Sophistication in Risk Management, Interest Rates, and Banking Stability*

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Abstract

We explore the impact of sophistication in risk management, as required by Basel II, on banking stability and market conditions. We compare a competitive banking system in which only average ratings are available with a competitive system in which banks are able to assess the default risk of individual firms. Sophistication in banking lowers deposit and loan-interest rates and hence decreases default probabilities of individual entrepreneurs. Sophistication decreases the default probability of banks only if the level of initial equity is sufficiently high. Otherwise banking stability declines.

Keywords: financial intermediation, macroeconomic risks, risk management, risk premiums, banking regulation, rating

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1 Introduction

The common assumption underlying banking regulation is that, along with capital requirements, more sophistication in rating and risk management will increase the stability of a banking system. This assumption is epitomized in the Basel II regulatory framework. In this paper we explore the extent to which this claim is valid.

We consider a competitive banking system embedded in a macroeconomic environment in which banks offer intermediation services to a population of entrepreneurs who produce subject to macroeconomic risk. Entrepreneurs and bank owners have an outside option for investing their resources. Banks compete for deposits and offer loans as delegated monitors but cannot adjust their initial equity, as would be the case in the short term. Risk premiums on loans are determined by free-exit and free-entry conditions for banks.

We compare two polar cases, a simple and a sophisticated banking system. In the simple system, banks are unable to individually rate the default risk of an investment project. They attribute the same default probability to each borrower and thus use the same rating. In the sophisticated system, banks use an infinitely fine rating system in which each borrower is individually rated. Loan interest rates are tailored according to the default probability of an entrepreneur. The goal of the paper is to investigate the influence of the rating ability of a banking system on market conditions and banking stability.

Our main findings are as follows: First, a sophisticated banking system rewards high-quality entrepreneurs who produce with low loan-interest rates. Aggregate repayments of entrepreneurs are therefore lower than in a simple system. As a consequence, the deposit rate and hence refinancing costs in a sophisticated banking system are lower than in a simple banking system. Second, the sophisticated banking system accumulates more equity in adverse macroeconomic environments. The intuition is as follows: Banks earn only the liquidation value of those entrepreneurs who default. If a sufficiently large number of bankruptcies occurs due to adverse shocks, the simple banking system's advantage of higher aggregate repayments does not outweigh its higher refinancing costs, as the latter is unaffected by a macroeconomic shock. Third, we show that the default probability of a sophisticated banking system is lower if the initial equity levels are sufficiently high. Otherwise the simple system is less likely to default, and sophistication in a banking system will decrease bank stability.

The approach taken in this paper is complementary to the work of Gehrig & Stenbacka (2004), who show that uncoordinated screening behavior of competing financial intermediaries creates a financial multiplier and may be responsible for macroeconomic fluctuations. In analyzing the systemic effects of screening activities by firms, this paper contributes to the literature on screening by banks, as surveyed in Freixas & Rochet (1997).

Our results are related to the literature on banking regulation. Comprehensive surveys with different emphases are given by Bhattacharya & Thakor (1993), Dewatripont & Tirole (1994), Hellwig (1994), Freixas & Rochet (1997), or Bhattacharya, Boot & Thakor (1998). Overall, our analysis suggests that the new regulatory policy for banking (Basel II) which requires banks to introduce more sophistication in assessing the default risk of their clients will only be beneficial if banks start with a sufficiently high level of bank capital.

A large body of literature has investigated the consequences of modern risk management techniques in capital markets, which have risen dramatically over the last few decades, cf. Carey & Stulz (2006). Although there is a large literature on sophistication in rating techniques[1], its aggregate consequences are unknown. This calls for a more detailed analysis of how sophisticated risk management tools affect the banking system and the macroeconomy.

The paper is organized as follows: In the next section we introduce the model and both types of banking systems. In section 3 we examine simple banks, and in section 4 we develop the mirror-image of the analysis for sophisticated banks. In section 5 we compare both systems, leading on from there to our main results. Section 6 concludes.

[1]For example, a set of intuitive rating principles has been developed by Krahnen & Weber (2001)

2 Model

2.1 Consumers and entrepreneurs

Consider an economy with two periods. The population of agents consists of a continuum indexed by $[0, 1]$. Each agent has individual wealth W in the first period. Agents are divided into two classes. One fraction of agents, indexed by $[0, \eta]$ with $0 < \eta < 1$, are potential entrepreneurs. The other fraction, indexed by $(\eta, 1]$, are consumers. Potential entrepreneurs and consumers differ in that only the former have access to investment technologies.

Consumers are endowed with intertemporal preferences on consumption in the two periods of their lives. Let $u(c_1, c_2)$ be a standard intertemporal utility function of a consumer, with c_1, c_2 denoting first-period and second-period consumption of a consumer, respectively. Given the endowment W in the first period and a deposit interest rate r^d, each young consumer saves the amount $s(r^d)$. Aggregate savings of all consumers are denoted by $S(r^d)$ and given by $S(r^d) = (1 - \eta)s(r^d)$. The alternative investment opportunity may be thought of as an outside option, such as government bonds or investments in other sectors of the economy that are not modeled explicitly.[2]

Potential entrepreneurs are assumed to be risk-neutral and to consume only in the second period. Each entrepreneur has to decide whether to invest in a production project that converts period-1 goods into period-2 goods or to invest her funds in the alternative project with return r_A. The funds required for each production project are fixed at $W + I$, so that an entrepreneur must borrow I additional units of the good from banks to undertake the production project. Entrepreneurs are heterogeneous in the quality of their production projects, which depends on their index i. The quality parameter of entrepreneur i is assumed to be private information. If an entrepreneur of type i obtains additional resources I and decides to invest, investment returns in the second period amount to

$$y = q(1 + i)f(W + I),$$

[2]For tractability we assume that consumers are not allowed to invest in the alternative project. This can be justified by liquidity services of deposits. However, without affecting the qualitative results, the model could be extended to the case in which consumers hold a portfolio of deposits and other assets. The saving function then takes the form $S = S(r^d, r_A)$.

where f denotes a standard atemporal neoclassical production function and $q > 0$ represents an exogenous macroeconomic productivity shock in the economy. Since W and I will remain fixed throughout the paper, we write $f = f(W+I)$. The distribution of shocks q is assumed to be given by a continuous density function $h(q)$ with support on a compact interval $[\underline{q}, \overline{q}]$ with $0 < \underline{q} < \overline{q}$.

Entrepreneurs are price-takers and operate under limited liability. Given a loan interest rate r^c, the expected profit for a producing entrepreneur i is

$$\Pi(i, r^c) := \int_{\underline{q}}^{\overline{q}} \max\{q(1+i)f - I(1+r^c), 0\}\, h(q)\mathrm{d}q. \tag{1}$$

Note that $\Pi(i, r^c)$ is non-decreasing in quality levels i and non-increasing in loan rates r^c. A risk-neutral entrepreneur with the quality parameter $i \in [0, \eta]$ will prefer to invest in the production project rather than the alternative project if the return on the production project is expected to be larger than the return on the alternative investment project, i.e., if

$$\Pi(i, r^c) \geq W\big(1 + r_A\big). \tag{2}$$

Finally, we assume that savings are never sufficient to fund all entrepreneurs. Since aggregate savings $S(r^d)$ are bounded by $(1-\eta)W$, this condition takes the form

$$(1-\eta)W < \eta I. \tag{3}$$

2.2 Simple and sophisticated banking

Depositors cannot observe the quality parameters of entrepreneurs and cannot verify whether or not an entrepreneur invests. The existence of such market frictions necessitates financial intermediation (see e.g. Hellwig 1994). To alleviate these informational problems, we assume that there are n banks, indexed by $j = 1, \ldots, n$ $(n > 1)$, which are owned by entrepreneurs. As bank owners are then risk-neutral and consume only in the second period, the objective of banks is to maximize expected profits accruing to current shareholders.[3] Banks monitor borrowers as delegated monitors in the sense of Diamond (1984), and their monitoring is assumed to be efficient in the sense that they

[3] We could also assume that banks are owned by a group of investors who are not modeled explicitly. The crucial assumption is that expected profit maximization is in the interest of shareholders.

are able to secure both the investment of an entrepreneur and the liquidation value in case of default, cf. Gersbach & Uhlig (2005).

For a comparison based on the same premises, we make the same assumptions regarding the competitive environment of the banking sector. First, both banking systems are perfectly competitive with free exit and free entry.[4] Bank owners have the opportunity to exit the banking industry and to invest their equity alternatively with return r_A ($r_A > 0$). Second, both systems start with the same amount of aggregate equity e_1, which consists of the value of physical capital k_1 and cash d_1. More precisely, each bank in each system starts with the same amount of initial cash holdings $\frac{d_1}{n}$ and the same value of physical capital $\frac{k_1}{n}$ in period 1. The physical capital stock allows banks to perform their intermediation services and consists of branches, IT systems, and other components of capital. To simplify the exposition, we assume that capital fully depreciates over both periods. The central assumption is that equity is given, so that our short-term perspective is justified.

Third, perfect competition among banks determines deposit and loan-interest rates. Each bank j can offer deposit contracts $D(r^d)$, where $1 + r^d$ is the repayment offered for one unit of resources. We distinguish between a *simple* and a *sophisticated* banking system that differ only in their ability to rate the quality of entrepreneurs.

1. *Simple Banking System.* The essential feature of the simple banking system is that banks are unable to rate entrepreneurs individually and to condition loan contracts on the quality parameter i of an entrepreneur. Banks only have an average rating of entrepreneurs and offer all entrepreneurs the same loan contract with interest rate r^c so that $1 + r^c$ is the repayment required from entrepreneurs for one unit of borrowed funds.

2. *Sophisticated Banking System.* In a sophisticated banking system, banks are able to rate each entrepreneur individually and to offer entrepreneur-specific loan contracts, denoted by $C(r_i^c)$, where r_i^c is the loan interest rate demanded from an entrepreneur of type i.

[4]The free-entry free-exit framework is a standard concept in industrial economics, see e.g. Vives (2004).

In both banking systems, banks operate under unlimited liability, and loans are only constrained by the amount of equity and deposits. We assume throughout that aggregate uncertainty is canceled out when depositors and entrepreneurs choose banks randomly.[5] As all banks are identical, they will obtain the same proportions of lenders and borrowers, respectively.

With these assumptions, the financial intermediation process in either system is as follows: In the first period, banks offer deposit and loan contracts, given by r^d and r^c (simple banking) or by r^d and $\{r_i^c\}_{i=0}^{\eta}$ (sophisticated banking), respectively. Each bank j obtains an amount of d^j in equity and an equal share of deposits from consumers. Entrepreneurs decide which contracts to accept. Resources are exchanged. In the second period, entrepreneurs who have chosen the production project produce subject to a macroeconomic shock and pay back loans with limited liability. Banks repay depositors.

The vulnerability of a banking system depends on its ability to accumulate equity. This motivates a comparison of equity accumulation in the two banking systems. Of particular interest are the probability distribution of equity and the downside equity risk in the second period. In particular, we will analyze the default probability for both banking systems, i.e., the probability of equity becoming negative.

3 Competitive Equilibria for Simple Banks

Consider first the case in which banks use simple risk-management tools and are unable to detect the quality parameter i of the production project. Recall that only one part of equity in the form of cash holdings $\frac{d_1}{n}$ can be used by a bank to finance loans. Physical capital cannot be used to finance loans, but it enables banks to collect deposits and grant loans. As all banks are assumed to be identical, we will formulate the equilibrium conditions for the whole banking system.

We assume d_1 to be less than $\overline{d} := \eta I - (1-\eta)W$. Since $(1-\eta)W + \overline{d} = \eta I$, a banking system with $d_1 > \overline{d}$ has more equity than is needed to finance all entrepreneurs. In this

[5] The exact construction of individual randomness so that this statement holds can be found in Alós-Ferrer (1999).

case, excess resources are available at any interest rate. We exclude this uninteresting case from our analysis. For each equity level $d_1 \in [0, \overline{d}]$ and each $r^d \geq 0$, there exists a unique *critical entrepreneur* $i_E \in [0, \eta]$, given by

$$i_E = i_E(d_1, r^d) := \frac{\eta I - d_1 - S(r^d)}{I}, \tag{4}$$

such that savings are balanced by investments, i.e.,

$$S(r^d) + d_1 = \big[\eta - i_E(d_1, r^d)\big]\, I. \tag{5}$$

3.1 Equilibrium concept

We motivate the equilibrium concept by the following considerations. Let $d_1 \in [0, \overline{d}]$ be the amount of equity in the form of cash in the first period. Banks raise deposits $S(r^d)$ that have to be paid back with interest at the end of the second period. We will require that in a competitive equilibrium loan demand equals loan supply, so that (5) holds. Since simple banks are unable to detect the quality parameter i, they charge all producing entrepreneurs the same loan interest rate r^c. Thus, simple banks lend $\big[\eta - i_E\big]I$ to firms and will receive payments $P = P\big(q, i_E, r^c\big)$ at the end of the second period, given by

$$P\big(q, i_E, r^c\big) = \int_{i_E}^{\eta} \min\Big\{q(1+i)f,\ I\big(1+r^c\big)\Big\} \mathrm{d}i, \tag{6}$$

with $i_E = i_E(d_1, r^d)$ as defined in (4). Given a pair of interest rates r^d, r^c, the second-period equity of the banking system is given by

$$G(q, d_1, r^d, r^c) := P\big(q, i_E(d_1, r^d), r^c\big) \ - \ S(r^d)(1 + r^d), \tag{7}$$

such that for each shock q and each $r^d, r^c \geq 0$, $d_2 = G(q, d_1, r^d, r^c)$ is the level of second-period equity of the banking system.[6] Banks are assumed to maximize expected profits. In a competitive equilibrium this means that existing banks must decide whether to offer their intermediation services at the current interest rates or to invest their equity $d_1 + k_1$ elsewhere. Given a pair of interest rates r^d and r^c, the expected second-period

[6]The corresponding profit is $G(q, d_1, r^d, r^c) - d_1$.

equity of the banking system is

$$\begin{aligned}\mathbb{E}\left[G(\cdot,d_1,r^d,r^c)\right] &= \mathbb{E}\big[P\big(\cdot,i_E(d_1,r^d),r^c\big)\big] - S(r^d)[1+r^d] \qquad (8)\\ &= \int_{\underline{q}}^{\overline{q}} P\big(q,i_E(d_1,r^d),r^c\big)\,h(q)\mathrm{d}q - S(r^d)[1+r^d].\end{aligned}$$

In order to offer their intermediation services to loan applicants, risk-neutral shareholders of banks will require that

$$\mathbb{E}[G(\cdot,d_1,r^d,r^c)] \geq d_1(1+r_A)+S(r^d)[r_A-r^d]. \qquad (9)$$

Otherwise banks will simply invest their funds in the alternative investment project.

We next define a competitive equilibrium with free entry for a simple banking system. Intuitively, a competitive equilibrium is a pair of interest rates $\big(r_*^d,r_*^c\big)$, such that

(i) it is optimal for banks to offer their intermediation services, i.e., no bank exits. Moreover, no new bank enters the market;

(ii) entrepreneurs take optimal investment decisions;

(iii) the market for loans is balanced.

Formally, a competitive equilibrium for a simple banking system is defined as follows:

Definition 1

Let $d_1 \in [0,\overline{d}]$. Assume that the value of physical capital satisfies $k_1 > (1-\eta)W\,r_A$. A competitive equilibrium of the banking system operating under unlimited liability is a pair of interest rates (r_*^d,r_*^c) such that the following conditions hold:

$$\begin{aligned}\mathbb{E}\left[G(\cdot,d_1,r_*^d,r_*^c)\right] &= [d_1+k_1]\,(1+r_A), & (10)\\ \Pi\big(i_E(d_1,r_*^d),r_*^c\big) &= W\,(1+r_A), & (11)\\ \big[\eta-i_E(d_1,r_*^d)\big]\,I &= S(r_*^d)\,+\,d_1, & (12)\\ r_*^d &< r_A. & (13)\end{aligned}$$

Equation (10) represents the optimality condition for existing banks and the free-entry free-exit condition. If the expected return on equity were lower than r_A, banks would exit and not offer their intermediation services. If the expected return were higher, new banks would enter until the expected return is again r_A. The condition on the value of the physical capital, namely that $k_1 > (1-\eta)Wr_A$, will ensure that (9) holds, so that it is profitable for banks to offer their intermediation services to loan applicants.

Equation (11) is the indifference condition for the critical quality level $i_* = i_E(d_1, r_*^d)$, which determines the demand for loans. In equilibrium, all entrepreneurs with sufficiently high quality parameters $i \geq i_*$ invest in their production projects, while all entrepreneurs with insufficient quality parameters $i < i_*$ invest in the alternative project. Equation (12) is the equilibrium condition for savings and investments at banks given in (5). The last condition (13) ensures that shareholders of banks and entrepreneurs who invest in the alternative project are not worse off than by depositing the money at banks.

The banking system operates under *unlimited liability* in the sense that banks, i.e., their bank managers internalize the default risk that would materialize in losses. This assumption can be justified in various ways. For instance, the non-pecuniary cost of defaults for managers can induce banks to behave as if they were maximizing expected profits. Alternatively, we might consider a banking system that operates under limited liability. Then the l.h.s. in (10) would have to be replaced by

$$\mathbb{E}\left[\max\{G(\cdot, d_1, r^d, r^c), 0\}\right].$$

From results in Gersbach & Wenzelburger (2008) one expects that the behavior of both models would be qualitatively the same. Finally, let us comment on how to adjust the equilibrium conditions to include fixed costs for monitoring loans. Suppose a bank has granted a loan with a face value of I and needs to spend $m \geq 0$ units of resources to secure the liquidation value of defaulting entrepreneurs. Then the free-exit and free-entry condition needs to be reformulated as

$$\mathbb{E}\left[G(\cdot, d_1, r_*^d, r_*^c)\right] - \left[\eta - i_*\right](I+m) = [d_1 + k_1]\,(1 + r_A).$$

3.2 Existence of competitive equilibria

We first establish the existence and uniqueness of a competitive equilibrium. We set

$$\underline{i} := i_E(d_1, r_A) \quad \text{and} \quad \bar{i} := i_E(d_1, 0). \tag{14}$$

Since S is assumed to be non-decreasing in deposit rates $r^d \in [0, r_A]$, the critical quality level i_E will vary between $\underline{i}$ and $\overline{i}$.

Proposition 1

Let $d_1 \in [0, \overline{d}]$ and $k_1 > (1-\eta)W r_A$ be given. Suppose, in addition, that the following conditions hold:

(i) The savings function S is non-decreasing and satisfies

$$S'(r^d) \cdot \frac{1+r^d}{S(r^d)} < 1, \quad r^d \in [0, r_A].$$

(ii) Productivity of entrepreneurs is such that

$$\begin{aligned}
&(a) \qquad \Pi(\underline{i}, 0) > W(1+r_A) > \Pi\big(\overline{i}, r_A\big),\\
&(b) \qquad (1+\underline{i})\mathbb{E}[q]f > (1+r_A)\Big(\frac{k_1}{\eta - \overline{i}} + I\Big),\\
&(c) \qquad (1+\overline{i})\mathbb{E}[q]f > (1+r_A)\Big(\frac{k_1}{\eta - \overline{i}} + I + W\Big).
\end{aligned}$$

Then a simple banking system admits a unique competitive equilibrium (r_*^d, r_*^c) with $0 < r_*^d < r_A < r_*^c$, so that the intermediation margin $r_*^c - r_*^d$ is positive. Moreover, $\underline{i} \leq i_* = i_E(d_1, r_*^d) \leq \overline{i}$.

In the following corollary we describe the special case of an inelastic savings function that allows a more explicit description of the equilibrium.

Corollary 1

If the savings function is inelastic such that $S(r^d) = \overline{S}$ for all $r^d \geq 0$, then $i_* = \frac{\eta I - d_1 - \overline{S}}{I}$ is independent of r_*^d, the loan interest rate is determined by

$$\Pi\big(i_*, r_*^c\big) = W(1+r_A),$$

and the deposit interest rate by

$$r_*^d = \frac{1}{\overline{S}}\Big(\mathbb{E}\big[P\big(\cdot, i_*, r_*^c\big)\big] - [d_1 + k_1](1+r_A)\Big) - 1. \qquad (15)$$

The proof of Proposition 1 is given in the appendix. condition (i) of Proposition 1 is a sufficient condition making it attractive for banks to finance entrepreneurs, whereas condition (iia) ensures that entrepreneurs will apply for loans. Conditions (iib) and (iic) relate expected liquidation values at the lowest and highest possible quality level for i_* to the return on funds. Condition (iib) ensures that condition (10) holds, and condition (iic) ensures that condition (13) holds.

Suppressing the dependency on k_1 for notational simplicity, set

$$P_*(q, d_1) := P(q, i_E(d_1, r_*^d), r_*^c)$$

for repayments in equilibrium. It follows from (10) that, in a competitive equilibrium, average repayments to banks are

$$\mathbb{E}\big[P_*(\cdot, d_1)\big] = e_1(1 + r_A) + S(r_*^d)(1 + r_*^d),$$

where $e_1 = d_1 + k_1$ as before. Hence the second-period equity of a simple banking system is

$$\begin{aligned} d_2 = G_*(q, d_1) \quad &:= \quad G(q, d_1, r_*^d, r_*^c) \\ &= \quad P_*(q, d_1) - S(r_*^d)(1 + r_*^d) \\ &= \quad P_*(q, d_1) - \mathbb{E}\big[P_*(\cdot, d_1)\big] + e_1(1 + r_A). \end{aligned} \tag{16}$$

Equation (16) is a compact representation of bank equity at the end of the intermediation process. Second-period equity is equal to initial equity plus the interest earned on equity plus the difference between realized and expected aggregate repayments.

3.3 Instability

In this section, we derive some intuitive characteristics of competitive equilibria in simple banking systems. Observe first that an entrepreneur with quality level i goes bankrupt if he is unable to fully pay back his loan, that is, if

$$I(1 + r_*^c) > q(1 + i)f.$$

Entrepreneurial bankruptcies will occur for all shocks below a critical shock q_{NB}, which is given by

$$q_{\text{NB}} := \frac{I(1 + r_*^c)}{(1 + i_*)f}.$$

From the indifference condition (11) we know that $\Pi(i_*, r_*^c) = W(1 + r_A) > 0$. This implies that in equilibrium the critical entrepreneur i_* and thus all producing entrepreneurs will fully repay their obligations with positive probability. On the other hand, bankruptcies will occur with positive probability if $q_{\rm NB} > \underline{q}$. The following lemma now states sufficient conditions under which producing entrepreneurs will default with positive probability.

Lemma 1

In a competitive equilibrium of a simple banking system, defaults of producing entrepreneurs occur with positive probability, that is, $q_{\rm NB} > \underline{q}$*, if*

$$W(1 + r_A) < (\mathbb{E}[q] - \underline{q})(1 + \underline{i})f. \tag{17}$$

The proof of Lemma 1 can be found in the appendix. We next analyze the question of how stable a simple banking system is. We infer from (16) that second-period equity is positive on average. The banking system defaults if its second-period equity is negative, so that $G_*(q, d_1) < 0$. Since P and hence G is non-decreasing in shocks q, there exists a unique critical shock $\underline{q} \le q_{\rm crit} < q$ such that the default probability of the simple banking system is given by

$$\pi_{\rm default} := \mathrm{Prob}(G_*(q, d_1) < 0) = \int_{\underline{q}}^{q_{\rm crit}} h(q)\mathrm{d}q. \tag{18}$$

Assuming that the probability density h is strictly positive on $[\underline{q}, \overline{q}]$, the default probability $\pi_{\rm default}$ is positive if $G_*(\underline{q}, d_1) < 0$ or, equivalently, if

$$P_*(\underline{q}, d_1) < S(r_*^d)(1 + r_*^d).$$

4 Competitive Equilibria for Sophisticated Banks

4.1 Equilibrium concept

We now turn to the case in which banks are sophisticated in their rating abilities so that they are able to detect the quality level i of an individual entrepreneur. They can thus determine entrepreneur-specific default probability. Of course, in practice rating individual entrepreneurs is a costly screening activity. At the cost of additional notational complexity, such expenditures for screening can easily be integrated into our analysis.[7] In accordance with the capital requirements of the first Basel Accord we will assume that the banks' debt/equity ratios are the same across loans. Since the loan size I is fixed, this implies that each loan is backed by the same amount of equity and deposits. Leaving unchanged all assumptions stated at the outset, the key difference to the simple banking system now is the requirement that sophisticated banks charge an *actuarially fair interest rate* for each loan. Let

$$\mathcal{R}(i, r_i^c) := \int_{\underline{q}}^{\overline{q}} \min\left\{q(1+i)f,\ I(1+r_i^c)\right\}\ h(q)\mathrm{d}q \tag{19}$$

denote the expected repayment from an entrepreneur with quality level i who has received a loan size I at the interest rate r_i^c. The highest possible average repayment of entrepreneur i is

$$\mathcal{R}(i, r_i^c) = \mathbb{E}[q](1+i)f,$$

which is attained for all loan-interest rates $r_i^c \geq \frac{\overline{q}(1+i)f}{I} - 1$.

In requiring banks to charge actuarially fair loan-interest rates, average repayments to banks have to be equal across entrepreneurs. Given some fixed amount R_0 to be repaid by producing entrepreneurs, an actuarially fair interest rate r_i^c for entrepreneur i has to be such that

$$\mathcal{R}\big(i, r_i^c\big) \stackrel{!}{=} R_0 \quad \text{for all} \quad i \in [i_{\text{low}}, \eta], \tag{20}$$

where $0 \leq i_{\text{low}} < \eta$ denotes the entrepreneur with the lowest quality level who is able to repay the amount R_0 on average. An average repayment R_0, which is attainable at least for entrepreneurs with sufficiently high qualities, must therefore satisfy

$$R_0 < \mathbb{E}[q](1+\eta)f.$$

Since $\mathcal{R}(i, r_i^c)$ in equation (19) is non-decreasing in quality levels, interest rates r_i^c satisfying (20) will have to be non-increasing for all $i \geq i_{\text{low}}$. Since all entrepreneurs $i < i_{\text{low}}$ will never be able to repay R_0 on average, we assume that $r_i^c = r_{i_{\text{low}}}^c$ for all $i \in [0, i_{\text{low}}]$. Sophisticated banks may thus reward high-quality entrepreneurs with low

[7] Such costs tend to increase the likelihood of banking instability, so our current set-up represents the best-case scenario for sophisticated banks.

interest rates while making it unattractive for low-quality entrepreneurs to apply for loan contracts.

To see that expected return on equity is equal across producing entrepreneurs, suppose for a moment that all entrepreneurs above some quality level $i^o \geq i_{\text{low}}$ apply for loans. The credit volume then is $(\eta - i^o)I$, which must be balanced by total equity and deposits so that $d^o + S(r^{do}) = \big(\eta - i^o\big)I$. The required amounts of equity and deposits per loan I are $d^o/(\eta - i^o)$ and $S(r^{do})/(\eta - i^o)$, respectively.

Intuitively, a competitive equilibrium for a sophisticated banking system with unlimited liability is a list consisting of a critical entrepreneur, an equity level, and loan-interest rates

$$\{i^o_*, r^{do}_*, \{r^{co}_*(i)\}^{\eta}_{i=0}\},$$

such that

(i) it is optimal for banks to offer their intermediation services at actuarially fair interest rates $r^c_i = r^{co}_*(i)$, $i \in [0, \eta]$

(ii) entrepreneurs take optimal investment decisions,

(iii) the market for loans is balanced.

Formally, a competitive equilibrium with financial intermediation for a sophisticated banking system is defined as follows:

Definition 2

Let $d_1 \in [0, \overline{d}]$ be given and assume that the value of physical capital satisfies $k_1 > (1 - \eta)W\, r_A$. A competitive equilibrium *of a sophisticated banking system is a list*

$$\Big\{i^o_*, r^{do}_*, \{r^{co}_*(i)\}^{\eta}_{i=0}\Big\}$$

consisting of a deposit-interest rate r^{do}_* and loan-interest rates $r^c_{*i} = r^{co}_*(i)$ with $i^o_* = i_E(d_1, r^{do}_*)$ such that

$$\mathcal{R}\big(i, r^{co}_*(i)\big) = \frac{d_1 + k_1}{\eta - i^o_*}(1 + r_A) + \frac{S(r^{do}_*)}{\eta - i^o_*}(1 + r^{do}_*), \quad i \in [i^o_*, \eta], \tag{21}$$

$$\Pi\big(i^o_*, r^{co}_*(i^o_*)\big) = W(1 + r_A), \tag{22}$$

$$\big[\eta - i^o_*\big]I = S(r^{do}_*) + d_1, \tag{23}$$

$$r^{do}_* < r_A. \tag{24}$$

The equilibrium notion derives naturally from the corresponding Definition 1 for a simple banking system. condition (21) states that on average banks must receive the same repayment on each loan. condition (22) is the indifference condition for the critical entrepreneur i^o_*. Since $\Pi(i, r^c)$ is increasing in quality levels i and decreasing in interest rates r^c locally around $(i^o_*, r^{co}(i^o_*))$ and $r^{co}_*(i)$ is non-increasing in i, all entrepreneurs $i < i^o_*$ either provide equity or invest in the alternative project, whereas all entrepreneurs $i \geq i^o_*$ apply for loans and invest in their production project. As before, (23) is the equilibrium condition in the loan market that determines the required equity level.

condition (21) is equivalent to the free-exit and free-entry condition, as a bank that offers its intermediation service for entrepreneurs will earn an expected return on equity of $1 + r_A$ if it employs $\frac{d_1+k_1}{\eta - i^o_*}$ as equity and $\frac{S(r^{do}_*)}{\eta - i^o_*}$ as deposits for funding an individual borrower. To verify that no banks exit or enter a sophisticated banking system in equilibrium, we proceed as follows: The repayments to banks in a sophisticated banking system are

$$P^o_*(q, d_1) = \int_{i_E(d_1, r^{do}_*)}^{\eta} \min\Big\{q(1+i)f,\ I\big[1 + r^{co}_*(i)\big]\Big\} \mathrm{d}i. \tag{25}$$

Taking expectations and using (21), the expected repayments in a sophisticated equilibrium are given by

$$\mathbb{E}\big[P^o_*(q, d_1)\big] = [\eta - i^o_*]\,\mathcal{R}\big(i^o_*, r^{co}_*(i^o_*)\big). \tag{26}$$

Using (21) and (25), second-period equity of the sophisticated system in equilibrium becomes

$$\begin{aligned} d^o_2 = G^o_*(q, d_1) \;&:=\; P^o_*(q, d_1) \;-\; S(r^{do}_*)\big[1 + r^{do}_*\big] \\ &=\; P^o_*(q, d_1) - \mathbb{E}\big[P^o_*(\cdot, d_1)\big] \;+\; e_1(1 + r_A). \end{aligned} \tag{27}$$

Thus

$$\mathbb{E}\left[G^o_*(\cdot, d_1)\right] = \; e_1(1 + r_A) \tag{28}$$

and the free-exit and free-entry condition is satisfied in equilibrium. Using (28) and the assumption in Definition 2 that $k_1 > (1 - \eta)W r_A$, it is straightforward to see that

$$\mathbb{E}[G^o_*(\cdot, d_1)] > d_1(1 + r_A) + S(r^{do}_*)(r_A - r^{do}_*) \tag{29}$$

holds in equilibrium. Thus (29) shows that banks are better off by financing entrepreneurs than by simply investing all of the funds they collect from savers in the risk-less alternative investment project.

4.2 Existence of sophisticated equilibria

The existence of sophisticated equilibria can be established as follows:

Proposition 2

Under the hypotheses of Proposition 1, a sophisticated banking system admits a unique sophisticated equilibrium

$$\left\{ i_*, r_*^{do}, \left\{ r_*^{co}(i) \right\}_{i=0}^{\eta} \right\}.$$

The equilibrium interest rates satisfy $0 < r_*^{do} < r_A < r_*^{co}(\eta)$ *and the intermediation margins for producing entrepreneurs are positive, that is,* $r_*^{co}(i) - r_*^{do} > 0$ *for all* $i \in [i_*^o, \eta]$*. Moreover,* $\underline{i} \leq i_*^o = i_E(d_1, r_*^d) \leq \overline{i}$*.*

Corollary 2

(i) Loan interest rates $r_*^{co}(i)$ *are decreasing in quality levels* $i \in [i_*^o, \eta]$ *that satisfy*

$$\underline{q}(1+i)f < I\left[1 + r_*^{co}(i)\right],$$

i.e., for all entrepreneurs who face a positive default risk.

(ii) If

$$W(1+r_A) > \left[(1+\overline{i})\mathbb{E}[q] - (1+\eta)\underline{q}\right]f,$$

then there exists an entrepreneur $i_{\text{NB}} \in [i_*^o, \eta]$ *with*

$$\underline{q}(1+i_{\text{NB}})f = I\left[1 + r_*^{co}(i_{\text{NB}})\right],$$

so that all entrepreneurs $i \geq i_{NB}$ *have zero default risk. For these entrepreneurs, interest rates are constant and satisfy* $r_*^{co}(i) = r_*^{co}(i_{\text{NB}})$ *for all* $i \in [i_{\text{NB}}, \eta]$*.*

The proofs of Proposition 2 and Corollary 2 are given in the appendix. Observe that Corollary 2 shows that a sophisticated banking system provides a floor for the loan-interest rates of entrepreneurs who will repay with certainty. All these entrepreneurs pay the same loan-interest rate. All other producing entrepreneurs pay higher interest, the loan-interest rate being monotonically decreasing with the quality of their production projects.

The special case of an inelastic savings function $S(r^d) = \overline{S}$ for all $r^d \geq 0$ is as follows: Observe that i_*^o is independent of r_*^{do} and equal to $i_* = \frac{\eta I - \overline{S} - d_1}{I}$. As a consequence, condition (22) coincides with condition (11) so that $r_*^{co}(i_*^o) = r_*^c$. This implies that the entrepreneur with the lowest quality level $i_* = i_*^o$ obtains the same loan contract in both systems. Using Proposition 1, we obtain

Lemma 2

If the savings function is inelastic such that $S(r^d) = \overline{S}$ for all $r^d \geq 0$, then $i_*^o = i_* = \frac{\overline{d} - d_1}{I}$ and the loan interest rates are determined by

$$\Pi\big(i_*^o, r_*^c\big) = W(1 + r_A)$$

and

$$\mathcal{R}\big(i, r_*^{co}(i)\big) = \mathcal{R}(i_*^o, r_*^c), \quad i \in [i_*^o, \eta]. \tag{30}$$

The deposit-interest rate is given by

$$r_*^{do} = \frac{1}{\overline{S}}\Big\{[\eta - i_*^o]\mathcal{R}(i_*^o, r_*^c) - [d_1 + k_1](1 + r_A)\Big\} - 1 \tag{31}$$

Note that in the case of an inelastic savings function, the equilibrium loan-interest rates $r_*^{co}(i)$ in Lemma 2 are independent of physical capital k_1, whereas the equilibrium deposit-interest rate r_*^{do} is not.

4.3 Instability

An entrepreneur with quality level i will go bankrupt if he is unable to fully pay back his credit, which is the case if

$$q(1 + i)f < I\big[1 + r_*^c(i)\big].$$

The critical shock below which entrepreneur i will go bankrupt in the sophisticated system is denoted by $q_{\mathrm{NB}}^o(i)$ and given by

$$q_{\mathrm{NB}}^o(i) := \frac{I\big[1 + r_*^{co}(i)\big]}{(1 + i)f}.$$

Since $r_*^{co}(i)$ is non-increasing in i, $q_{\mathrm{NB}}^o(i)$ is strictly decreasing in i.

It follows immediately from the indifference condition (22) that the critical entrepreneur i_*^o does not go bankrupt in equilibrium with positive probability. Hence, in equilibrium all producing entrepreneurs will fully repay their obligations with positive probability. On the other hand, bankruptcies will only occur with positive probability if $q_{\mathrm{NB}}^o(i_*^o) > \underline{q}$. If $q_{\mathrm{NB}}^o(i_*^o) \leq \underline{q}$, we infer from Corollary 2 that $r_*^{co}(i) = r_*^{co}(i_*^o)$ is constant for all $i \in [i_*^o, \eta]$. The following lemma now gives sufficient conditions for the case in which at least some entrepreneurs will default with positive probability.

Lemma 3

In a sophisticated equilibrium, bankruptcies of producing entrepreneurs occur with positive probability, i.e. $q_{\mathrm{NB}}^o(i_*^o) > \underline{q}$, *if*

$$W(1 + r_A) < (\mathbb{E}[q] - \underline{q})(1 + \underline{i})f.$$

The proof of Lemma 3 is analogous to that of Lemma 1. We next examine the question of how stable a sophisticated banking system is. We infer from (27) that second-period equity is positive on average. Default occurs if $G_*^o(q, d_1) < 0$. Since P^o and therefore G^o is non-decreasing in shocks q, there exists a unique critical shock $\underline{q} \leq q_{\mathrm{crit}}^o < q$ such that the default probability of the sophisticated banking system is given by

$$\pi_{\mathrm{default}}^o := \mathrm{Prob}(G_*^o(q, d_1) < 0) = \int_{\underline{q}}^{q_{\mathrm{crit}}^o} h(q)\mathrm{d}q. \tag{32}$$

Assuming that probability density h is strictly positive on $[\underline{q}, \overline{q}]$, the default probability π_{default}^o is positive if $G_*^o(\underline{q}, d_1) < 0$ or, equivalently, if

$$P_*^o(\underline{q}, d_1) < S(r_*^{do})(1 + r_*^{do}).$$

5 Comparison of the Two Systems

5.1 The general case

For the comparison of the two banking systems, let us first focus on interest rates. We assume throughout this section that (17) holds, so that by Lemmas 1 and 3 both systems allow for entrepreneurial bankruptcies.

In the following theorem we compare the loan-interest rate in the simple banking system with the schedule of loan-interest rates in a sophisticated banking system.

Theorem 1

Let the hypotheses of Proposition 1 be satisfied and assume that (17) holds, so that in both systems entrepreneurial bankruptcies occur with positive probability. Then the following holds:

(i) Deposit interest rates satisfy $r_*^{do} < r_*^d$.

(ii) $i_*^o \geq i_*$, *where the inequality is strict for* $S' > 0$.

(iii) If the savings function S is sufficiently inelastic, then there exists an entrepreneur $i_{\mathrm{ER}} \in [i_*^o, \eta)$ *with*

$$\begin{aligned}
&(a) \quad r_*^c < r_*^{co}(i), \quad i \in [0, i_{\mathrm{ER}}),\\
&(b) \quad r_*^c = r_*^{co}(i_{\mathrm{ER}}),\\
&(c) \quad r_*^c > r_*^{co}(i), \quad i \in (i_{\mathrm{ER}}, \eta].
\end{aligned}$$

In particular, $i_{\mathrm{ER}} = i_*^o$ *if* $S' = 0$ *and* $i_{\mathrm{ER}} > i_*^o$ *if* $S' > 0$.

The intuition for Theorem 1 is as follows: In a sophisticated banking system, banks tailor loan-interest rates to the quality level of entrepreneurs. All entrepreneurs with $i > i_{\mathrm{ER}}$ obtain lower loan-interest rates in a sophisticated banking system than in a simple banking system. Since in the sophisticated system the average return on loans is lower than in the simple system, deposit rates in a sophisticated system must be lower than in a simple banking system, i.e., $r_*^{do} < r_*^d$.

To illustrate the consequences of Theorem 1, note that aggregate output in entrepreneurial production in response to a shock by $q \in [\underline{q}, \overline{q}]$ is

$$Y_*(q) = qf \int_{i_*}^{\eta} (1+i)\mathrm{d}i$$

for the simple banking system and

$$Y_*^o(q) = qf \int_{i_*^o}^{\eta} (1+i)\mathrm{d}i$$

for the sophisticated banking system. Since $i_*^o \geq i_*$ by Theorem 1 (ii), we have

$$Y_*^o(q) \leq Y_*(q) \quad \text{for all} \quad q \in [\underline{q}, \overline{q}]. \tag{33}$$

This implies that sophisticated banks will crowd out investment in entrepreneurial production. Note that the inequality in (33) is strict if consumers do not save inelastically.

We next address the question of which of the two banking systems will accumulate more second-period equity. It is readily seen from (16) and (27) that expected second-period equity of the simple banking system is the same as in the sophisticated system, namely equal to $e_1(1 + r_A)$. These two equations also imply the following result, which shows that on average sophisticated banks will have lower repayments than simple banks:

Proposition 3

Under the hypotheses of Proposition 1, expected repayments to the simple banking system are higher than to the sophisticated system, that is,

$$\mathbb{E}\big[P_*\big] - \mathbb{E}[P_*^o] = S(r_*^d)[1 + r_*^d] - S(r_*^{do})[1 + r_*^{do}] > 0. \tag{34}$$

We next analyze the conditions under which a simple banking system accumulates more capital than a sophisticated system. The banking system with lower bank capital will be less likely to collapse if macroeconomic shocks are adverse. As long as $S' \geq 0$ is sufficiently small so that the volume effect outweighs the price effect, one consequence of Theorem 1 is that repayments to simple banks are always higher than repayments to sophisticated banks:

$$P_*(q, d_1) \geq P_*^o(q, d_1) \quad \text{for all} \quad q \in \big[\underline{q}, \overline{q}\big]\,. \tag{35}$$

In order to compare the banking systems' ability to accumulate capital, recall that future bank capital of the simple banking system is determined by $d_2 = G_*(q, d_1)$ with G_* given in (16), while the future bank capital of a sophisticated system is determined by $d_2^o = G_*^o(q, d_1)$ with G_*^o given in (27). Given the same initial equity level $e_1 = d_1 + k_1$ and the same interest rate r_A, it follows from (16) and (27) that $d_2 \leq d_2^o$ if and only if

$$P_*(q, d_1) - P_*^o(q, d_1) \leq \mathbb{E}[P_*(\cdot, d_1)] - \mathbb{E}[P_*^o(\cdot, d_1)]. \tag{36}$$

The following proposition shows that the sophisticated banking system will accumulate more bank capital than the simple system for all shocks below a certain *'break-even'* value q_{BE}. As a consequence, a sophisticated system is better able to cope with highly adverse shocks than a simple system. The reverse is true for positive shocks. Here a simple system will accumulate more bank capital.

Proposition 4

Under the hypotheses of Theorem 1, assume that (17) holds such that in both banking systems entrepreneurial bankruptcies occur with positive probability. Then for given $d_1 \in [0, \overline{d}]$ *and* $k_1 > 0$*, there exists a critical shock* $q_{\mathrm{BE}} \in (\underline{q}, \overline{q})$ *such that*

$$\begin{aligned} &(i) \quad G_*(q, d_1) < G_*^o(q, d_1), \quad q \in [\underline{q}, q_{\mathrm{BE}}), \\ &(i) \quad G_*(q_{\mathrm{BE}}, d_1) = G_*^o(q_{\mathrm{BE}}, d_1), \\ &(iii) \quad G_*(q, d_1) > G_*^o(q, d_1), \quad q \in (q_{\mathrm{BE}}, \overline{q}], \end{aligned}$$

The proof of Proposition 4 is given in the appendix. Proposition 4 indicates that the sophisticated banking system outperforms the simple system for all sufficiently low shocks, while the simple system outperforms the sophisticated system for sufficiently high shocks. To illustrate this result, consider an extreme case in which all firms go bankrupt, thus causing a default of the sophisticated banking system. Such an adverse macroeconomic shock means that all firms will be bankrupt under a simple banking system as well. Since banks earn only the liquidation values, revenues in both banking systems are identical in this case. However, deposit rates are higher in a simple banking system, so their aggregate losses are higher. This explains the lower bank capital for a simple banking system if macroeconomic shocks are below the critical level q_{BE}.

On the other hand, the simple banking system accumulates more capital for sufficiently high shocks. Indeed, since $r_*^{co}(i) < r_*^c$ for $i > i_{\mathrm{ER}}$, we have

$$q_{\mathrm{NB}}^o(i) < q_{\mathrm{NB}} \quad \text{for all } i > i_{\mathrm{ER}}$$

with equality holding for the critical entrepreneur $i = i_{\mathrm{ER}}$. Hence the default risk of an individual entrepreneur is lower in a sophisticated system. Observe that (35) holds with strict inequality for all shocks $q > q_{\mathrm{NB}}^o(\eta)$. This means that repayments to simple banks are higher, provided the macroeconomic environment is sufficiently favorable. It follows from the equilibrium condition (22) that $q_{\mathrm{NB}}^o(\eta) < \overline{q}$ so that this scenario will occur with positive probability.

Our main theorem now shows that in terms of the default probabilities as defined in (18) and (32), the stability of the two banking systems depend on the initial level of equity.

Theorem 2

Let the hypotheses of Proposition 4 be satisfied.

(i) If

$$G^o_*(q_{\mathrm{BE}}, d_1) = G_*(q_{\mathrm{BE}}, d_1) > 0, \tag{37}$$

then the default probability of the sophisticated banking system is lower than the default probability of the simple banking system, i.e., $\pi^o_{\text{default}} < \pi_{\text{default}}$.

(ii) If, on the contrary,

$$G^o_*(q_{\mathrm{BE}}, d_1) = G_*(q_{\mathrm{BE}}, d_1) < 0, \tag{38}$$

then the default probability of the sophisticated banking system is higher than the default probability of the simple banking system, i.e. $\pi^o_{\text{default}} > \pi_{\text{default}}$.

In view of (16), condition (37) is clearly satisfied if initial equity e_1 is sufficiently high. To illustrate result (i) of Theorem 2, suppose a sufficiently adverse macroeconomic shock occurs. Although loan-interest rates are higher in a simple banking system, revenues do not fully reflect the interest-rate differentials with respect to the sophisticated banking system, as under both systems banks will only earn liquidation values for a substantial set of entrepreneurs. However, simple banks face higher refinancing costs $r^d_* > r^{do}_*$ that are unaffected by a macroeconomic shock. Hence simple banks are more likely to default than sophisticated banks when sufficiently adverse shocks occur.

On the other hand, (37) may be violated for a low level of initial equity e_1. Then banks may default if moderate adverse macroeconomic shocks and a small number of firm bankruptcies occur. In this case, a simple banking system has more benefits from higher average loan rates, which may outweigh the higher refinancing costs in comparison to a sophisticated system. While entrepreneurs benefit from lower loan interest rates in the sophisticated system, the system itself may lack a sufficient number of repayments. As a consequence, sophistication may decrease bank stability.

We conclude the comparison with a welfare discussion. Consider first the expected aggregate consumption of entrepreneurs in both systems. For an economy with a simple banking system, expected consumption of all entrepreneurs is

$$\mathbb{E}[C_E] \;=\; i_* W(1 + r_A) + \int_{i_*}^{\eta} \Pi(i, r^c_*)\mathrm{d}i, \tag{39}$$

whereas expected aggregate consumption of entrepreneurs in an economy with a sophisticated banking system is

$$\mathbb{E}[C_E^o] \quad = \quad i_*^o W(1 + r_A) + \int_{i_*^o}^{\eta} \Pi\big(i, r_*^c(i)\big) \mathrm{d}i. \tag{40}$$

We obtain

Theorem 3

Let the hypotheses of Proposition 1 be satisfied and assume that bankruptcies occur with positive probability. Then expected aggregate consumption of entrepreneurs in an economy with a simple banking system is lower than in an economy with a sophisticated system.

The proof of the theorem is given in the appendix. Theorem 3 shows that in general, risk-neutral entrepreneurs will always prefer an economy with a sophisticated banking system. Hence sophistication in banking may entail a trade-off between stability and expected aggregate consumption of entrepreneurs. However, entrepreneurs with intermediate quality levels who obtain funding from a simple banking system are worse off with a sophisticated system. Since $r_*^d > r_*^{do}$ consumers, by contrast, will always prefer the simple system.

Any further analysis regarding welfare and income-distributional consequences of sophistication in rating depends on how banks are bailed out in the case of defaulting positive probability. Let us briefly discuss two scenarios assuming that case (i) of Theorem 2 holds. Suppose first that banks are bailed out by future (unmodeled) generations. This scenario is well-known in the banking crisis literature, when countries have responded to large-scale banking problems by drastically lowering future short-term interest rates. This can generate large profits for banks and may thus allow them to recover (see e.g. Hellman, Murdock & Stiglitz (2000) and Hoshi & Kashyap 2004). Under such circumstances, the current generation in our model does not bear the cost of a bail-out. In this scenario, expected aggregate consumption and aggregate welfare of the current generation is higher with a sophisticated system than with a simple banking system, while future generations are worse off. A complete welfare analysis, however, calls for a fully fledged OLG model and must be left for future research.

Suppose second that the current generation bears the cost of the bail-out. This implies that the current generation has to provide rescue funds for a defaulting bank. A priori, consumers in such a scenario tend to be worse off with a sophisticated banking system than with a simple banking system if they have to contribute to rescue funds, whereas the effect on entrepreneurs is ambiguous. If bail-out schemes are anticipated by agents, their decision problems have to be adapted before a welfare analysis can be conducted. This must be left to future research.

5.2 Uniformly distributed shocks

To derive more tractable results and obtain explicit loan-interest rates, we will assume that the macroeconomic productivity shocks are uniformly distributed on $[\underline{q}, \overline{q}]$ and that the savings function is inelastic. Throughout this section we assume that condition (17) is satisfied so that $q_{\mathrm{NB}} > \underline{q}$. This condition now takes the form

$$W(1 + r_A) \leq (1 + \underline{i})f\left(\tfrac{\overline{q}-\underline{q}}{2}\right).$$

The following lemma gives explicit interest rates for this case:

Lemma 4

Under the hypotheses of Proposition 1, assume that savings are inelastic and that the macroeconomic shocks are uniformly distributed, i.e. $h(q) := \frac{1}{\overline{q}-\underline{q}}$. *Then the competitive loan-interest rate takes the form*

$$1 + r_*^c = \frac{(1+i_*)f}{I}\left[\overline{q} - \sqrt{\frac{2(\overline{q}-\underline{q})W}{(1+i_*)f}(1+r_A)}\right], \tag{41}$$

where $i_* = \frac{\eta I - \overline{S} - d_1}{I}$.

The proof of Lemma 4 is given in the appendix. Setting $\underline{r}(i) = (1+i)\underline{q}f/I - 1$ and $\overline{r}(i) = (1+i)\overline{q}f/I - 1$, the expected repayment from entrepreneur i, given an interest rate r^c, is

$$\mathcal{R}(i, r^c) = \begin{cases} I(1+r^c) & \text{if } r^c < \underline{r}(i), \\ \frac{1}{\overline{q}-\underline{q}}\left\{\overline{q}I(1+r^c) - \frac{I^2(1+r^c)^2}{2(1+i)f} - \frac{1}{2}\underline{q}^2(1+i)f\right\} & \text{if } \underline{r}(i) \leq r^c \leq \overline{r}(i), \\ \frac{\overline{q}+\underline{q}}{2}(1+i)f & \text{if } \overline{r}(i) < r^c. \end{cases} \tag{42}$$

Inserting the loan-interest function given by Lemma 4 into (42) and using the fact that loan-interest rates of both banking systems coincide for the lowest-quality entrepreneur as in Lemma 2, we obtain

$$\mathcal{R}\big(i_*, r_*^c\big) = \tfrac{\overline{q}+\underline{q}}{2}(1+i_*)f - W(1+r_A). \tag{43}$$

We can now solve condition (30) for $r_*^{co}(i)$ for each producing entrepreneur i to obtain the following lemma:

Lemma 5

Under the hypotheses of Proposition 2, assume that the macroeconomic shocks are uniformly distributed such that $h(q) := \frac{1}{\overline{q}-\underline{q}}$. Then the deposit-interest rate takes the form

$$1 + r_*^{do} = \frac{1}{\overline{S}}\Big([\eta - i_*]\tfrac{\overline{q}+\underline{q}}{2}(1+i_*)f - \big([\eta - i_*]W + e_1\big)(1+r_A)\Big). \tag{44}$$

The critical quality level i_{NB} above which no entrepreneur goes bankrupt is given by

$$1 + i_{\mathrm{NB}} = (1+i_*)\tfrac{\overline{q}+\underline{q}}{2\underline{q}} - \tfrac{W(1+r_A)}{\underline{q}f} \tag{45}$$

The loan-interest rates for producing entrepreneurs in a sophisticated banking equilibrium are given by

$$1 + r_*^{co}(i) = \begin{cases} \frac{(1+i)f}{I}\left[\overline{q} - \sqrt{\frac{i-i_*}{1+i}(\overline{q}^2 - \underline{q}^2) + \frac{2(\overline{q}-\underline{q})W}{(1+i)f}(1+r_A)}\right] & \text{if } i \in [i_*, i_{\mathrm{NB}}], \\ \frac{\mathcal{R}(i_*, r_*^c)}{I} & \text{otherwise.} \end{cases} \tag{46}$$

We observe that $r_*^{co}(i)$ exhibits the typical pattern. The loan interest is decreasing in the quality level i for $i \in [i_*, i_{\mathrm{NB}}]$, while it is constant for $i \geq i_{\mathrm{NB}}$.

6 Conclusion

This paper demonstrates that more sophistication in the assessment of individual default risks of entrepreneurs will only increase banking stability if initial equity is sufficiently high. We have shown that sophistication in risk management rewards high-quality entrepreneurs with lower loan rates at the expense of depositors facing lower returns. Sophistication in rating techniques thus has distributional implications for both sides of the market. Our analysis suggests that regulatory policies for banking such as Basel II, which require banks to introduce more sophistication in assessing the credit-worthiness their clients, will only be beneficial if banks are sufficiently healthy in terms of equity.

7 Appendix

Proof of Proposition 1.

To simplify notation, we will throughout the proof take d_1 as a constant and write $i_E(r^d)$ instead of $i_E(d_1, r^d)$. Furthermore, we denote the expected profit of the critical investor by Π_E, i. e.

$$\Pi_E(r^d, r^c) := \Pi\big(i_E(r^d), r^c\big).$$

Since each $\Pi(i, r^c)$ is non-decreasing in i and $i_E(r^d)$ is non-increasing in r^d, $\Pi_E(r^d, r^c)$ is non-increasing as a function of r^d (with r^c being held constant). We immediately observe that $\Pi_E(r^d, r^c)$ is non-increasing as a function of r^c (with r^d being held constant). The proof now proceeds in four steps.

Step 1. Consider first condition (11), which takes the form

$$\Pi_E(r^d, r^c) - W\,(1 + r_A) \overset{!}{=} 0. \tag{47}$$

Noting that $\underline{i} \le i_E(r^d) \le \overline{i}$, condition (ii,a) implies that for each $r^d \in [0, r_A]$,

$$\Pi_E(r^d, 0) \ge \Pi_E(r_A, 0) > W\,(1 + r_A).$$

For each $i \in [0, \eta]$ we have $\Pi(i, r^c) = 0$ for all $r^c \ge \overline{r}^c(i)$, where

$$\overline{r}^c(i) := \frac{(1+i)\overline{q}f}{I} - 1, \quad i \in [0, \eta]. \tag{48}$$

Note that each $\Pi_E(r^d, r^c)$ is strictly decreasing for $r^c \le \overline{r}^c(i)$, as is the case for each $\Pi(i, r^c)$. Thus, for each $r^d \in [0, r_A]$ there exists a uniquely determined $r^c = h(r^d)$ solving (47). Since $\Pi_E(r^d, r^c)$ is non-increasing in r^d, $h(r^d)$ is non-increasing in r^d as well. The map $r^d \mapsto h(r^d)$ is continuous and defines a curve in the r^d-r^c plane. From condition (ii,a) and the monotonicity of Π_E it follows that

$$\Pi_E(0, 0) \ge \Pi_E(r_A, 0) > W\,(1 + r_A) > \Pi_E(0, r_A) \ge \Pi_E(r_A, r_A),$$

which implies $h(0) > 0$ and $h(r_A) < r_A$.

Step 2. In order to examine the no-entry no-exit condition (10), consider the function

$$F(r^d, r^c) := \frac{\mathbb{E}\Big[P\big(\cdot, i_E(r^d), r^c\big)\Big]}{\eta - i_E(r^d)} - \frac{(d_1 + k_1)(1 + r_A) - S(r^d)(1 + r^d)}{\eta - i_E(r^d)}, \tag{49}$$

where $r^d \in [0, r_A]$ and $r^c \geq 0$. condition (10) then takes the form $F(r^d, r^c) \stackrel{!}{=} 0$. As $i_E(r^d)$ is non-increasing in r^d, it follows from Gersbach & Wenzelburger (2007, Lemma 2) that the first term in (49) is non-increasing in r^d. Differentiating the second term in (49) with respect to r^d, we get

$$\frac{\mathrm{d}}{\mathrm{d}r^d}\left(\frac{(d_1+k_1)(1+r_A)+S(r^d)(1+r^d)}{\eta - i_E(r^d)}\right) > 0, \quad r^d \in (0, r_A), \tag{50}$$

if and only if

$$\frac{S(r^d)+d_1}{k_1(1+r_A)+d_1(r_A-r^d)} > \frac{S'(r^d)}{S(r^d)}, \quad r^d \in (0, r_A).$$

This condition is implied by condition (i). Hence F is strictly decreasing in r^d.

Since by definition of i_E, $\big[\eta - i_E(r^d)\big]I = S(r^d)+d_1$ and since repayments P are always less than $\big[\eta - i_E(r^d)\big]\, I(1+r^c)$, we obtain

$$F(r^d, r^d) \leq -\left(\frac{k_1(1+r_A)+d_1(r_A-r^d)}{\eta - i_E(r^d)}\right) < 0. \tag{51}$$

In view of (48), we have for each $i \in [0, \eta]$,

$$\mathbb{E}\big[P\big(\cdot, i, r^c\big)\big] \geq [\eta - i](1+i)\mathbb{E}[q]f \quad \text{for all} \;\; r^c \geq \overline{r}^c(i).$$

Using this inequality, again the fact that by construction $d_1 = \big[\eta - i_E(r^d)\big]I - S(r^d)$, and condition (ii,b), we obtain for each $r^c \geq \overline{r}^c\big(i_E(r^d)\big)$, $r^d \in [0, r_A]$,

$$\begin{aligned}
&\mathbb{E}\big[P(\cdot, i_E(r^d), r^c)\big] - (d_1+k_1)(1+r_A) - S(r^d)(1+r^d) \\
&\geq \big[\eta - i_E(r^d)\big]\big[1+i_E(r^d)\big]\mathbb{E}[q]f - (d_1+k_1)(1+r_A) - S(r^d)(1+r^d) \\
&\geq \big[\eta - i_E(r^d)\big]\big[1+i_E(r^d)\big]\mathbb{E}[q]f - k_1(1+r_A) - \big[\eta - i_E(r^d)\big]I(1+r_A) \\
&> 0.
\end{aligned}$$

Hence $F(r^d, r^c) > 0$ for all $r^c \geq \overline{r}^c\big(i_E(r^d)\big)$, $r^d \in [0, r_A]$. Since F is continuous, the Intermediate Value Theorem implies the existence of a continuous map g such that

$$F\big(r^d, g(r^d)\big) = 0, \quad r^d \in [0, r_A].$$

Equation (51) implies $g(r^d) > r^d$ for all $r^d \in [0, r_A]$. The map g is strictly increasing because F is strictly decreasing in r^d. Thus $g(r^d)$, $r^d \in [0, r_A]$ describes a strictly upward-sloping curve in the r^d-r^c plane which lies above the 45°-line.

Step 3. Any competitive equilibrium (r^d_*, r^c_*) is determined by an intersection point of the two previously defined curves h and g given by

$$r^c_* = h(r^d_*) = g(r^d_*).$$

It follows from the monotonicity properties of h and g that a competitive equilibrium is unique once it exists. Since $g(r_A) > r_A > h(r_A)$, it remains to show that $g(0) < h(0)$. This condition is satisfied if $\Pi_E\big(0, g(0)\big) > \Pi_E\big(0, h(0)\big)$ or, using (47), if

$$\Pi_E\big(0, g(0)\big) > W(1 + r_A). \tag{52}$$

Note to this end that

$$\mathcal{R}\big(i, r^c\big) + \Pi\big(i, r^c\big) = (1 + i)\mathbb{E}[q]f \quad \text{for all} \;\; i \in [0, \eta], \;\; r^c \geq 0, \tag{53}$$

and

$$\mathbb{E}\big[P(\cdot, i, r^c)\big] \geq (\eta - i)\mathcal{R}(i, r^c) \quad \text{for all} \;\; i \in [0, \eta], \;\; r^c \geq 0. \tag{54}$$

Using (53), (54), (10) and the definition of i_E, we obtain

$$\begin{aligned}
\Pi_E\big(0, g(0)\big) &\geq \big[1 + \bar{i}\big]\mathbb{E}[q]f - \frac{\mathbb{E}\big[P\big(\cdot, \bar{i}, g(0)\big)\big]}{\eta - \bar{i}} \\
&= \big[1 + \bar{i}\big]\mathbb{E}[q]f - \frac{(d_1 + k_1)(1 + r_A) + S(0)}{\eta - \bar{i}} \\
&\geq \big[1 + \bar{i}\big]\mathbb{E}[q]f - \frac{k_1(1 + r_A)}{\eta - \bar{i}} - I(1 + r_A).
\end{aligned}$$

Condition (ii,c) now implies (52). Hence $g(0) < h(0)$.

Step 4. From Step 1 we see that $0 < r^d_* < r_A$. Since $i_E(r^d)$ is strictly decreasing in r^d, we have $\underline{i} \leq i_* \leq \bar{i}$. The fact that $r^c_* > r_A$ follows directly from (10) and the assumption that $k_1 > (1 - \eta)W r_A$, so that the intermediation margin is positive. This completes the proof.

■

Proof of Lemma 1.

Observe first that entrepreneur $i \in [0, \eta]$ defaults with positive probability for all loan-interest rates

$$r^c > \underline{r}^c(i) := \frac{\underline{q}(1 + i)f}{I} - 1.$$

For the critical entrepreneur i_* we have

$$\Pi\big(i_*, \underline{r}^c(i_*)\big) = (\mathbb{E}[q] - \underline{q})(1 + i_*)f.$$

Since $\Pi(i_*, \cdot)$ is decreasing in $r^c \leq \overline{r}^c(i_*)$, the equilibrium interest rate r^c_* is greater than $\underline{r}^c(i_*)$ if and only if $W(1 + r_A) < (\mathbb{E}[q] - \underline{q})(1 + i_*)f$. Since in equilibrium $i_* \geq \underline{i}$, a sufficient condition for this event is (17).

■

Proof of Proposition 2.

Consider the function $h : [0, \eta] \to [0, \overline{r}^c_\eta]$ with $r^c = h(i)$ as defined in the first step of the proof of Proposition 1 and note that by condition (ii,a) we have

$$h(\underline{i}) < r_A \quad \text{and} \quad 0 < h(i) < \tfrac{\overline{q}(1+i)f}{I} - 1 \quad \text{for each } i \in [\underline{i}, \eta]. \tag{55}$$

The rest of the proof proceeds in four steps.

Step 1. As regards the no-entry no-exit condition (21), consider the function

$$\tilde{F}(r^d, r^c) := \mathcal{R}\big(i_E(d, r^d), r^c\big) - \frac{(d_1 + k_1)(1 + r_A) + S(r^d)(1 + r^d)}{\big[\eta - i_E(d_1, r^d)\big]}, \tag{56}$$

where $r^d \in [0, r_A]$ and $r^d \geq 0$. condition (21) then takes the form $\tilde{F}(r^d, r^c) \overset{!}{=} 0$. The the first term in (56) is non-increasing in r^d, as $i_E(d_1, r^d)$ is non-increasing in r^d and $\mathcal{R}(i, r^c)$ non-decreasing in i. By the same reasoning as in Step 2 of the proof of Proposition 1, the second term in (56) is increasing with respect to r^d. Hence $\tilde{F}$ is decreasing in r^d.

Since repayments $\mathcal{R}$ are always less than $I(1 + r^c)$, we obtain

$$\tilde{F}(r^d, r^d) \leq -\frac{k_1(1 + r_A) + d_1(r_A - r^d)}{\big[\eta - i_E(d_1, r^d)\big]} < 0. \tag{57}$$

On the other hand, we have

$$\mathcal{R}\big(i, r^c\big) = \big[1 + i\big]\mathbb{E}[q]f \quad \text{for all} \;\; r^c \geq \overline{r}^c(i), \;\; i \in [0, \eta]$$

and Assumption (ii,b) implies

$$\tilde{F}(r^d, r^c) > 0 \quad \text{for all} \;\; r^c \geq \overline{r}^c\big(i_E(d_1, r^d)\big), \;\; r^d \in [0, r_A].$$

Since $\tilde{F}$ is increasing in r^c, the Intermediate Value Theorem then implies for each the existence of a continuous map $\tilde{g}$ such that

$$\tilde{F}\big(r^d, \tilde{g}(r^d)\big) = 0, \quad r^d \in [0, r_A].$$

Equation (57) implies $\tilde{g}(r^d) > r^d$ for all $r^d \in [0, r_A]$. The map $\tilde{g}$ is increasing because $\tilde{F}(r^d, r^c)$ is strictly decreasing in r^d. Hence $\tilde{g}(r^d)$, $r^d \in [0, r_A]$ describes an upward-sloping curve in the r^d-r^c plane that lies above the 45°-line.

Step 2. In any sophisticated equilibrium, the interest rates $(r_*^{do}, r_{i_*^o}^c)$ are determined by an intersection point of the two previously defined curves h and $\tilde{g}$ given by

$$r_{i_*^o}^c = h(r_*^{do}) = \tilde{g}(r_*^{do}).$$

Since $\tilde{g}(r_A) > r_A > h(r_A)$, it remains to show that $\tilde{g}(0) < h(0)$. This condition is satisfied if $\Pi\big(i_E(d_1, 0), \tilde{g}(0)\big) > \Pi\big(i_E(d_1, 0), h(0)\big)$ or, using (22), if

$$\Pi\big(i_E(d_1, 0), \tilde{g}(0)\big) > W(1 + r_A) \tag{58}$$

Using (53), we see that (58) holds if

$$\big[1 + i_E(d_1, 0)\big]\mathbb{E}[q]f - \mathcal{R}\big(i_E(d_1, 0), \tilde{g}(0)\big) > W(1 + r_A).$$

The latter equation is implied by Assumption (ii,c), using (21).

Step 3. From Step 2 we know that there exists a unique loan interest rate $r_{i_*^o}^c = h(i_*^o)$ such that (21) holds for $i_*^o = i_E(d, r_*^{do})$. We show now that (21) is satisfied for all $i \in [i_*^o, \eta]$. It follows from the second inequality in (55) that

$$\overline{q}(1 + i)f > I(1 + r_{i_*^o}^c) \quad \text{for all} \;\; i \in [i_*^o, \eta].$$

Note that $\mathcal{R}(i, r^c)$ is strictly increasing in r^c as long as $\overline{q}(1+i)f > I(1+r^c)$ and strictly increasing in i as long as $\underline{q}(1 + i)f < I(1 + r^c)$. This implies the existence of a unique function $r_*^{co}(i)$, $i \in [i_*^o, \eta]$ with $r_*^{co}(i_*^o) = r_{i_*^o}^c$ such that

$$\mathcal{R}\big(i, r_*^{co}(i)\big) = \mathcal{R}(i_*^o, r_{i_*^o}^c), \quad i \in [i_*^o, \eta]. \tag{59}$$

Thus condition (21) is satisfied.

Step 4. We need to define $r_*^{co}(i)$ for all quality levels $i \in [0, \eta]$. Note that $r_*^{co}(i)$ can be defined via (59) for all $i \in [0, i_*^o]$ that satisfy

$$\mathbb{E}[q](1+i)f \geq \mathcal{R}\big(i_*^o, r_{i_*^o}^c\big). \tag{60}$$

Suppose there exists $i_{\text{low}} \geq 0$ such that (60) holds with equality. Setting $r_*^{co}(i) = r_*^{co}(i_{\text{low}})$ for all $i \in [0, i_{\text{low}}]$, the function $r_*^{co}(i)$ is then extended to a non-increasing function on all of $[0, \eta]$. Clearly $i \mapsto r_*^{co}(i)$ is continuous on $[0, \eta]$ since $\mathcal{R}$ is continuous. Hence

$$\Pi\big(i, r_*^{co}(i)\big) < W(1 + r_A), \quad \text{for all } \; i \in [0, i_*^o),$$

showing that all entrepreneurs $i < i_*^o$ either provide equity or invest in the alternative project, whereas all entrepreneurs $i \geq i_*^o$ invest in their production project.

Step 5. It follows from $\tilde{g}(0) < h(0)$ and Assumption (ii,a) that $0 < r_*^{do} < r_A$ and hence $\underline{i} \leq i_*^o \leq \bar{i}$. The fact that $r_*^{co}(\eta) > r_A$ follows directly from (21) and the assumption that $k_1 > (1-\eta)Wr_A$ so that the intermediation margins are positive for producing entrepreneurs. This completes the proof.

■

Proof of Theorem 1.

(i) By the mean value theorem for integrals, for each $r^d \in [0, r_A]$, $r^c \geq 0$ there exists $\tilde{i} \in [i_E(d_1, r^d), \eta]$, such that

$$\frac{\mathbb{E}\Big[P\big(\cdot, i_E(d_1, r^d), r^c\big)\Big]}{\big[\eta - i_E(d_1, r^d)\big]} = \mathcal{R}(\tilde{i}, r^c).$$

Since $\mathcal{R}$ is non-decreasing in i, $\mathcal{R}\big(i_E(d_1, r^d), r^c\big) \leq \mathcal{R}(\tilde{i}, r^c)$. Hence $\tilde{F}(r^d, r^c) \leq F(r^d, r^c)$ and since both $\tilde{F}$ and F are increasing in r^c, $g(r^d) \leq \tilde{g}(r^d)$. Since $r^d \in [0, r_A]$ was arbitrary, $r_*^{do} \leq r_*^d$ and $r_*^c \leq r_*^{co}(i_*^o)$. Suppose now $r_*^{do} = r_*^d$. Then $i_*^o = i_*$, and the equilibrium conditions (10) and (21) imply

$$\frac{\mathbb{E}\Big[P\big(\cdot, i_*, r_*^c\big)\Big]}{\big[\eta - i_*\big]} = \mathcal{R}(i_*, r_*^c). \tag{61}$$

Since $\mathcal{R}$ is non-decreasing in i, (61) can only hold if $\underline{q}(1 + i_*)f \geq I(1 + r_*^c)$. However, this was ruled out by assumption, so that $r_*^{do} < r_*^d$.

(ii) This follows immediately from (i), using $i_* = i_E(d_1, r_*^d)$ and $i_*^o = i_E(d_1, r_*^{do})$.

(iii) If $S' = 0$, then $i_{\text{ER}} = i_* = i_*^o$, and the assertion follows from the fact that bankruptcies occur in equilibrium. In this case, $r_*^{co}(\eta) < r_*^{co}(i_*) = r_*^c$. If $S' > 0$, then $r_*^{co}(i_*^o) > r_*^c$ and by continuity $r_*^{co}(\eta) < r_*$ if S' is sufficiently small.

■

Proof of Proposition 4.

Using (16) and (27), $d_2 \leq d_2^o$ if and only if

$$P_*\big(q, d_1\big) - P_*^o\big(q, d_1\big) \leq \mathbb{E}\big[P(\cdot, d_1)\big] \; - \; \mathbb{E}\big[P_*^o(\cdot, d_1)\big]. \tag{62}$$

Since $r_*^{co}(i) < r_*^c$ for all $i > i_*$, the l.h.s. of (62) is always non-negative and, using the assumption regarding $q_{\text{NB}}^o(\eta)$, positive for sufficiently high shocks q. Since $P_*(q, d_1) - P_*^o(q, d_1)$ is increasing in q, there exists a unique q_{BE} such that (62) hold with equality. This implies that (62) and hence (36) hold if and only if $q \leq q_{\text{BE}}$. Since q_{BE} depends on i_* and r_A, this proves the proposition.

■

Proof of Theorem 2.

We need to show that $q_{\text{crit}}^o < q_{\text{crit}}$ in case (i) and $q_{\text{crit}}^o > q_{\text{crit}}$ in case (ii). It follows from Proposition 4 that $G_*(q, d_1) < G_*^o(q, d_1)$ for $q < q_{\text{BE}}$ and $G_*(q, d_1) > G_*^o(q, d_1)$ for $q > q_{\text{BE}}$. Since $G_*(q, d_1)$ and $G_*^o(q, d_1)$ are monotonically increasing in q with $G_*(\underline{q}, d_1) < G_*^o(\underline{q}, d_1) < 0$, the critical values must therefore satisfy $q_{\text{crit}}^o < q_{\text{crit}}$ in case (i) and the reverse inequality must hold in case (ii). This proves the theorem.

■

Proof of Theorem 3.

We need to show that

$$\mathbb{E}[C_E^o] > \mathbb{E}[C_E]. \tag{63}$$

Taking integrals, equation (53) implies

$$\int_{i_*}^{\eta} \Pi(i, r_*^c)\mathrm{d}i = \mathbb{E}[q]f \int_{i_*}^{\eta} (1+i)\mathrm{d}i \; - \; \mathbb{E}[P_*] \tag{64}$$

and

$$\int_{i_*}^{\eta} \Pi\big(i, r_*^{co}(i)\big)\mathrm{d}i = \mathbb{E}[q]f \int_{i_*^o}^{\eta} (1+i)\mathrm{d}i \; - \; \mathbb{E}[P_*^o]. \tag{65}$$

Substituting (64) into (39) and (65) into (40), we see after calculating the integrals that condition (63) holds if and only if

$$\mathbb{E}[P_*] - \mathbb{E}[P_*^o] + [i_*^o - i_*]W(1+r_A) > [i_*^o - i_*]\left(1 + \tfrac{i_* + i_*^o}{2}\right)\mathbb{E}[q]f. \tag{66}$$

If $S' = 0$, then $i_*^o = i_*$ by Theorem 1 (ii), and it follows from Proposition 3 that (66) holds. By continuity, (66) holds for sufficiently inelastic savings functions.

■

Proof of Lemma 4.

Setting $\underline{r}(i) = (1+i)\underline{q}f/I - 1$ and $\overline{r}(i) = (1+i)\overline{q}f/I - 1$, the expected profit of entrepreneur i given an interest rate r^c is

$$\Pi(i, r^c) = \begin{cases} (1+i)f\frac{\overline{q}+\underline{q}}{2} - I(1+r^c) & \text{if } r^c < \underline{r}(i), \\ \frac{(1+i)f}{2(\overline{q}-\underline{q})}\left[\overline{q} - \frac{I(1+r^c)}{(1+i)f}\right]^2 & \text{if } \underline{r}(i) \le r^c \le \overline{r}(i), \\ 0 & \text{if } \overline{r}(i) < r^c. \end{cases}$$

Then (41) follows from condition (11) by observing that in the resulting quadratic equation for $r_*^c(i)$ only the smaller solution is economically viable.

■

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