

# Sophistication in Risk Management, Bank Equity, and Stability\*

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## Abstract

We investigate the question of whether sophistication in risk management fosters banking stability. We compare a simple banking system in which an average rating is used with a sophisticated banking system in which banks are able to assess the default risk of entrepreneurs individually. Both banking systems compete for deposits, loans, and bank equity. While a sophisticated system rewards entrepreneurs with low default risks with low loan interest rates, a simple system acquires more bank equity and finances more entrepreneurs. Expected repayments in a simple system are always higher and its default risk is lower if productivity is sufficiently high. Expected aggregate consumption of entrepreneurs, however, is higher in a sophisticated banking system.

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# 1 Introduction

The popular view of banking regulation is that more sophistication in rating and risk management increases the stability of banking systems. This view has motivated the introduction of modern risk management techniques during the last decade. While such techniques which transform risks of an economy are well established in capital markets<sup>1</sup>, the application of sophisticated rating tools by commercial banks is a more recent development. From the perspective of a single institution, it is clear that costless sophistication of risk management techniques is always beneficial. However, whether or not this holds for the economy as a whole is a largely unresolved issue.

In this paper we argue that the conclusions drawn from a systemic perspective may be quite different from those derived from the perspective of a single institution. We investigate the issue of to what extent the investment behavior of banks and the stability of the banking system will change with the introduction of more sophistication in rating and risk management. We consider a competitive banking system in which banks offer their intermediation services to a population of entrepreneurs who have three investment opportunities. They may either invest their initial wealth into a production project which is subject to macroeconomic risks, into bank equity, or into an alternative investment opportunity with an exogenously given return. Banks compete for equity and deposits and offer loans as delegated monitors. Risk premia on loans are determined by the equity market as banks need to offer sufficient returns in order to attract equity. We compare two polar cases, a simple and a sophisticated banking system. In the simple system banks are unable to rate the risk of a production project individually. They attribute the same default probability to each borrower and thus use the same rating. In the sophisticated system banks use an infinitely fine rating system in which each borrower is individually rated. Loan interest rates are tailored according to the default probability of an entrepreneur.

Our main findings are as follows. First, relative to the simple banking system in which all entrepreneurs are offered the same credit terms, a sophisticated system rewards high-quality entrepreneurs with low interest rates and penalizes low-quality entrepreneurs with high interest rates. Since credit terms for low-quality entrepreneurs are more favorable, loan demand in a simple banking system is higher than in a sophisticated system. Second, simple banks attract more equity while in a sophisticated system more resources are invested into the alternative investment opportunity. This confirms

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<sup>1</sup>A large body of literature investigates the consequences of modern risk management techniques for capital markets which have risen dramatically over the last decades, cf. Carey & Stulz (2006).

concerns that sophisticated rating as imposed by the Basel II regulatory framework will make it difficult for middle-sized firms to obtain loans.

Third, aggregate repayments to the simple banking system are on average higher than to the sophisticated system. As simple banks acquire more equity, their capital buffer against adverse macroeconomic productivity shocks is larger. Hence, although sophisticated banks have higher average-quality projects, they are less capable of absorbing adverse productivity shocks. Moreover, the default risk of the sophisticated system may even be higher. Sophistication in risk management may therefore have a destabilizing effect on the banking system. Fourth, expected aggregate consumption of entrepreneurs in an economy with a sophisticated banking system is on average higher than aggregate consumption in an economy with a simple banking system. This shows that sophistication in rating may entail a tradeoff between stability and expected aggregate consumption of entrepreneurs.

The approach of this paper is complementary to the work of Gehrig & Stenbacka (2004) who show that uncoordinated screening behavior of competing financial intermediaries creates a financial multiplier and may be an independent source of fluctuations. In analyzing the systemic effects of screening activities by firms, this paper contributes to the literature on screening by banks surveyed, for example, in Freixas & Rochet (1997). The focus of this paper is more on the consequences for market conditions and systemic defaults when banks introduce more sophisticated rating tools. An interesting question for future research is how even more sophisticated risk management techniques such as the securitization of bank loans including the use of derivative products affect systematic risks as discussed in Franke & Krahen (2006).

Our results are related to the literature on banking regulation. Comprehensive surveys with different emphases are given by Bhattacharya & Thakor (1993), Dewatripont & Tirole (1994), Hellwig (1994), Freixas & Rochet (1997), or Bhattacharya, Boot & Thakor (1998). Overall, we suggest that increased sophistication in rating<sup>2</sup> as advocated in the new banking regulatory framework (Basel II) may produce unintended negative side effects. Indeed, the analysis of our paper indicates that increased sophistication in banking may create more instability in the long run.

The paper is organized as follows: In the next section we introduce the model and both types of banking systems. In Section 3 we examine simple banks, and in Section 4 we perform the mirror-image of the analysis for sophisticated banks. In Section 5 both systems are compared and our main results are presented. Section 6 concludes.

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<sup>2</sup>Krahen & Weber (2001) develop a comprehensive set of intuitive rating principles.

## 2 Model

### 2.1 Consumers and entrepreneurs

We consider a two-period model with periods  $t = 1$  and  $t = 2$ . The population of agents consists of a continuum indexed by  $[0, 1]$ . Each agent has individual wealth  $W$  in the first period. Agents are divided into two classes. One fraction of agents, indexed by  $[0, \eta]$  with  $0 < \eta < 1$ , are potential entrepreneurs. The other fraction, indexed by  $(\eta, 1]$ , are consumers. Potential entrepreneurs and consumers differ in that only the former have access to investment technologies.

Consumers are endowed with intertemporal preferences over consumption in the two periods of their lives. Let  $u(c_1, c_2)$  be a standard intertemporal utility function of a consumer, with  $c_1, c_2$  denoting first-period and second-period consumption of a consumer, respectively. Given the endowment  $W$  in the first period and a deposit interest rate  $r^d$ , each young consumer saves the amount  $s(r^d)$ . Aggregate savings of all consumers are then denoted by  $S(r^d)$  and given by  $S(r^d) = (1 - \eta)s(r^d)$ . Throughout this paper, we assume that the deposit rate  $r^d$  is equal to an exogenously given return  $r_A > 0$  of an alternative investment opportunity<sup>3</sup>, so that aggregate savings become  $\bar{S} = S(r_A)$ . The alternative investment opportunity may be thought of as an outside option, such as government bonds or investments in other sectors of the economy that are not modeled explicitly.

Potential entrepreneurs are assumed to be risk-neutral and to consume only in the second period. Each entrepreneur has to decide whether to invest in a production project that converts period-1 goods into period-2 goods, to provide bank equity for banks, or to invest her funds in the alternative project with return  $r_A$ . The funds required for each production project are fixed at  $W + I$ , so that an entrepreneur must borrow  $I$  additional units of the good from banks to undertake the production project. Entrepreneurs are heterogeneous in the quality of their production projects which depends on their index  $i$ . The quality parameter of entrepreneur  $i$  is assumed to be private information. If an entrepreneur of type  $i$  obtains additional resources  $I$  and decides to invest, investment returns in the second period amount to

$$y = q(1 + i)f(W + I),$$

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<sup>3</sup>This assumption can also be justified by arbitrage opportunities banks would have in case  $r^d < r_A$ . Under such circumstances, banks could simply collect funds from consumers and invest them into the alternative investment opportunity. Granting loans could then become non-profitable and financial intermediation would break down.

where  $f$  denotes a standard atemporal neoclassical production function and  $q > 0$  represents an exogenous macroeconomic productivity shock in the economy. Since  $W$  and  $I$  will remain fixed throughout the paper, we write  $f = f(W + I)$ . The distribution of shocks  $q$  is assumed to be given by a continuous density function  $h(q)$  with support on a compact interval  $[\underline{q}, \bar{q}]$  with  $0 < \underline{q} < \bar{q}$ .

Entrepreneurs are price-takers and operate under limited liability. Given a loan interest rate  $r^c$ , the expected profit of an investing entrepreneur  $i$  is

$$\Pi(i, r^c) := \int_{\underline{q}}^{\bar{q}} \max\{q(1+i)f - I(1+r^c), 0\} h(q) dq. \quad (1)$$

Note that  $\Pi(i, r^c)$  is non-decreasing in quality levels  $i$  and non-increasing in loan rates  $r^c$ . Denote by  $r^e$  the expected return on equity banks offer to their potential equity holders. A risk-neutral entrepreneur with the quality parameter  $i \in [0, \eta]$  will prefer to invest in the production project rather than into equity or the alternative project if the return on the production project is expected to be larger than the return on either equity or the alternative investment project, i.e., if

$$\Pi(i, r^c) \geq W(1 + \max\{r^e, r_A\}). \quad (2)$$

Finally, we assume that savings are never sufficient to fund all entrepreneurs. Since aggregate savings  $\bar{S} = S(r_A)$  is bounded by  $(1 - \eta)W$ , this condition takes the form

$$(1 - \eta)W < \eta I. \quad (3)$$

## 2.2 Banking Sector

Following Gersbach & Wenzelburger (2004), we assume that depositors cannot observe the quality parameters of entrepreneurs and cannot verify whether or not an entrepreneur invests. The existence of such market frictions necessitates financial intermediation (see e.g. Hellwig 1994). To alleviate these information problems, we assume that there are  $n$  (commercial) banks, indexed by  $j = 1, \dots, n$  ( $n > 1$ ). They monitor loans as delegated monitors in the sense of Diamond (1984) and their monitoring is assumed to be efficient in that they are able to secure both the investment of an entrepreneur and the liquidation value in case of default, cf. Gersbach & Uhlig (2005).

To avoid that properties other than the ability to rate entrepreneurs determine the results, we make the same assumption regarding the competitive structure of the banking

sector. First, both banking systems compete for equity and loans while facing a given deposit interest rate  $r_A$ . Each bank  $j$  can offer deposit contracts  $D(r_A)$ , where  $1 + r_A$  is the repayment offered for one unit of resources. Deposits are implicitly insured by bail-out guarantees of a government.

Second, banks raise equity by issuing equity contracts. An equity contract specifies that the holder is entitled to obtain a share of dividends in proportion to the resources he has given to a particular bank. By providing equity, an entrepreneur becomes a shareholder of a bank. A bank becomes a legal entity and can only operate if it obtains a sufficiently positive amount of equity.<sup>4</sup> As entrepreneurs own banks, banks are risk neutral as well. Third, both banking systems are perfectly competitive. Bank owners have the opportunity to invest in the alternative project with return  $r_A > 0$ .

Competition among banks determines the level of equity and loan interest rates. We distinguish between a simple and a sophisticated banking system which differ only in their ability to rate the quality of an entrepreneur's production project.

1. *Simple Banking System.* The essential feature of the simple banking system is that banks are unable to rate entrepreneurs individually and to adjust loan contracts to the quality parameter  $i$  of an entrepreneur. Banks only have an average rating of entrepreneurs and offer all entrepreneurs the same loan contract  $C(r^c)$ , where  $1 + r^c$  is the repayment required from entrepreneurs for one unit of borrowed resources.
2. *Sophisticated Banking System.* In a sophisticated banking system, banks are able to rate each entrepreneur individually and to offer entrepreneur-specific loan contracts, denoted by  $C(r_i^c)$ , where  $r_i^c$  is the loan interest rate demanded from an entrepreneur of type  $i$ .

In both banking systems, banks operate under unlimited liability and loans are only constrained by the available amount of equity and deposits. We assume throughout that aggregate uncertainty is canceled out when depositors and entrepreneurs randomly choose banks.<sup>5</sup> As all banks are identical, they will obtain the same amount of equity and deposits in a particular banking system.

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<sup>4</sup>Otherwise the legal entity is not founded as there are no owners. As banks take all the equity they can obtain, the equilibrium value will be determined by the supply of equity.

<sup>5</sup>The exact construction of individual randomness so that this statement holds can be found in Alós-Ferrer (1999). We could also rely on the weaker forms of the strong law of large numbers developed in Al-Najjar (1995) and Uhlig (1996), where independence of individual random variables can be assumed and aggregate stability is the limit of an economy with finite characteristics.

With these assumptions, the financial intermediation process in either system is as follows. Given  $r_A$ , banks in the first period offer equity contracts and loan contracts  $r^c$  (simple banking) or loan contracts  $\{r_i^c\}_{i=0}^\eta$  (sophisticated banking), respectively. Each bank  $j$  obtains an amount of  $d^j$  in equity and an equal share of deposits from consumers. Entrepreneurs decide which contracts to accept. Resources are exchanged. In the second period, funded entrepreneurs produce subject to a macroeconomic shock and pay back loans with limited liability. Banks repay depositors and shareholders.

We are now ready to investigate to what extent the ability of a competitive banking system to rate firms reduces its vulnerability to entrepreneurial bankruptcies. To this end, we will compare how equity develops in both types of banking systems. We are particularly interested in the distribution and downside risk of equity in the second period. It is intuitively clear that the lower accumulated equity is in period 2, the more the stability of the banking sector is endangered.

### 3 Competitive Equilibria for Simple Banks

We motivate the equilibrium concept for a simple banking system in which banks are unable to rate entrepreneurs individually. Recall that each entrepreneur owns  $W$  units of funds that she can invest either into a production project, into equity of the banking system or into the alternative investment project. If the total investment of entrepreneurs into the banking system is  $d_*$ , the amount of equity of an individual bank is  $\frac{d_*}{n}$ . As all banks are assumed to be identical, the equilibrium conditions will be formulated for the whole banking system.

There are two restrictions on the amount of equity entrepreneurs have to supply to banks. First, equity has to balance loan demand and second, it has to meet capital adequacy rules. Formally, these conditions can be described as follows. Let  $i_* \in [0, \eta]$  be the equilibrium critical quality level such that all entrepreneurs  $i \in [i_*, \eta]$  invest in their production project and all entrepreneurs  $i \in [0, i_*)$  either provide bank equity or invest in the alternative investment project. If banks want to satisfy loan demand, their equity  $d_*$  will have to satisfy

$$\bar{S} + d_* = [\eta - i_*]I, \quad (4)$$

stating that loan demand equals supply, and

$$d_{\text{reg}} \leq d_* \leq i_*W, \quad (5)$$

where  $d_{\text{reg}} > 0$  is a prescribed minimum equity level banks must acquire in order to be allowed to operate. We assume that  $d_{\text{reg}}$  is prescribed by some *capital-adequacy rule* which is understood to be the requirement that the banking system has to fulfill

$$\frac{d_*}{[\eta - i_*]I} \geq \alpha, \quad (6)$$

where  $0 \leq \alpha \leq 1$  is some small positive number (e.g.  $\alpha = 8\%$  as in Basel I). The capital requirement (6) defines a minimum equity level of the banking system as a percentage  $\alpha$  of the credit volume  $[\eta - i_*]I$ . Inserting (4) into (6), we see that the minimum equity level the banking system has to acquire is given by

$$d_{\text{reg}} = \frac{\alpha}{(1-\alpha)}\bar{S}. \quad (7)$$

Using (4), the constraints in (5) translate into two restrictions on the critical quality level  $i_*$ , namely that  $i_* \in [\underline{i}, \bar{i}]$  with

$$\underline{i} := \frac{\eta I - \bar{S}}{I + W} \quad \text{and} \quad \bar{i} = \eta - \frac{\bar{S}}{(1-\alpha)I}. \quad (8)$$

Thus, at least  $\underline{i}$  entrepreneurs must provide their endowments for equity in order to meet (4). Given  $i_* \in [\underline{i}, \bar{i}]$ , the remaining resources  $i_*W - d_* \geq 0$  will be invested into the alternative investment project, where  $d_* = \underline{i}W$  for  $i_* = \underline{i}$ .

### 3.1 Equilibrium Concept

We now describe the equilibrium concept. Let  $r^d = r_A \geq 0$  be the deposit interest rate of the banking system which is the same for all banks and identical to the interest rate of the alternative investment project. Banks receive funds  $\bar{S}$  from consumers which have to be paid back with interest at the end of the second period. Given a return on equity  $r^e$  and a loan interest rate  $r^c$  which is charged to all investing entrepreneurs, (2) determines a critical quality level  $i_*$  so that all entrepreneurs  $i < i_*$  provide equity or invest into the alternative investment project and all entrepreneurs  $i \geq i_*$  ask for loans at banks. Hence, aggregate loan demand of firms becomes  $[\eta - i_*]I$  so that payments to the banking system  $P = P(q, i_*, r^c)$  at the end of the second period are

$$P(q, i_*, r^c) = \int_{i_*}^{\eta} \min\{q(1+i)f, I(1+r^c)\} di \quad (9)$$

and their equity level in the second period is

$$G(q, i_*, r^c) = P(q, i_*, r^c) - \bar{S}(1+r_A). \quad (10)$$

This equity level is payed out to the banks' equity holders. Given a critical quality level  $i_*$  and a loan interest rate  $r^c$ , the expected second-period equity of the banking system is

$$\begin{aligned}\mathbb{E}[G(\cdot, i_*, r^c)] &= \mathbb{E}[P(\cdot, i_*, r^c)] - \bar{S}(1 + r_A) \\ &= \int_{\underline{q}}^{\bar{q}} P(q, i_*, r^c) h(q) dq - \bar{S}(1 + r_A)\end{aligned}\tag{11}$$

and the expected return on equity is given by the function<sup>6</sup>

$$g(i_*, r^c) = \frac{\mathbb{E}[G(\cdot, i_*, r^c)] - d_*}{d_*}.\tag{12}$$

After these preparations, we may define a competitive equilibrium for a simple banking system with unlimited liability, that is, a system in which banks are fully accountable for possible losses. Intuitively, a competitive equilibrium is a critical quality level, an equity level, and a loan interest rate  $(i_*, d_*, r_*^c)$  such that

- (i) it is optimal for banks to offer their intermediation services,
- (ii) entrepreneurs take optimal investment decisions,
- (iii) the market for loans is balanced.

Formally, a competitive equilibrium for a simple banking system with unlimited liability is defined as follows.<sup>7</sup>

**Definition 1**

Let  $r_A \geq 0$  be given. A competitive equilibrium of a simple banking system which operates under unlimited liability is a triplet  $(i_*, d_*, r_*^c)$  such that

$$\Pi(i_*, r_*^c) = W [1 + g(i_*, r_*^c)],\tag{13}$$

$$r_A \leq g(i_*, r_*^c),\tag{14}$$

$$[\eta - i_*] I = \bar{S} + d_*,\tag{15}$$

$$d_{\text{reg}} \leq d_* \leq i_* W.\tag{16}$$

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<sup>6</sup>Expected net profit of shareholders is  $\mathbb{E}[G(\cdot, i_*, r^c)] - d_*$ .

<sup>7</sup>An equilibrium notion for a simple banking system with limited liability is found in Gersbach & Wenzelburger (2004). While the interest rate margins in such systems will generally be larger, they are more complex to analyze. The qualitative results of this paper, however, should not be affected.

Equation (13) is the indifference condition for the critical quality level  $i_*$ , which determines the demand for loans. The equilibrium conditions force a spread  $r_*^c - r_A$  that accounts for the risk of loan losses. This risk premium compensates banks for the risk of defaulting entrepreneurs. Equation (15) is the equilibrium condition for loan demand and deposits already given in (4) which determines the required equity level. Condition (16) ensures that enough entrepreneurs will provide equity so that the banking system has enough funds to finance production projects and ensures minimum capital requirements in equilibrium.

Equation (14) requires banks' expected return on equity to be at least as high as the return of the alternative project.<sup>8</sup> If the expected return on equity were lower than  $r_A$ , no equity would be supplied. In this case no bank could operate, since the minimum capital requirement in (16) were violated. If the expected return on equity is strictly higher than  $r_A$ , no entrepreneur will invest into the alternative project, so that, as argued above,  $d_* = \underline{i}W$ . Equilibria with positive investment in the alternative project are those in which  $i_*W > d_*$  and in which the expected return on equity is equal to  $r_A$ , i.e., (14) holds with equality.

### 3.2 Existence of competitive equilibria

Throughout the remainder of this paper, we will focus on competitive equilibria with positive investment into the alternative project because these are economically the more plausible ones. Existence and uniqueness of these equilibria are stated in the following Proposition whose proof is found in the appendix.

#### Proposition 1

Let  $r_A \geq 0$  be given and suppose that the following conditions are satisfied:

(i)  $\Pi(\underline{i}, 0) > W(1 + r_A) > \Pi(\underline{i}, r_A)$ .

(ii) The average output of the entrepreneur with the quality level  $\bar{i}$  as given in (8) satisfies

$$[W + I](1 + r_A) < \mathbb{E}[q](1 + \bar{i})f.$$

Then a simple banking system admits a unique competitive equilibrium  $(i_*, d_*, r_*^c)$ . Moreover,  $d_{\text{reg}} < d_* < i_*W$  and  $g(i_*, r_*^c) = r_A$ , so that the investment into the alternative project is strictly positive.

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<sup>8</sup>Alternatively, this equation may be interpreted as a free-exit condition for banks, cf. Gersbach & Wenzelburger (2006).

Given the productivity of entrepreneurs, Conditions (i) and (ii) involve two restrictions on  $r_A$ . Condition (i) of Proposition 1 requires that entrepreneur  $\underline{i}$  is willing to invest in her production project at a zero loan interest rate while she will invest into equity or into the alternative project if the loan interest rate is higher than  $r_A$ . The proof of the proposition reveals that the second inequality in Assumption (i) forces the return on equity to be equal to  $r_A$ , so that in equilibrium entrepreneurs will invest into the alternative project.

Assumption (ii) states that the average output of entrepreneur  $\bar{i}$  is higher than the return on  $W + I$  at the interest rate of the alternative project  $r_A$ . Hence it is profitable for banks to finance at least high-quality production projects. In the proof of the proposition it will become apparent that the second inequality in Assumption (i) implies  $\mathbb{E}[q](1 + \underline{i})f < [W + I](1 + r_A)$ , showing that the relation between the productivity of entrepreneurs and  $r_A$  determines the investment behavior of entrepreneurs and banks.

### 3.3 Instability

We are now in a position to calculate the default probability of a simple banking system. Writing  $P_*(q) := P(q, i_*, r_*^c)$  for repayments in a competitive equilibrium, average repayments of entrepreneurs are

$$\mathbb{E}[P_*] = [\eta - i_*]I(1 + r_A) \quad (17)$$

and hence are positive. We next consider aggregate losses of the banking system in equilibrium. Using (9), these are formally defined as

$$\begin{aligned} L_*(q) &:= [\eta - i_*]I[1 + r_*^c] - P_*(q) \\ &= \int_{i_*}^{\eta} \max\left\{I[1 + r_*^c] - q(1 + i)f, 0\right\} di. \end{aligned} \quad (18)$$

Expected aggregate losses in equilibrium are

$$\begin{aligned} \mathbb{E}[L_*] &= [\eta - i_*]I[1 + r_*^c] - \mathbb{E}[P_*] \\ &= [\eta - i_*]I[r_*^c - r_A]. \end{aligned} \quad (19)$$

Inserting (15) and (17) and then (18) and (19) into (10), second-period equity of a simple banking system is

$$\begin{aligned} G_*(q) &:= G(q, i_*, r_*^c) = P_*(q) - \bar{S}(1 + r_A) \\ &= P_*(q) - \mathbb{E}[P_*] + d_*(1 + r_A) \\ &= \mathbb{E}[L_*] - L_*(q) + d_*(1 + r_A). \end{aligned} \quad (20)$$

An entrepreneur with quality level  $i$  goes bankrupt if she is unable to fully pay back her credit, that is, if

$$I(1 + r_*^c) > q(1 + i)f.$$

The entrepreneur with the lowest quality level who is not bankrupt after encountering the shock  $q$  is given by

$$i_B = i_B(q) := \begin{cases} \frac{I(1 + r_*^c)}{qf} - 1 & \text{if } q_{\text{TB}} \leq q < q_{\text{NB}}, \\ i_* & \text{if } q \geq q_{\text{NB}}, \end{cases} \quad (21)$$

where

$$q_{\text{NB}} := \frac{I[1 + r_*^c]}{[1 + i_*]f} \quad \text{and} \quad q_{\text{TB}} := \frac{I(1 + r_*^c)}{(1 + \eta)f}. \quad (22)$$

If shocks are sufficiently positive  $q \geq q_{\text{NB}}$ , then no firm goes bankrupt and aggregate losses of banks are zero. For shocks  $q_{\text{TB}} \leq q < q_{\text{NB}}$ , all investing entrepreneurs with quality levels  $i_* < i < i_B(q)$  enter bankruptcy, whereas entrepreneurs with quality levels  $i \geq i_B(q)$  pay back their loans fully. Finally, all entrepreneurs will enter bankruptcy if  $q < q_{\text{TB}}$  and losses are maximal.

It follows directly from (20) that second-period equity is on average positive, so that a simple banking system will not default on average. The probability of a system-wide default by banks can now be calculated as follows. An individual bank is bankrupt if second-period equity is negative. Due to the assumed symmetry of banks, this is equivalent to the condition  $G_*(q) < 0$  stating that the whole banking system is bankrupt. Using (20), this condition takes the form

$$L_*(q) > \mathbb{E}[L_*] + d_*(1 + r_A). \quad (23)$$

By (21) a necessary condition for the default of a bank is  $q < q_{\text{NB}}$ . The default probability for banks can now be determined as follows.

### Proposition 2

*Under the hypotheses of Proposition 1, assume that*

$$L_*(\underline{q}) > \mathbb{E}[L_*] + d_*(1 + r_A) > L_*(\bar{q}),$$

*so that banks may default. Then there exists a unique critical level  $\underline{q} < q_{\text{crit}} < \bar{q}$  for macroeconomic shocks, such that the banking system defaults if and only if  $q < q_{\text{crit}}$ . The default probability is*

$$\pi_{\text{default}} := \text{Prob}\left(L_*(q) > \mathbb{E}[L_*] + d_*(1 + r_A)\right) = \int_{\underline{q}}^{q_{\text{crit}}} h(q) dq. \quad (24)$$

Proposition 2, the proof of which is given in Appendix A, shows that banks default with positive probability as soon as the buffer  $d_*(1 + r_A)$  is too small to insure against negative macroeconomic shocks. Observe that  $q_{\text{crit}} < q_{\text{NB}}$ .

The preceding results allows us to characterize the default probability of the banking system in terms of the underlying exogenous parameters and distributions. The equation (24) is a value-at-risk formula for the banking system and for an individual bank. It can be interpreted in a different way. Suppose that  $\pi_{\text{default}}$  is predetermined, e.g., by banking regulation. Equation (24) determines then the required equity level  $d_*$  such that the default risk is equal to  $\pi_{\text{default}}$ .

## 4 Competitive Equilibria for Sophisticated Banks

### 4.1 Equilibrium concept

We turn to the other polar case in which banks are sophisticated in their rating abilities so that they are able to detect the quality level  $i$  of an individual entrepreneur. They can thus determine the entrepreneur-specific default probability. In accordance with the capital requirements of the first Basel Accord we will assume that the banks' debt/equity ratios are the same across loans. Since the loan size  $I$  is fixed, this implies that each loan is backed by the same amount of equity and deposits. Leaving all assumptions as stated at the outset unchanged, the key difference to the simple banking system now is the requirement that sophisticated banks charge an *actuarially fair interest rate* for each loan. To do so, let

$$\mathcal{R}(i, r_i^c) := \int_{\underline{q}}^{\bar{q}} \min \{q(1+i)f, I(1+r_i^c)\} h(q) dq \quad (25)$$

denote the expected repayment from an entrepreneur with quality level  $i$  who has received a loan size  $I$  at the interest rate  $r_i^c$ . The highest possible average repayment of entrepreneur  $i$  is

$$\mathcal{R}(i, r_i^c) = \mathbb{E}[q](1+i)f$$

which is attained for all loan-interest rates  $r_i^c \geq \frac{\bar{q}(1+i)f}{I} - 1$ .

In requiring banks to charge actuarially fair loan-interest rates, average repayments to banks have to be equal across entrepreneurs. Given some fixed amount  $R_0$  to be repayed by investing entrepreneurs, an actuarially fair interest rate  $r_i^c$  for entrepreneur

$i$  has to be such that

$$\mathcal{R}(i, r_i^c) \stackrel{!}{=} R_0 \quad \text{for all } i \in [i_{\text{low}}, \eta], \quad (26)$$

where  $0 \leq i_{\text{low}} < \eta$  denotes the entrepreneur with the lowest quality level who is able to repay the amount  $R_0$  on average. An average repayment  $R_0$ , which is attainable at least for entrepreneurs with sufficiently high qualities, must therefore satisfy

$$R_0 < \mathbb{E}[q](1 + \eta)f.$$

Since (25) is non-decreasing in quality levels, any interest rates  $r_i^c$  satisfying (26) will have to be non-increasing for all  $i \geq i_{\text{low}}$ . Since all entrepreneurs  $i < i_{\text{low}}$  will never attain  $R_0$ , we assume that  $r_i^c = r_{i_{\text{low}}}^c$  for all  $i \in [0, i_{\text{low}}]$ . Sophisticated banks may thus reward high-quality entrepreneurs with low interest rates while making it unattractive for low-quality entrepreneurs to apply for loan contracts.

To see that expected return on equity is equal across investing entrepreneurs, suppose for a moment that all entrepreneurs above some quality level  $i^o \geq i_{\text{low}}$  apply for loans. The credit volume then is  $[\eta - i^o]I$  which must be balanced by total equity and deposits, so that  $d^o + \bar{S} = [\eta - i^o]I$ . The required amounts of equity and deposits per loan  $I$  are  $d^o/[\eta - i^o]$  and  $\bar{S}/[\eta - i^o]$ , respectively. Given  $r_i^c$ , the expected return  $g^o(i, r_i^c)$  on equity  $d^o/[\eta - i^o]$  for each loan is given by

$$g^o(i, r_i^c) = \frac{[\eta - i^o]\mathcal{R}(i^o, r_{i^o}^c) - \bar{S}(1 + r_A) - d^o}{d^o}, \quad i \in [i^o, \eta]. \quad (27)$$

Clearly, each return  $g^o(i, r_i^c)$ ,  $i \in [i^o, \eta]$ , must be greater or equal than  $r_A$  so that risk-neutral entrepreneurs with low-quality projects are willing to supply equity.

Intuitively, a competitive equilibrium for a sophisticated banking system with unlimited liability is a list consisting of a critical entrepreneur, an equity level, and loan interest rates

$$\{i_*, d_*, \{r_*^{co}(i)\}_{i=0}^\eta\},$$

where  $r_i^c = r_*^{co}(i)$ ,  $i \in [0, \eta]$  such that

- (i) it is optimal for banks to offer their intermediation services at actuarially fair interest rates,
- (ii) entrepreneurs take optimal investment decisions,
- (iii) the market for loans is balanced.

Formally, a competitive equilibrium with financial intermediation for a sophisticated banking system is defined as follows.

**Definition 2**

Let  $r_A \geq 0$  be given. A sophisticated (competitive) equilibrium of a sophisticated banking system is a list

$$\left\{ i_*^o, d_*^o, \{r_*^{co}(i)\}_{i=0}^\eta \right\}$$

consisting of a critical entrepreneur  $i_*^o$ , an equity level  $d_*^o$  and a non-increasing loan interest rate function  $r_*^{co}(i)$ ,  $i \in [0, \eta]$ , such that

$$\mathcal{R}(i, r_*^{co}(i)) = R(i_*^o, r_*^{co}(i_*^o)), \quad i \in [i_*^o, \eta], \quad (28)$$

$$\Pi(i_*^o, r_*^{co}(i_*^o)) = W[1 + g^o(i_*^o, r_*^{co}(i_*^o))], \quad (29)$$

$$r_A \leq g^o(i_*^o, r_*^{co}(i_*^o)), \quad (30)$$

$$[\eta - i_*^o]I = \bar{S} + d_*^o, \quad (31)$$

$$d_{\text{reg}} \leq d_*^o \leq i_*^o W. \quad (32)$$

Condition (28) states that on average banks must receive the same repayment on each loan. Condition (29) is the indifference condition for the critical entrepreneur. Since  $\Pi(i, r^c)$  is increasing in quality levels  $i$  and decreasing in interest rates  $r^c$  locally around  $(i_*^o, r_*^{co}(i_*^o))$  and  $r_*^{co}(i)$  is non-increasing in  $i$ , all entrepreneurs  $i < i_*^o$  either provide equity or invest into the alternative project, whereas all entrepreneurs  $i \geq i_*^o$  apply for loans and invest into their production project. As before, (31) is the equilibrium condition in the loan market which determines the required equity level. Condition (32) guarantees that the required equity for banks is available and that the minimum capital requirement stated at the outset of the paper is satisfied.

Equation (30) requires banks' return on equity to be at least as high as the return of the alternative project. If the expected return on equity were lower than  $r_A$ , no equity would be supplied. In this case no bank could operate, since the minimum capital requirement in (32) as stipulated at the outset of the paper were violated. If the expected return on equity is strictly higher than  $r_A$ , no entrepreneur will invest into the alternative project, so that, as argued above,  $d_*^o = \underline{i}W$ . Again, we focus on equilibria with positive investment in the alternative project in which  $i_*^o W > d_*^o$  and in which the return on equity is equal to  $r_A$ , i.e., (30) holds with equality.

## 4.2 Existence of sophisticated equilibria

The existence of sophisticated equilibria is established as follows, where the focus is again on equilibria with positive investment into the alternative project.

### Proposition 3

Let  $r_A \geq 0$  be arbitrary and suppose that the conditions of Proposition 1 hold. Then a sophisticated banking system admits a unique sophisticated equilibrium

$$\left\{ i_*^o, d_*^o, \{r_*^{co}(i)\}_{i=0}^\eta \right\}.$$

Moreover,  $d_{\text{reg}} < d_*^o < i_*^o W$  and  $g^o(i_*^o, r_*^{co}(i_*^o)) = r_A$ , so that the investment into the alternative project is strictly positive.

### Corollary 1

(i) Loan interest rates  $r_*^{co}(i)$  are decreasing in quality levels  $i \in [i_*^o, \eta]$  which satisfy

$$\underline{q}(1+i)f < I[1+r_*^{co}(i)],$$

that is, for all entrepreneurs who face a positive default risk.

(ii) If there exists an entrepreneur  $i_{\text{NB}} \in [i_*^o, \eta]$  with

$$\underline{q}(1+i_{\text{NB}})f = I[1+r_*^{co}(i_{\text{NB}})],$$

so that all entrepreneurs  $i \geq i_{\text{NB}}$  have zero default risk, then loan interest rates are constant satisfying  $r_*^{co}(i) = r_A$  for all  $i \in [i_{\text{NB}}, \eta]$ .

The proofs of Proposition 3 and Corollary 1 are given in Appendix A. Using (27) and the fact that (30) holds with equality, we see that the expected repayment of entrepreneur  $i_*^o$  is

$$\mathcal{R}(i_*^o, r_*^{co}(i_*^o)) = \left[ \frac{d_*^o + \bar{S}}{\eta - i_*^o} \right] (1+r_A) = I(1+r_A). \quad (33)$$

Hence by (28) the repayment of all investing entrepreneurs is equal to  $I(1+r_A)$ . Observe that Corollary 1 not only shows that a sophisticated banking system provides a floor  $r_A$  for the loan-interest rates but also that the interest rate margin  $r_*^{co}(i) - r_A$  is zero for entrepreneurs who will repay with certainty. All these entrepreneurs pay the same loan interest rate  $r_A$ , all other entrepreneurs pay a higher loan-interest rate. For the latter entrepreneurs the loan-interest rate is monotonically decreasing with the quality of their production projects.

### 4.3 Instability

Similar to the case of simple banking, we can derive the default probability of an individual bank, which is equal to the probability of a system-wide collapse of the banking system. Aggregate repayments to banks in an sophisticated equilibrium are

$$P_*^o(q) = \int_{i_*^o}^{\eta} \min\left\{q(1+i)f, I[1+r_*^{co}(i)]\right\} di. \quad (34)$$

Taking expectations and using (33), the expected (aggregate) repayments in a sophisticated equilibrium are

$$\mathbb{E}[P_*^o] = \int_{i_*^o}^{\eta} \mathcal{R}(i, r_*^{co}(i)) di = [\eta - i_*^o] I(1 + r_A). \quad (35)$$

This equation together with (31) implies that second-period equity of the sophisticated system in equilibrium is

$$\begin{aligned} G_*^o(q) &:= P_*^o(q) - \bar{S}[1 + r_A] \\ &= P_*^o(q) - \mathbb{E}[P_*^o] + d_*^o(1 + r_A). \end{aligned} \quad (36)$$

Aggregate losses of the sophisticated system are formally defined by

$$L_*^o(q) = \int_{i_*^o}^{\eta} I[1 + r_*^{co}(i)] di - P_*^o(q). \quad (37)$$

Expected aggregate losses are

$$\mathbb{E}[L_*^o] = \int_{i_*^o}^{\eta} I[1 + r_*^{co}(i)] di - \mathbb{E}[P_*^o]. \quad (38)$$

Inserting (37) and (38) into (36) yields

$$G_*^o(q) = \mathbb{E}[L_*^o] - L_*^o(q) + d_*^o(1 + r_A). \quad (39)$$

Taking expectations, expected second-period equity becomes

$$\mathbb{E}[G_*^o] = d_*^o(1 + r_A),$$

confirming that the return on equity is  $r_A$ . The default condition for an individual bank and for the banking system is  $G_*^o(q) < 0$  which, using (39), takes the form

$$L_*^o(q) > \mathbb{E}[L_*^o] + d_*^o(1 + r_A). \quad (40)$$

It can readily be seen from (39) that the future equity of the banking system is positive for sufficiently high shocks  $q$ . It follows from (37) that  $L_*^o(q)$  is decreasing in  $q$ . Therefore, the following proposition is proven analogously to Proposition 2.

#### Proposition 4

Under the hypotheses of Proposition 3, assume that

$$L_*^o(\underline{q}) > \mathbb{E}[L_*^o] + d_*^o(1 + r_A) > L_*^o(\bar{q}),$$

so that banks may default. Then there exists a unique critical level  $\underline{q} < q_{\text{crit}}^o < \bar{q}$  for macroeconomic shocks, such that a sophisticated banking system defaults if, and only if,  $q < q_{\text{crit}}^o$ . The default probability is

$$\pi_{\text{default}}^o := \text{Prob}\left(L_*^o(q) > \mathbb{E}[L_*^o] + d_*^o(1 + r_A)\right) = \int_{\underline{q}}^{q_{\text{crit}}^o} h(q) dq. \quad (41)$$

Similar to the case of a simple banking system, (41) is a value-at-risk formula for a sophisticated banking system. Proposition 4 states that banks default with positive probability as soon as the buffer  $d_*^o(1+r_A)$  is too small to buffer negative macroeconomic shocks.

## 5 Comparison of the two Systems

For a comparison of the two banking systems, let us first consider the special case in which no entrepreneurial bankruptcies occur in the simple banking system. This will occur if

$$\underline{q}(1 + i_*)f \geq I[1 + r_*^c].$$

It follows from (17) that  $r_*^c = r_A$  and the expected repayment to banks from an investing entrepreneur  $i$  then is

$$\mathcal{R}(i, r_*^c) = I[1 + r_A] \quad \text{for all } i \in [i_*, \eta].$$

By virtue of Corollary 1 we have

$$i_*^o = i_* \quad \text{and} \quad r_*^{co}(i) = r_*^c = r_A \quad \text{for all } i \in [i_*, \eta],$$

implying that the simple and the sophisticated banking system charge the same loan-interest rates and finance the same number of entrepreneurs.

This situation changes as soon as entrepreneurial bankruptcies are possible. In the next theorem we compare the loan interest rate in the simple banking system with the schedule of loan interest rates in a sophisticated banking system.

**Theorem 1**

Let the hypotheses of Proposition 1 be satisfied and assume that firm bankruptcies occur with positive probability, i.e.,  $\underline{q}(1+i_*)f < I[1+r_*^c]$ . Then there exists  $i_{\text{ER}} \in (i_*^o, \eta)$  with

- (i)  $r_*^c < r_*^{\text{co}}(i)$ ,  $i \in [0, i_{\text{ER}})$ ,
- (ii)  $r_*^c = r_*^{\text{co}}(i_{\text{ER}})$ ,
- (iii)  $r_*^c > r_*^{\text{co}}(i)$ ,  $i \in (i_{\text{ER}}, \eta]$ .

The proof of Theorem 1 is given in the appendix. Theorem 1 shows that loan interest rates for high-quality borrowers fall below the loan interest rate that is obtained in a simple banking system if entrepreneurial bankruptcies occur. This is a result of competition which enforces the same return on equity in both systems. Sophisticated banks charge higher interest rates from intermediate-quality borrowers in order to compensate for higher default risk and reward high-quality borrowers with low loan interest rates. Theorem 1 illustrates that sophisticated banks eliminate cross-subsidization between their borrowers by requiring the same average repayment from each borrower. The introduction of sophistication in rating, therefore, has distributional effects: high-quality entrepreneurs benefit, while intermediate-quality entrepreneurs are worse off. By charging the same loan interest rate, average repayments to simple banks on the contrary are increasing with the quality level of the entrepreneur.

The ability of the two systems to attract equity is compared next.

**Theorem 2**

Let hypotheses of Proposition 1 be satisfied and assume that  $\underline{q}(1+i_*)f < I[1+r_*^c]$ . Then the following properties hold:

- (i)  $i_* < i_*^o$ ,
- (ii)  $d_* > d_*^o$ ,
- (iii)  $[i_*W - d_*] < [i_*^oW - d_*^o]$ .

The proof of Theorem 2 is given in the appendix. The intuition for Theorem 2 may be described as follows. In order to adjust loan interest rates to the quality level of entrepreneurs and to generate equity returns of  $1+r_A$  at the same time, sophisticated banks charge low-quality entrepreneurs higher loan interest rates, making it less attractive for them to invest into their production projects. Hence, simple banks invite more entrepreneurs to invest into their production projects than sophisticated banks, i.e.,  $i_* < i_*^o$ . Market clearing in the loan and equity market, i.e. a sufficiently high margin  $r_*^c - r_A$ , enables a simple banking system to attract the necessary equity so that more production projects are financed, i.e.,  $\eta - i_*^o < \eta - i_*$ . As a consequence, with

a sophisticated banking system more resources are invested in the alternative project, while at the same time high-quality entrepreneurs pay lower loan interest rates.

To further illustrate the consequences of Theorem 2, notice that aggregate output in entrepreneurial production in response to a shock by  $q \in [\underline{q}, \bar{q}]$  is

$$Y_*(q) = qf \int_{i_*}^{\eta} (1+i)di$$

for the simple banking system and

$$Y_*^o(q) = qf \int_{i_*^o}^{\eta} (1+i)di$$

for the sophisticated banking system. Since  $i_* < i_*^o$  by Theorem 2 (i), we have

$$Y_*(q) > Y_*^o(q) \quad \text{for all } q \in [\underline{q}, \bar{q}].$$

This implies that sophisticated banks will crowd out investment in entrepreneurial production. Note that this effect is only present if investment into the alternative project is profitable, so that  $i_*^o > i_* > \underline{i}$ , where  $\underline{i}$  was defined in (8).

We next address the question of which of the two banking systems will accumulate more second-period equity. It is straightforward to infer from Theorem 2 that expected second-period equity of the simple banking system is higher than in the sophisticated system. Using (20) and (36), we obtain the following result, showing that on average sophisticated banks will have less capital than simple banks.

**Proposition 5**

*Under the hypotheses of Proposition 1, expected second-period equity in the simple banking system is higher than in the sophisticated system and expected repayments to the simple banking system is higher than expected repayments to the sophisticated system, that is,*

$$\mathbb{E}[P_*] - \mathbb{E}[P_*^o] = \mathbb{E}[G_*] - \mathbb{E}[G_*^o] = [d_* - d_*^o](1 + r_A) > 0. \quad (42)$$

For our further analysis, we introduce the representation

$$P_*(q) - P_*^o(q) = \int_{i_*}^{i_*^o} \min\{q(1+i)f, I[1+r_*^c]\}di + \int_{i_*^o}^{\eta} z(i, q)di, \quad (43)$$

where

$$z(i, q) := \min\{q(1+i)f, I[1+r_*^c]\} - \min\{q(1+i)f, I[1+r_*^{co}(i)]\}. \quad (44)$$

The first term on the r.h.s. of (43) reflects the repayments from additional loans that are granted under a simple banking system. Hence, this term is a volume effect. Under the hypotheses of Theorem 1, there exists a quality level  $i_{\text{ER}} \in (i_*^o, \eta)$  such that  $r_*^{co}(i_{\text{ER}}) = r_*^c$ . Since the default risk for investing entrepreneurs is positive,  $r_*^c > r_A$ . This implies that there must exist a critical shock  $q_{\text{ER}} > \underline{q}$  such that

$$q_{\text{ER}}(1 + i_{\text{ER}})f = I[1 + r_*^c] = I[1 + r_*^{co}(i_{\text{ER}})]. \quad (45)$$

For the shock  $q_{\text{ER}}$  entrepreneurs with quality levels  $i \geq i_{\text{ER}}$  fully meet their repayment obligations in both systems, implying that  $z(i, q_{\text{ER}}) \geq 0$  for  $i \geq i_{\text{ER}}$ . On the other hand, in both systems all entrepreneurs  $i < i_{\text{ER}}$  default at the shock  $q_{\text{ER}}$ , so that  $z(i, q_{\text{ER}}) = 0$  for  $i < i_{\text{ER}}$ . In view of (20) and (36), this implies that

$$G_*(q) - G_*^o(q) = P_*(q) - P_*^o(q) > 0 \quad \text{for all } q \in [\underline{q}, q_{\text{ER}}).$$

Hence the simple banking system outperforms the sophisticated system for adverse shocks in accumulating more second-period equity. This result is illustrated in Figure 1. The gray area of this figure describes the repayments to the simple banking system encountering the shock  $q_{\text{ER}}$ . As can be seen from the figure, the repayments to the sophisticated system are lower at least for all shocks  $q \leq q_{\text{ER}}$ . Formally, the result is summarized in the following proposition.

### Proposition 6

*Let the hypotheses of Proposition 1 be satisfied and assume  $\underline{q}(1 + i_*)f < I[1 + r_*^c]$ . Then there exists a shock  $q_{\text{BE}}^{(1)} \geq q_{\text{ER}}$  with  $q_{\text{ER}}$  as given in (45), such that*

$$G_*(q) > G_*^o(q) \quad \text{for all } q \in [\underline{q}, q_{\text{BE}}^{(1)}).$$

Proposition 6 states that that the simple banking system accumulates more equity than the sophisticated system for all shocks below a ‘break-even’ value  $q_{\text{BE}}^{(1)}$ . Without loss of generality, we may assume that  $q_{\text{BE}}^{(1)}$  is the highest value such that the assertion of Proposition 6 holds. This proposition reveals that the introduction of sophistication in rating to an economy may create a positive default risk when  $G(\underline{q}) > 0 > G^o(\underline{q})$ .

Observe that  $q_{\text{BE}}^{(1)}$  depends significantly on the quality level  $i_{\text{ER}}$  which in turn is determined by the distribution of the macroeconomic shocks. The lower  $i_{\text{ER}}$  is, that is, the more entrepreneurs are rewarded by low loan interest rates in the sophisticated system, the higher  $q_{\text{BE}}^{(1)}$  is. A priori, it cannot be ruled out that  $q_{\text{BE}}^{(1)} \geq \bar{q}$ . In this case the simple banking system outperforms the sophisticated system for all shocks and

its default probability  $\pi_{\text{default}}$  is lower than the default probability of the sophisticated system  $\pi_{\text{default}}^o$ .

Matters are different if  $q_{\text{BE}}^{(1)} < \bar{q}$ . Then there may exist a lowest possible ‘break-even’ value  $q_{\text{BE}}^{(2)}$  with  $q_{\text{BE}}^{(1)} \leq q_{\text{BE}}^{(2)} < \bar{q}$  such that  $G_*^o(q) > G_*(q)$  for all  $q > q_{\text{BE}}^{(2)}$  meaning that repayments in the sophisticated system are higher than in a simple system for sufficiently high shocks. A sufficient condition for such a  $q_{\text{BE}}^{(2)}$  to exist is that no entrepreneurial bankruptcies may occur in both systems and that in such a case repayments to the sophisticated banks are higher than to simple banks. The latter case occurs for

$$\int_{i_*^o}^{\eta} I[1 + r_*^{co}(i)] di > [\eta - i_*] I[1 + r_*^c]. \quad (46)$$

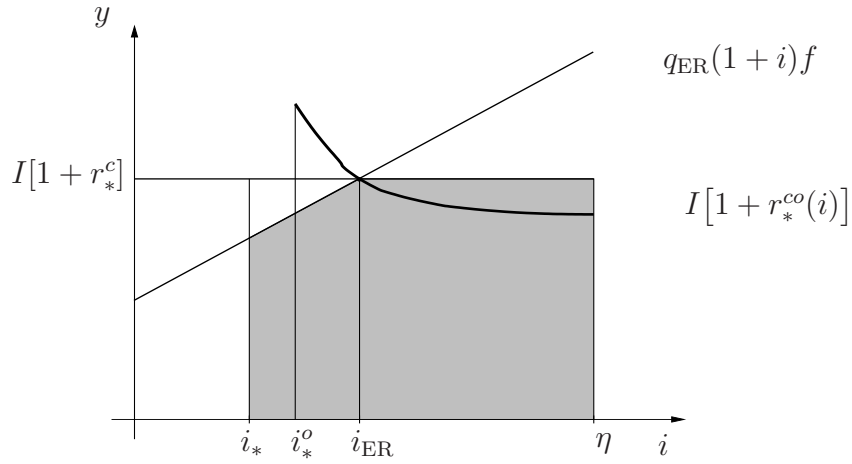


Figure 1: Repayments of entrepreneurs.

Our last theorem states conditions under which the default probability of a simple banking system is lower than the default probability of the sophisticated system and vice versa. Given the entrepreneurs’ productivity it will be shown in the appendix that the amount of banks’ repayment obligations determines which default probability is higher. Note to this end that given a fixed deposit interest rate  $r_A$ , an increase in savings  $\bar{S}$  due to a change in consumers’ endowments in either system will only affect its equilibrium equity level, whereas loan interest rates and credit volumes remain unaltered.

### Theorem 3

Let the hypotheses of Proposition 1 be satisfied and assume that  $\underline{q}(1 + i_*)f < I[1 + r_*^c]$ . Moreover, let the probability density be strictly positive, i.e.,  $h(q) > 0$  for all  $q \in [\underline{q}, \bar{q}]$ .

(i) Let  $q_{\text{BE}}^{(1)}$  be given by Proposition 6 and assume, in addition, that

$$G^o(q_{\text{BE}}^{(1)}) = P_*(q_{\text{BE}}^{(1)}) - \bar{S}(1 + r_A) > 0.$$

Then the default probability of the simple banking system is lower than the default probability of the sophisticated system, i.e.,  $\pi_{\text{default}} < \pi_{\text{default}}^o$ .

(ii) Suppose that there exists  $q_{\text{BE}}^{(2)} < \bar{q}$  such that

$$G^o(q) > G(q) \quad \text{for all } q > q_{\text{BE}}^{(2)}$$

and assume, in addition, that

$$G_*(q_{\text{BE}}^{(2)}) = P_*(q_{\text{BE}}^{(2)}) - \bar{S}(1 + r_A) < 0.$$

Then the default probability of the simple banking system is higher than the default probability of the sophisticated system, i.e.,  $\pi_{\text{default}} > \pi_{\text{default}}^o$ .

We conclude the comparison with a welfare discussion. We first consider expected aggregate consumption of entrepreneurs in both systems. For an economy with a simple banking system, expected consumption of all entrepreneurs is

$$\mathbb{E}[C] = i_* W(1 + r_A) + \int_{i_*}^{\eta} \Pi(i, r_*^c) di, \quad (47)$$

whereas expected aggregate consumption of entrepreneurs in an economy with a sophisticated banking system is

$$\mathbb{E}[C^o] = i_*^o W(1 + r_A) + \int_{i_*^o}^{\eta} \Pi(i, r_*^c(i)) di. \quad (48)$$

We obtain:

#### Theorem 4

*Let hypotheses of Proposition 1 be satisfied and assume that bankruptcies occur with positive probability. Then, expected aggregate consumption of entrepreneurs in an economy with a simple banking system is lower than in an economy with a sophisticated system.*

The proof of the theorem is given in the appendix. Theorem 4 shows that risk-neutral entrepreneurs in the aggregate will always prefer an economy with a sophisticated banking system. Hence, sophistication in banking may entail a tradeoff between stability and expected aggregate consumption of entrepreneurs. However, entrepreneurs with

intermediate quality levels who obtain funding from a simple banking system are worse off with a sophisticated system.

Any further analysis regarding welfare and income-distributional consequences of sophistication in rating depends on how banks are bailed out in case they may default with positive probability. Let us briefly discuss two scenarios assuming that case (i) of Theorem 3 holds. Suppose first that banks are bailed out by future (unmodeled) generations. This scenario is well-known in the banking crises literature when countries responded to large-scale banking problems by drastically lowering future short-term interest rates. This may generate large profits for banks and may thus allow them to recover (see e.g. Hellman, Murdock & Stiglitz (2000) and Hoshi & Kashyap 2004). Under such circumstances, the current generation in our model does not bear the cost of a bail-out. As in this scenario expected aggregate consumption of the current generation is higher in a sophisticated system, so is their aggregate welfare as the utilitarian sum of all utilities while future generations are worse off. A complete welfare analysis, however, calls for a fully fledged OLG model and must be left for future research.

As for the second scenario, suppose that the current generation bears the cost of the bail-out. This implies that the current generation has to provide rescue funds in case of a bank default which allow banks to at least partially pay back depositors. A priori, consumers in such a scenario tend to be worse off with a sophisticated banking system should they have to contribute to rescue funds while the effect on entrepreneurs appears to be ambiguous. If bail-out schemes are anticipated by agents, their decision problems have to be adapted before a welfare analysis can be conducted. This is left for future research.

## 6 Conclusions

We have shown that sophistication in risk management benefits high-quality entrepreneurs by lowering loan rates at the expense of intermediate-quality entrepreneurs. Sophisticated banks reduce the credit access of entrepreneurs with intermediate-quality levels and attract less equity than simple banks. As a consequence, expected repayments of the simple system are always higher and its default risk is lower for sufficiently low repayment obligations. These results suggest that more sophistication in the assessment of individual entrepreneurs' default risk may decrease banking stability if entrepreneurs' productivity is sufficiently high. This may be a serious concern for the impact of banking regulation. Moreover, sophistication in banking introduces a variety of income-distributional effects in the economy.

# A Appendix

## Proof of Proposition 1.

*Step 1.* We show first that in equilibrium (14) must hold with equality, i.e.,  $g(i_*, r_*^c) = r_A$ . Assume, on the contrary, that  $g(i_*, r_*^c) > r_A$ . Then as argued above  $i_* = \underline{i}$ . It follows from (13) and from the second inequality of Assumption (i) that  $\Pi(\underline{i}, r_*^c) > \Pi(\underline{i}, r_A)$ . This implies  $r_*^c < r_A$  and thus

$$g(\underline{i}, r_*^c) \leq \frac{[\eta - \underline{i}]I(1 + r_*^c) - \bar{S}(1 + r_A) - d_*}{d_*} < r_A$$

which is a contradiction.

*Step 2.* We determine the relationship between interest rates and quality levels that makes an entrepreneur indifferent between investing into production and into the alternative project. Set

$$r_{\max}^c := \frac{\bar{q}(1 + \eta)f}{I} - 1,$$

with  $\bar{q}$  denoting the highest possible shock. Then expected profits of entrepreneur  $i$  are zero, i.e.,  $\Pi(i, r^c) = 0$  for  $r^c \geq r_{\max}^c$  with  $\Pi$  given in (1). Consider the condition

$$\Pi(i, r^c) - W(1 + r_A) \stackrel{!}{=} 0, \quad i \in [\underline{i}, \eta]. \quad (49)$$

Since by Assumption (i)

$$\Pi(i, 0) \geq \Pi(\underline{i}, 0) > W(1 + r_A) > \Pi(i, r_{\max}^c) = 0, \quad i \in [\underline{i}, \eta]$$

for each  $i \in [\underline{i}, \eta]$ , equation (49) has a solution  $r^c = \varphi(i)$  such that

$$\Pi(i, \varphi(i)) - W(1 + r_A) = 0 \quad \text{for all } i \in [\underline{i}, \eta]. \quad (50)$$

Since  $\Pi(i, r^c)$  is continuous and strictly increasing in  $i$ , this solution is uniquely determined. Since  $\Pi(i, r^c)$  is decreasing in  $r^c \in [0, r_{\max}^c]$ ,  $\varphi$  is increasing in  $i$ .

*Step 3.* By Step 1 we can assume that (14) must hold with equality. Consider the function  $F : [\underline{i}, \eta] \rightarrow \mathbb{R}$ , defined by

$$F(i) := \frac{\mathbb{E}[P(\cdot, i, \varphi(i))]}{[\eta - i]} - I(1 + r_A). \quad (51)$$

Using (11), (12), and Condition (15), Condition (14) as an equality takes the form  $F(i_*) \stackrel{!}{=} 0$ .

By Assumption (i),  $W(1 + r_A) > \Pi(\underline{i}, r_A)$  so that  $\varphi(\underline{i}) < r_A$ . Hence,

$$F(\underline{i}) \leq I[1 + \varphi(\underline{i})] - I[1 + r_A] < 0. \quad (52)$$

On the other hand, in view of (25) it follows from

$$[\eta - i] \mathcal{R}(i, r^c) < \mathbb{E}[P(\cdot, i, r^c)], \quad i \in [\underline{i}, \eta], \quad r^c \geq 0, \quad (53)$$

that  $F(\bar{i}) > 0$  provided that

$$F(\bar{i}) \geq R(\bar{i}, \varphi(\bar{i})) - I(1 + r_A) > 0. \quad (54)$$

To see that the second inequality in (54) holds, observe that for each  $r^c \geq 0$ ,

$$\Pi(i, r^c) + \mathcal{R}(i, r^c) = \mathbb{E}[q](1 + i)f, \quad i \in [0, \eta]. \quad (55)$$

It follows from (50) and (55) that Assumption (ii) is equivalent to (54). Then (52), (54) and the continuity of  $F$  imply the existence of  $i_*$  so that  $F(i_*) = 0$ . We next infer from Step 2 that  $F(i)$  is increasing so that  $i_*$  is uniquely determined. Clearly,  $r_*^c = \varphi(i_*)$  and since by construction  $\underline{i} < i_* < \bar{i}$ , Conditions (13) and (16) are satisfied with  $d_{\text{reg}} < d_* < i_*W$ . This completes the proof. ■

### Proof of Proposition 2.

By assumption, we have  $G_*(\underline{q}) < 0$ . Since  $G_*(q) > 0$  for  $q \geq q_{\text{NB}}$  and since the function  $G_*$  is continuous and strictly increasing in  $q$ , there exists a unique critical shock  $\underline{q} < q_{\text{crit}} < q_{\text{NB}}$  so that  $G_*(q_{\text{crit}}) = 0$ . ■

### Proof of Proposition 3.

*Step 1.* The argument to show that (30) must hold with equality is analogously to Step 1 of Proposition 1 and therefore omitted. Consider next the function  $\varphi : [\underline{i}, \eta] \rightarrow [0, r_{\text{max}}^c]$  with  $r^c = \varphi(i)$  as defined in the second step of the proof of Proposition 1 and note that by Assumption (i), we have

$$\varphi(\underline{i}) < r_A \quad \text{and} \quad 0 < \varphi(i) < \frac{\bar{q}(1+i)f}{I} - 1 \quad \text{for each } i \in [\underline{i}, \eta]. \quad (56)$$

*Step 2.* Consider the function  $H : [\underline{i}, \eta] \rightarrow \mathbb{R}$ , defined by

$$H(i) := \mathcal{R}(i, \varphi(i)) - I(1 + r_A). \quad (57)$$

Using the definition of  $\varphi(i)$ , (27), and Condition (31), Condition (30) for  $i_*^o$  then takes the form  $H(i_*^o) \stackrel{!}{=} 0$ . The first inequality in (56) implies

$$H(\underline{i}) = \mathcal{R}(\underline{i}, \varphi(\underline{i})) - I(1 + r_A) < 0.$$

Analogously to (54), Assumption (ii) implies

$$H(\bar{i}) = \mathcal{R}(\bar{i}, \varphi(\bar{i})) - I(1 + r_A) > 0.$$

Since  $\varphi(i)$  is increasing in  $i$  and  $\mathcal{R}(i, r^c)$  is non-decreasing in both arguments, it follows from the second condition in (56) that  $H(i)$  is increasing in  $i$ . Hence, there exists a unique  $i_*^o$  such that  $H(i_*^o) = 0$ . Since by construction  $\underline{i} < i_*^o < \bar{i}$ , Conditions (29) and (32) are satisfied with  $d_{\text{reg}} < d_*^o < i_*^o W$ .

*Step 3.* We show that (28) is satisfied. From Step 2 we know that there exists a unique loan interest rate  $r_{i_*^o}^c = \varphi(i_*^o)$  for entrepreneur  $i_*^o$  such that  $\mathcal{R}(i_*^o, r_{i_*^o}^c) = I(1 + r_A)$ . It follows from the second inequality in (56) that

$$\bar{q}(1 + i)f > I(1 + r_{i_*^o}^c) \quad \text{for all } i \in [i_*^o, \eta]. \quad (58)$$

Notice that  $\mathcal{R}(i, r^c)$  is strictly increasing in  $r^c$  as long as  $\bar{q}(1 + i)f > I(1 + r^c)$  and strictly increasing in  $i$  as long as  $\underline{q}(1 + i)f < I(1 + r^c)$ . This implies the existence of a unique function  $r_*^{co}(i)$ ,  $i \in [i_*^o, \eta]$  with  $r_*^{co}(i_*^o) = r_{i_*^o}^c$  such that

$$\mathcal{R}(i, r_*^{co}(i)) = I(1 + r_A), \quad i \in [i_*^o, \eta]. \quad (59)$$

Thus Condition (28) is satisfied.

*Step 4.* We need to define  $r_*^{co}(i)$  for all quality levels  $i \in [0, \eta]$ . Notice that  $r_*^{co}(i)$  can be defined via (59) for all  $i \geq 0$  that satisfy  $\mathbb{E}[q](1 + i)f \geq I(1 + r_A)$ . Suppose there exists  $i_{\text{low}} \geq 0$  such that  $\mathbb{E}[q](1 + i_{\text{low}})f = I(1 + r_A)$ . Setting  $r_*^{co}(i) = r_*^{co}(i_{\text{low}})$  for all  $i \in [0, i_{\text{low}}]$ , the function  $r_*^{co}(i)$  is then extended to a non-increasing function on all of  $[0, \eta]$ . Clearly,  $i \mapsto r_*^{co}(i)$  is continuous on  $[0, \eta]$  since  $\mathcal{R}$  is continuous. Hence

$$\Pi(i, r_*^{co}(i)) < W(1 + r_A), \quad \text{for all } i \in [0, i_*^o),$$

showing that all entrepreneurs  $i < i_*^o$  either provide equity or invest into the alternative project, whereas all entrepreneurs  $i \geq i_*^o$  invest into their production project. This completes the proof. ■

**Proof of Corollary 1.**

Since  $\mathcal{R}(i, r^c)$  is increasing in  $r^c$  as long as  $\bar{q}(1+i)f > I[1+r^c]$ , the loan interest function  $r_*^{co}(i)$  is decreasing unless there exists  $i_{NB} \in [i_*^o, \eta]$  such that  $\underline{q}(1+i_{NB})f = I[1+r_*^{co}(i_{NB})]$ . In this case it follows from (33) that  $r_*^{co}(i) = r_A$  for all  $i \geq i_{NB}$ . ■

**Proof of Theorem 1.**

From (17) we know that

$$\mathbb{E}[P_*] = \int_{i_*}^{\eta} \mathcal{R}(i, r_*^c) di = [\eta - i_*]I(1 + r_A). \quad (60)$$

By the mean value theorem for integrals, there exists  $i_{ER} \in [i_*, \eta]$  such that

$$\mathbb{E}[P_*] = [\eta - i_*]\mathcal{R}(i_{ER}, r_*^c). \quad (61)$$

Since  $\underline{q}(1 + i_*)f < I(1 + r_*^c)$  by assumption,  $\mathcal{R}(i, r_*^c)$  is increasing with respect to  $i$  at least locally around  $i_*$  and otherwise non-decreasing in  $i$ . Hence  $i_{ER} \in (i_*, \eta)$ . Now (60) and (61) imply  $\mathcal{R}(i_{ER}, r_*^c) = I(1 + r_A)$  such that  $r_*^{co}(i_{ER}) = r_*^c$  by construction of a sophisticated equilibrium. Since  $i_{ER} > i_*$ , we have

$$\Pi(i_{ER}, r_*^c) > \Pi(i_*, r_*^c) = W(1 + r_A)$$

and thus  $i_{ER} > i_*^o$ , implying that  $i_{ER} \in (i_*^o, \eta)$ . Assumption  $\underline{q}(1 + i_*)f < I(1 + r_*^c)$  implies  $r_*^c > r_A$  so that  $r_*^{co}(i)$  must be decreasing locally around  $i_{ER}$  by Corollary 1. This completes the proof. ■

**Proof of Theorem 2.**

It follows from the proofs of Propositions 1 and 3 that the critical entrepreneurs  $i_*$  and  $i_*^o$  are given by the conditions  $F(i_*) = 0$  and  $H(i_*^o) = 0$ , respectively, where  $F$  and  $H$  were defined in (51) and (57), respectively. By (53) we have  $H(i) < F(i)$  for all  $i \in [i_*, \eta)$ , so that  $i_* < i_*^o$ . This proves the first claim. The second claim follows directly from (15) and (31) yielding  $d_*^o < d_*$ . The third claim is an immediate consequence of the first two assertions. ■

**Proof of Theorem 3.**

(i) Since both systems face a positive default risk,

$$\bar{S}(1 + r_A) > P_*(\underline{q}) > P_*^o(\underline{q}).$$

On the other hand

$$P_*(q_{\text{BE}}^{(1)}) \geq P_*^o(q_{\text{BE}}^{(1)}) > \bar{S}(1 + r_A).$$

Since  $P_*$  and  $P_*^o$  are increasing in  $q$ , it follows from Propositions 5 and 6 that  $q_{\text{crit}} < q_{\text{crit}}^o$ , where the critical values  $q_{\text{crit}}$  and  $q_{\text{crit}}^o$  are given by

$$P_*(q_{\text{crit}}) = P_*^o(q_{\text{crit}}^o) = \bar{S}(1 + r_A).$$

Recalling the definitions of the default probabilities (24) and (41), this proves (i).

(ii) In this case we have

$$P_*(q_{\text{BE}}^{(2)}) \leq P_*^o(q_{\text{BE}}^{(2)}) < \bar{S}(1 + r_A).$$

Since  $P_*$  and  $P_*^o$  are increasing in  $q$ , it follows from Propositions 5 and 6 that  $q_{\text{crit}} > q_{\text{crit}}^o$ . This proves (ii). ■

**Proof of Theorem 4.**

We need to show that

$$\mathbb{E}[C] < \mathbb{E}[C^o]. \tag{62}$$

Taking integrals, equation (55) implies

$$\begin{aligned} \int_{i_*}^{\eta} \Pi(i, r_*^c) di &= \mathbb{E}[q]f \int_{i_*}^{\eta} (1 + i) di - \mathbb{E}[P_*] \\ &= \mathbb{E}[q]f \left[ \eta - i_* + \frac{1}{2}(\eta^2 - i_*^2) \right] - [\eta - i_*]I(1 + r_A) \end{aligned} \tag{63}$$

and

$$\begin{aligned} \int_{i_*}^{\eta} \Pi(i, r_*^{co}(i)) di &= \mathbb{E}[q]f \int_{i_*^o}^{\eta} (1 + i) di - \mathbb{E}[P_*^o] \\ &= \mathbb{E}[q]f \left[ \eta - i_*^o + \frac{1}{2}(\eta^2 - (i_*^o)^2) \right] - [\eta - i_*^o]I(1 + r_A). \end{aligned} \tag{64}$$

Substituting (63) into (47) and (64) into (48), we see after rearranging that Condition (62) holds if and only if

$$\mathbb{E}[q]f \left( 1 + \frac{i_* + i_*^o}{2} \right) < (W + I)(1 + r_A). \tag{65}$$

On the other hand, using (55), in a sophisticated equilibrium

$$\mathbb{E}[q](1 + i_*^o)f = \Pi(i_*^o, r_*^{co}(i_*^o)) + \mathcal{R}(i_*^o, r_*^{co}(i_*^o)) = (W + I)[1 + r_A].$$

Since by Theorem 1 (i)  $i_* < i_*^o$ , this shows that (65) holds, completing the proof. ■

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