones. This means, as stated in Proposition 1, that on average the optimal factor amount decreases if f' is concave and increases if f' is convex. If f' is linear on $[\underline{x}, \bar{x}]$, the marginal changes resulting from an increase or decrease in the expected price are of equal magnitude and therefore cancel each other out. Thus, more transparency has no effect on the ex ante factor use in this case.

As will be shown next, the curvature of the production function is of critical importance for the effect of higher transparency on the ex ante production level. Define the function $R_f : \mathbb{R}_+ \to \mathbb{R}_+, x \mapsto R_f(x)$ by

$$R_f(x) := -\frac{f''(x)}{f'(x)} \quad \forall x \in \mathbb{R}_+, \tag{21}$$

which measures the concavity of the production function f. In addition define the function $P_f : \mathbb{R}_+ \to \mathbb{R}, x \mapsto P_f(x)$ by

$$P_f(x) := -\frac{f'''(x)}{f''(x)} \quad \forall x \in \mathbb{R}_+, \tag{22}$$

which measures the curvature of the marginal product function f'. Because f'' is assumed to be negative, it will be negative if and only if f' is concave and positive if and only if it is convex.[4]

Definition 3 *The production function f exhibits decreasing (increasing) concavity if R_f is decreasing (increasing).*

Depending on the trend of the concavity of the production function the ex ante level of production may increase or decline with more transparency.

Proposition 2 *When the factor price transparency becomes higher, the ex ante expected level of production will increase (decrease) if the production function f exhibits decreasing (increasing) concavity on $[\underline{x}, \bar{x}]$.*

The link between the change of the production level and decreasing versus increasing concavity of f depends on two interacting effects. This can be seen with the help of the following fact, which can be found in Kimball (1990).

Lemma 2 *The production function f exhibits decreasing (increasing) concavity if and only if P_f is greater (smaller) than R_f.*

By assumption the production function exhibits diminishing marginal returns. That means an increase in factor use has a smaller impact on production than a decrease. Therefore the spread in the optimal factor amounts caused by more transparency has a negative effect on average production. The strength of this effect can be associated with R_f, since it measures the concavity of f in terms of the relative decline of the marginal product. On the other hand, with a higher level

[4]Note that R_f is formally equivalent to the measure of absolute risk-aversion of Pratt (1964) and Arrow (1971) and P_f to the measure of absolute prudence of Kimball (1990).

of transparency the average factor use for production may increase or decrease according to Proposition 1. The sign and absolute value of P_f can be associated with the direction and magnitude of this change.[5] Now, when P_f is negative, the average factor use declines with higher transparency and both effects work in the same direction, decreasing the ex ante level of production. When P_f is positive, the two effects counteract because the average factor use increases. Proposition 2 states that in this case the dominating effect can be determined by directly comparing P_f and R_f. If P_f is smaller than R_f the effect of diminishing marginal returns is dominating and the ex ante production level is therefore decreasing. If P_f is greater than R_f the effect of increasing average factor use is dominating and the level of production increases. If P_f and R_f are equal on $[\underline{x}, \bar{x}]$, however, both effects will cancel each other out so that ex ante production will not change. This is for example the case if the production function is assumed to be $f(x) = -\exp(-x) + 1$, where $P_f(x) = 1 = R_f(x)$ for all $x \in \mathbb{R}_+$.

Next, the impact of a more transparent factor price on the average cost level will be characterized. As it turns out, the curvature of the production function is again of critical importance. Define the function $\hat{R}_f : \mathbb{R}_+ \to \mathbb{R}_+, x \mapsto \hat{R}_f(x)$ by

$$\hat{R}_f(x) := \frac{R_f(x)}{f'(x)} = -\frac{f''(x)}{(f'(x))^2} \quad \forall x \in \mathbb{R}_+. \tag{23}$$

It is an alternative measure for the concavity of f, different from R_f. Following Eckwert and Zilcha (2008), this allows for a refinement of the concept of decreasing concavity.

Definition 4 *The production function f exhibits moderately (strongly) decreasing concavity if it exhibits decreasing concavity and $\hat{R}_f$ is increasing (decreasing).*

Depending on the type of curvature of the production function average expenditure for the used input factor amount may increase or decrease in response to higher transparency.

Proposition 3 *The ex ante average cost increases (decreases) with higher transparency if the production function f exhibits strongly decreasing (moderately decreasing or increasing) concavity on $[\underline{x}, \bar{x}]$.*

[5]Remember that f' is concave (convex) if P_f is uniformly negative (positive). Moreover, the effect stated in Proposition 1 is stronger if f' is more concave or more convex, what can be measured in the sense of P_f being lower or higher.

Moderately and strongly decreasing concavity can equivalently be formulated in terms of R_f and P_f. This is again helpful to understand the result in terms of two interacting effects.

Lemma 3 *The production function f exhibits moderately decreasing concavity if and only if P_f is greater than R_f and smaller than $2R_f$. It exhibits strongly decreasing concavity if and only if P_f is greater than $2R_f$.*

With higher transparency the transformation of the input factor into the good becomes more efficient because factor use and factor price are better aligned: for high factor prices the firm chooses a smaller factor amount and for low prices it chooses a higher one. Consequently, this effect decreases the average expenditure. On the other hand, the average factor use may increase or decline. This may therefore cause a positive or negative effect on the average cost of the optimal factor amount. If P_f is negative, average factor use decreases and both effects work in the same direction, decreasing ex ante cost. By contrast, if P_f is positive, the two effects counteract because the average factor use increases. Proposition 3 reveals that when P_f is smaller than $2R_f$, the efficiency-effect is dominating and the ex ante cost level decreases. For the case P_f being greater than $2R_f$, the effect of increasing average factor use is dominating and therefore average cost increases. More transparency has no effect on average cost if both effects cancel each other out. This is the case if P_f is equal to R_f on $[\underline{x}, \bar{x}]$. This holds for example for the function $f(x) = ln(x + exp(-1)) + 1$.

Surprisingly in view of the results already presented, the ex ante expected profit always benefits from higher factor price transparency.[6]

Proposition 4 *The ex ante expected profit increases, when the factor price transparency becomes higher.*

Due to (8) profit is given by the difference of production amount and cost. From ex ante perspective this gives

$$P((G, Y)) = F((G, Y)) - C((G, Y)). \tag{24}$$

As already discussed above, increasing (decreasing) average factor use has a positive (negative) effect on average production and average cost. But the total

[6]Notice again that the result is not just an implication of the famous theorem by Blackwell (1951, 1953). This is not applicable here because the observed signal determines not only the posterior belief but also the futures price depends on it.

effect on the production level is also affected by the negative effect of diminishing marginal returns and on the cost level by the efficiency-effect. According to propositions 2 and 3, the efficiency-effect is dominating an increase in average factor use for a larger range of values of P_f than the effect of diminishing marginal returns. Because it is always the stronger effect, the difference of ex ante production and cost, i.e. average profit, always increases, as stated in Proposition 4.

According to propositions 1 to 4, the impact of higher factor price transparency can be summarized in four different cases, classified by the relation of P_f and R_f. This is illustrated in Figure 2. A decrease of ex ante factor use X, production F, cost C and profit P is marked dark grey, while an increase is marked light grey.

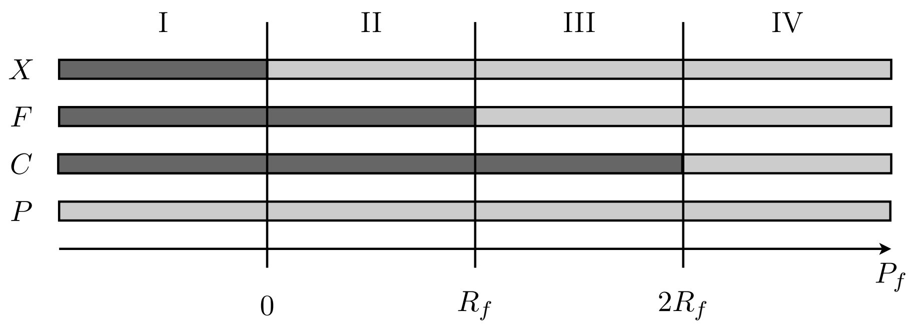

Figure 2: Impact of higher transparency.

When $P_f(x) \in \mathbb{R}_-$ for all $x \in [\underline{x}, \bar{x}]$, case I, the average factor use decreases. Supplemented by diminishing marginal returns and efficiency-effect, also ex ante production and cost are decreasing on average. Because the latter effect is the stronger one, average profit increases. When $P_f \in [0, R_f]$, case II, average factor use is increasing but due to the effect of diminishing marginal returns, production is still declining. Moreover, because of the strong efficiency-effect, also the average cost of the used factor amount is going down. If $P_f \in [R_f, 2R_f]$, case III, average factor use is increasing stronger and ex ante production is also increasing. Average cost is still declining because of the dominating efficiency-effect. In the last case, when $P_f \in [2R_f, \infty)$, case IV, average factor use is heavily increasing. This leads not only to a higher ex ante production level but also to higher average cost. This is because the efficiency gain is no longer high enough to cover the cost for the additional amount of the factor used.

4 Conclusion

The purpose of this paper was to examine the role of information about random input prices for a competitive firm. It turned out that the impact of more transparency on optimal factor use, production amount and cost depends crucially on the strength of the curvatures of total and marginal product. By contrast, ex ante profit always increases.

If the production function is considered to be of the Cobb-Douglas type, one can easily use the presented results to show that all terms under consideration increase. This means that the other three cases are ruled out under this often used specification. Hence, a more general production function may lead to considerable new effects in related models. Moreover, for a Cobb-Douglas type production function with decreasing returns to scale the analysis can easily be extended to two input factors where the price of one of them is random. More transparency leads in this case also to increasing factor demand, production, cost and profit.

In view of the presented results differences in production technologies of different economic sectors may account for converse changes in different firms' factor use, production amount and cost in reaction to more transparency. The results do not necessarily support the common conjecture that more transparency leads to more economic activity per se.

A Appendix

A.1 Proofs

Lemma 1 is used to prove propositions 1 to 4. Further, a mean preserving spread increases (decreases) the expectation of any convex (concave) function (Rothschild and Stiglitz 1970). Therefore it suffices to derive conditions under which the terms under consideration are convex (concave) in the expected future factor price $z(y)$. For notational convenience a number of arguments is omitted.

Proof of Proposition 1: Implicit differentiation of (6) leads to

$$x' = \frac{\partial x(z(y))}{\partial z(y)} = \frac{1}{f''} < 0, \tag{25}$$

so that

$$x'' = \frac{\partial^2 x(z(y))}{\partial (z(y))^2} = -\frac{f'''}{(f'')^3} \tag{26}$$

and therefore

$$sign\{x''\} = sign\{f'''\}. \tag{27}$$

This yields that $x(z(y))$ is concave (convex) in $z(y)$ if the marginal product function f' is concave (convex). □

Proof of Proposition 2: Differentiating the production function at optimal factor use leads to

$$\frac{\partial f(x(z(y)))}{\partial z(y)} = f'x' = \frac{f'}{f''} = -\frac{1}{R_f(x(z(y)))}. \tag{28}$$

Since $x(z(y))$ is decreasing in $z(y)$, it follows that $f(x(z(y)))$ is concave (convex) in $z(y)$ if R_f is increasing (decreasing). □

Proof of Lemma 3: To prove the claim, take the logarithmic derivative

$$\frac{\hat{R}'_f(x)}{\hat{R}_f(x)} = \frac{d}{dx}\ln\left(\frac{-f''(x)}{(f'(x))^2}\right) = \frac{f'''(x)}{f''(x)} - \frac{2f''(x)}{f'(x)} = 2R_f(x) - P_f(x) \tag{29}$$

and recall that $\hat{R}_f$ is positive by assumption. □

Proof of Proposition 3: Differentiating (16) yields

$$\frac{\partial c(z(y))}{\partial z(y)} = x(z(y)) + z(y)x', \tag{30}$$

so that with (6) and (26) one gets

$$\begin{aligned}\frac{\partial^2 c(z(y))}{\partial (z(y))^2} &= 2x' + z(y)x'' \\ &= x'\left(2 - \frac{f'''f'}{f''^2}\right) \\ &= x'\left(2 - \frac{P_f(x(z(y)))}{R_f(x(z(y)))}\right).\end{aligned} \tag{31}$$

Thus, having in mind that R_f is positive by assumption and x' negative by (25), it follows

$$\frac{\partial^2 c(z(y))}{\partial (z(y))^2} \gtreqless 0 \quad \Leftrightarrow \quad P_f(x(z(y))) \gtreqless 2R_f(x(z(y))). \tag{32}$$

With Lemma 3 this shows that $c(z(y))$ is convex (concave) in $z(y)$ if f exhibits strongly decreasing (moderately decreasing or increasing) concavity. □

Proof of Proposition 4: Observe that (8) is maximized profit. Therefore the Envelope Theorem yields

$$\frac{\partial \Pi(z(y))}{\partial z(y)} = -x(z(y)) < 0. \tag{33}$$

Further differentiation and (25) lead to

$$\frac{\partial^2 \Pi(z(y))}{\partial (z(y))^2} = -x' > 0. \tag{34}$$

Thus $\Pi(z(y))$ is convex in $z(y)$. □

References

Arrow, K. (1971). The theory of risk aversion. In K. Arrow (Ed.), *Essays in the Theory of Risk Bearing*. Amsterdam: North-Holland.

Athey, S. (2000). Investment and information value for a risk averse firm. MIT Working Paper No. 00-30, Massachusetts Institute of Technology, Cambridge.

Batra, R. N. and A. Ullah (1974). Competitive firm and the theory of input demand under price uncertainty. *Journal of Political Economy 82*, 537–548.

Blackwell, D. (1951). The comparison of experiments. In J. Neymann (Ed.), *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, Berkeley, 93–102. University of California Press.

Blackwell, D. (1953). Equivalent comparison of experiments. *Annals of Mathematical Statistics 24*, 265–272.

Broll, U. and B. Eckwert (2006). Transparency in the foreign exchange market and the volume of international trade. *Review of International Economics 14*, 571–581.

Coes, D. (1977). Firm output and changes in uncertainty. *American Economic Review 67*, 249–251.

Eckwert, B. and I. Zilcha (2003). Incomplete risk sharing arrangements and the value of information. *Economic Theory 21*, 43–58.

Eckwert, B. and I. Zilcha (2008). Improvement in information and private investment in education. Mimeo.

Feder, G., R. E. Just and A. Schmitz (1980). Futures markets and the theory of the firm under price uncertainty. *Quarterly Journal of Economics 94*, 317–328.

Hermelingmeier, C. (2008). A note on comparing information structures using posterior state distributions. EBIM Working Paper 2008-2, Bielefeld University, Bielefeld.

Kimball, M. S. (1990). Precautionary saving in the small and in the large. *Econometrica 58*, 53–73.

Milgrom, P. R. (1981). Good news and bad news: Representation theorems and applications. *Bell Journal of Economics 12*, 380–391.

Milgrom, P. R. (1994). Comparing optima: Do simplifying assumptions affect conclusions? *Journal of Political Economy 102*, 609–615.

Pratt, J. W. (1964). Risk aversion in the small and in the large. *Econometrica 32*, 122–136.

Rothschild, M. and J. E. Stiglitz (1970). Increasing risk: I. A definition. *Journal of Economic Theory 2*, 225–243.

Sandmo, A. (1971). On the theory of the competitive firm under price uncertainty. *American Economic Review 61*, 65–73.