

# Improvement in Information and Private Investment in Education\*

by

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**Abstract** This paper uses the framework of an OLG economy for an analysis of the dynamic interaction between the precision of information about individual skills, investment in education, human capital accumulation and social welfare. The human capital of an individual depends on both his (subjectively) random ability and his investment in education. Individual investment in education is financed through a loan contract with income-contingent terms of repayment. Investment decisions are based on public signals (test outcomes) which screen all agents for their abilities. We find that better information, which allows more efficient screening, enhances aggregate human capital formation but may, at the same time, stifle aggregate investment in education. Moreover, social welfare may increase or decline depending on the transformation technology and on the relative measure of risk aversion.

**Keywords and Phrases:** Information system, higher education, human capital, welfare.

**JEL Classification Numbers:** D80, J24, I22.

## 1 Introduction

In recent decades ample cross-country empirical evidence has demonstrated the important role of higher education for generating personal incomes and for promoting economic growth (see Card and Krueger, 1992, Barro, 1998, Bassinini and Scarpenta, 2001, Restuccia and Urrutia, 2004). In line with this evidence, the size of educational investment has soared in the OECD countries during the second half of the twentieth century (Greenaway and Haynes, 2003, Checchi, 2006). This development, to the extent that higher education has been financed through public subsidies, may have contributed to inequality of the income distribution, because public subsidies of higher education constitute an implicit monetary transfer from the poor towards the more affluent individuals (Friedman, 1962). In fact, reference to an egalitarian income distribution is a common justification for policies which shift the financial burden of an expanding higher education sector away from public funding towards private funding. Yet, policies aimed at strengthening private funding of educational investment must come along with suitable financing schemes which remove financial barriers for young individuals to participate in the higher education system.

Friedman (1962) has pointed out that, due to imperfections in the capital market, investment in human beings cannot be financed on the same terms as investment in physical capital. These imperfections originate from two peculiarities of human capital. First, individual human capital is not collateralizable. Lenders are therefore hard to find because they get little or no security for their loans. Second, human capital is affected not only by educational investment but also by random individual ability. In principle, individual ability risks are diversifiable. However, market economies have failed to provide institutions for insuring those risks due to moral hazard problems and the existence of informational asymmetries between a student and an insurance company. Nerlove (1972) argues that this uncertainty about individual ability, if left uninsured, may reduce investment in education below socially optimal levels.

In order to remedy the consequences of these capital market imperfections, Friedman (1962) suggested an income-contingent loan-repayment program for the financing of higher education. Under such a program, which mimicks a special type of

equity financing of the acquisition of human capital, students would sell a share in their future earning prospects to a financial institution. Ideally, the loan program would also provide some diversification of the risky individual income prospects. Since students cannot (or only at inordinately high cost) be individually rated with respect to their abilities or future income prospects, they must be combined into groups for which such rating is practicable. All agents in the same group will be offered education loans on the same terms. This means, of course, that high incomes achieved by some members of the group, and their higher-than-average payments, will be used to offset low incomes earned by others and their consequently lower-than-average payments.<sup>1</sup>

Naturally, grouping students into different categories of future income prospects requires screening information upon which this process can be based. In this paper we analyze how the precision of such screening information affects investment in education, human capital formation, and economic welfare. In the early stage of life, when individual ability is still unknown, each agent is subjected to a test. The test produces a signal which contains some noisy information about the agent's ability. Based on their test outcomes (signals) all agents will be grouped into different categories of future income prospects. Our analysis abstracts from problems of moral hazard and adverse selection by assuming that both the test outcomes and individual abilities (or incomes) are publicly observable at the time when they have realized. Thus, there exists no discrepancy between the information possessed by an individual student and that which could be known to an insurance company.

Instead, in this paper we concentrate on a different issue: from the viewpoint of an efficient transformation of investment in education into human capital it would be appropriate, if agents with good ability prospects invest more aggressively in education than agents with poor ability prospects. Does a better information system, which produces more reliable test outcomes, lead to a better alignment of individual ability prospects and investment levels? We investigate this question with regard to both the efficiency of the human capital formation process and economic

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<sup>1</sup>Several countries have already established income-contingent student loan programs. Australia led the way in 1989 and was followed by Ghana, Sweden, Chile, New Zealand and the UK (for a complete survey, see Lleras, 2004). Recently, the US have also introduced income-sensitive components into existing student loan repayment plans, and Israel is considering similar steps.

welfare. Our analysis does not address the problem of whether the services of the higher education sector should be subsidized. This issue is quite different from the question how well income-contingent loan repayment plans work and whether their performance can be improved through a better information system.

We consider an overlapping generations economy with endogenous human capital formation depending on investment in education as well as random innate ability. When young, agents choose their (private) investment in education under uncertainty about their abilities and, hence, under uncertainty about their future incomes. Prior to making this decision, each agent receives a signal which is correlated to his ability and which allows him to update the belief about his future income. Educational investment is financed through a loan contract. The contract specifies a repayment obligation which is contingent on the agent's signal and on his income during the working period. The design of these loan contracts allows a pooling of individual income risks within each signal group. Under a better information system risk pooling is less effective, because the signal groups shrink in size when the signals become more reliable. On the other hand, better information may lead to a more efficient transformation of educational investment into human capital. In this setting we find that better information enhances aggregate human capital formation but may, at the same time, stifle aggregate investment in education. Moreover, economic welfare may increase or decline with better information, depending on the transformation technology and on the relative measure of risk aversion.

We organize the paper as follows. In the next section we present the model and define the fundamental concepts. In Section 3 we study the effects of better information on investment in education and on human capital formation under an income-contingent loan-repayment program. In Section 4 we examine the welfare implications of better information, and Section 5 concludes the paper. All proofs are relegated to a separate Appendix.

## 2 The Model

Our overlapping generations economy has a continuum of agents in each generation. Individuals live for three periods: the youth period in which education and skills are acquired, the working period followed by the retirement period. Generation  $t$  contains all individuals born at time  $t - 1$  and it is denoted by  $G_t$ ,  $t = 0, 1, \dots$ . In his youth period, following his public schooling, an agent takes out a loan and makes a capital investment in education in order to acquire skills to be used in the next period. In the working period individual labor income depends on the agent's skills, or human capital, which is assumed to be observable. Labor income will be used for three purposes: to repay the education loan, consumption in the working period, and savings for the retirement period. Finally, in the retirement period the agent consumes all his savings; hence, we do not include intergenerational transfers in our model. Furthermore, there exists a single commodity which can either be consumed or invested into a production process.

Nature assigns at birth an ability level  $A^i \in \mathcal{A} = [\underline{A}, \overline{A}] \subset \mathbb{R}_{++}$  to each agent  $i \in G_t$ ,  $t = 0, 1, \dots$  which becomes known only at the working period. The formation of human capital, at the youth period, depends on several inputs in our model: public investment in education (at the early stage, say, compulsory schooling), private investment (for example, in higher education) and innate ability assigned randomly at birth. In the second period, these investments along with the agents' abilities jointly determine their levels of human capital. Although innate ability is determined at birth, during the youth period an agent's ability level is yet unknown. Therefore the decision about the private investment in education,  $x^i$ , is made under uncertainty. The level of human capital, or skills, of agent  $i$  (and his ability  $A^i$ ) will be revealed only at the outset of the working period. Let  $\nu(A)$  be the (time invariant) density of agents with ability  $A$ . From each individual perspective, ability is the realization of a random variable with distribution  $\nu(\cdot)$ . We assume that ability risks are identical across agents and that there is no aggregate uncertainty, i.e., the ex post distribution of the stochastic ability variable is exactly  $\nu$ .<sup>2</sup>

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<sup>2</sup>Feldman and Gilles (1985, p. 29, Proposition 2) have shown that a probabilistic setting exists, where this version of a law of large numbers for large economies holds. In this setting, though, the individual risks are not independent.

We assume that public investment in educating each child is the same regardless of ability. Moreover, to simplify our analysis we take this investment to be *invariant over time*. Thus, the human capital production process can be represented as follows: for each agent  $i$ , his human capital  $h^i$ , is a function of ability,  $A^i$ , and his private investment in education,  $x^i \in \mathbb{R}_+$ ,

$$\tilde{h}^i = \tilde{A}^i g(x^i). \quad (1)$$

The human capital ‘accumulation function’,  $g(x)$ , contains the public spending on education and it is invariant over time. Since during the youth age innate ability and human capital are perceived as random, we have marked these variables by a  $\tilde{\cdot}$ .

**Assumption 1**  $g(x)$  is twice differentiable, strictly increasing and concave, and satisfies  $\lim_{x \rightarrow 0} g'(x) = \infty$ . Furthermore,  $g(x)$  exhibits decreasing concavity, meaning that

$$K(x) := -\frac{g''(x)}{g'(x)} \quad (2)$$

is a decreasing function.

$K(x)$  is a measure of concavity of the accumulation function  $g$ . Our assumption implies that this measure of concavity is decreasing in  $x$  and, hence, that  $g'(x)$  is a convex function. Thus, the marginal product of investment in education decreases at a declining rate. Most functional forms commonly used in the literature to describe the formation of human capital satisfy this restriction.

Each agent  $i$  chooses private investment in education after he has learned a publicly observable signal  $y^i \in Y \subset \mathbb{R}$  of his ability  $A^i$ . Students receive such signals before they enter higher education. Examples include personality tests and matriculation examinations used by universities to screen the field of applicants. The test results are noisy but they are correlated with the characteristics that have been tested.

The signals assigned to agents with ability  $A$  are distributed according to the density  $\nu_A(y)$ . The function  $\nu_A(\cdot)$  is also the ex post distribution of signals across

agents with ability  $A$ .<sup>3</sup> By construction, the distributions of signals and abilities are correlated. This implies that the signal assigned to an agent reveals some information about his ability and can, therefore, be used as a screening device. Based on the screening information conveyed by the signal, the agent forms expectations about his ability in a Bayesian way. As a consequence, the agent's investment decision takes into account the conditional distribution of his ability (perceived as random) given the observed signal.

The distribution of signals in the same generation has the density

$$\mu(y) = \int_{\mathcal{A}} \nu_A(y) \nu(A) dA. \quad (3)$$

Average ability of all agents who have received the signal  $y$  is

$$\bar{A}_y := E[\tilde{A}|y] = \int_{\mathcal{A}} A \nu_y(A) dA, \quad (4)$$

where  $\nu_y(A)$  denotes the conditional density of  $A$  given the signal  $y$ .

All agents are expected utility maximizers with von-Neumann Morgenstern lifetime utility function

$$U(c_1, c_2) = u_1(c_1) + u_2(c_2). \quad (5)$$

$c_1$  and  $c_2$  denote consumption in the second and third period of life, respectively. In his first period of life each agent makes a capital investment in education, but he does not consume. The utility functions  $u_i : \mathbb{R}_+ \rightarrow \mathbb{R}$ ,  $i = 1, 2$ , are strictly increasing and strictly concave.

In each period, competitive firms produce a commodity that can be used either for consumption or for production. The firms use physical capital,  $K$ , and human capital,  $H$ , as production factors. We assume that physical capital fully depreciates in the production process. The production process is given by an aggregate production function  $F(K, L)$ , which exhibits constant returns to scale. In his 'working period' each agent  $i$  inelastically supplies  $l$  units of labor and, hence, his supply of human capital is  $lh^i$ . For simplicity we take  $l = 1$ . The production function has the following properties:

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<sup>3</sup>Again, this assumption is justified by the aforementioned result in Feldman and Gilles (1985, p. 29, Proposition 2).

**Assumption 2**  $F(K, H)$  is concave, homogeneous of degree 1, and satisfies  $F_K > 0$ ,  $F_H > 0$ ,  $F_{KK} < 0$ ,  $F_{HH} < 0$ .

We also assume that physical capital is internationally mobile while human capital is immobile. This implies that the interest rate,  $\bar{r}_t$ , is exogenously given at each date (‘small country’ assumption). Due to the full depreciation of physical capital in each period, marginal productivity of aggregate physical capital,  $K_t$  equals  $1 + \bar{r}_t$ . Thus, given the aggregate stock of human capital at date  $t$ ,  $H_t$ , the stock of physical capital,  $K_t$ , adjusts such that the following condition holds:

$$R_t := 1 + \bar{r}_t = F_K(K_t, H_t), \quad \text{for } t = 1, 2, 3, \dots \quad (6)$$

This implies, by Assumption 1, that  $K_t/H_t$  is determined by the international rate of interest  $\bar{r}_t$ . Hence, due to the competitive factor prices, the wage rate (price of one unit of human capital), is given by the marginal product of effective labor:  $w_t = F_L(K_t/H_t, 1)$ , which is determined once  $\bar{r}_t$  is given.

Let us now consider the optimization problem that each  $i \in G_t$  faces, given  $\bar{r}_t, w_t$ , and  $H_{t-1}$ . At date  $t - 1$ , when ‘young’, this individual chooses investment in education,  $x^i$ , while his ability is still unknown. The investment decision will be based on the noisy information about the agent’s ability that is conveyed by the signal  $y^i$ . The investment is financed through a conditionally insured loan contract provided by a financial institution (to be called Students Loans Institution or SLI).

The terms of repayment differ for agents in different signal groups: each dollar borrowed and invested in education by individual  $i$  in  $G_t$  with signal  $y^i$  and income  $I^i := w_t h^i = w_t A^i g(x^i)$  involves an obligation to pay back  $R_t I^i / \bar{I}_{y^i}$  dollars in the working period, where  $\bar{I}_{y^i} := w_t \bar{A}_{y^i} g(x^i)$  denotes the average income in the group of all agents in  $G_t$  with signal  $y^i$ .<sup>4</sup> Thus the SLI uses the publicly observable signals as a screening device. It provides loans that allow individual income risks to be shared on fair terms within the various signal groups. In particular, the financial

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<sup>4</sup>In principle, if the terms of a loan contract depend on an agent’s signal, i.e., on his performance in test scores, there might be a problem of misrepresentation if it pays for a bright person to pretend being stupid. In our framework this problem does not arise because agents with better signals get more favorable terms of repayment. The reverse problem is ruled out by assumption: we assume that stupid people are unable to pretend being smart.

institution makes zero profit on the loans extended to all agents in the same signal group: it pays a gross interest rate,  $R_t$ , in the capital market which is just equal to the rate realized on total loans within each signal group, i.e.,  $E[R_t \tilde{I} / \bar{I}_y | y] = R_t$ .

The optimal decisions each consumer takes are done in two consecutive steps. At date  $t-1$ , after the signal  $y^i$  has been observed, our agent  $i \in G_t$  chooses an optimal level of investment in education,  $x^i$ . When choosing the investment level, the agent perceives his ability to be randomly distributed according to  $\nu_{y^i}(\cdot)$ . The optimal savings,  $s^i$ , are chosen at date  $t$  after his income (and hence, ability  $A^i$ ) has been observed. At this time,  $x^i$  (which has been chosen at date  $t-1$ ) is predetermined.

For any given levels of  $A^i$ ,  $h^i$ ,  $x^i$ ,  $w_t$ ,  $R_t$ , and  $R_{t+1}$  the optimal *saving* decision is determined by

$$\max_{s^i} u_1(c_1^i) + u_2(c_2^i) \quad (7)$$

$$\text{s.t. } c_1^i = w_t h^i - x^i \frac{R_t A^i}{\bar{A}_{y^i}} - s^i \quad (8)$$

$$c_2^i = R_{t+1} s^i \quad (9)$$

and satisfies the necessary and sufficient first order condition

$$u_1' \left[ w_t h^i - x^i \frac{R_t A^i}{\bar{A}_{y^i}} - s^i \right] = R_{t+1} u_2'(R_{t+1} s^i), \quad \forall A^i. \quad (10)$$

Thus, the optimal level of investment in education  $x^i$  is determined by:

$$\max_{x^i} E \left[ u_1(\tilde{c}_1^i) + u_2(\tilde{c}_2^i) \middle| y^i \right] \quad (11)$$

$$\text{s.t. } \tilde{c}_1^i = w_t \tilde{h}^i - x^i \frac{R_t \tilde{A}^i}{\bar{A}_{y^i}} - \tilde{s}^i \quad (12)$$

$$\tilde{c}_2^i = R_{t+1} \tilde{s}^i, \quad (13)$$

where  $\tilde{h}^i$  is given by equation (1) and  $\tilde{s}^i$  satisfies equation (10). Due to the Envelope theorem and the strict concavity of the utility functions, problem (11)-(13) has a

unique solution determined by the first order condition

$$w_t g'(x^i) = R_t / \bar{A}_y^i. \quad (14)$$

At date  $t - 1$ , the members of  $G_t$  differ only by the signals they have received. Therefore, all individuals in the same signal group,  $G_t(y)$ , choose the same investment level, denoted  $x_t(\bar{A}_y)$ .<sup>5</sup> Similarly, at date  $t$  the members of  $G_t$  differ by their abilities and by the signals they have received one period earlier. All agents in the same ability/signal group  $G_t(A, y)$  make the same savings and consumption decisions, denoted by  $s_t(A, \bar{A}_y)$ ,  $c_t^1(A, \bar{A}_y)$ ,  $c_t^2(A, \bar{A}_y)$ .

According to (14), optimal investment in education is a strictly increasing function of  $\bar{A}_y$ ,

$$\frac{\partial x_t(\bar{A}_y)}{\partial \bar{A}_y} = -\frac{R_t}{w_t(\bar{A}_y)^2 \hat{g}''(x_t)} > 0. \quad (15)$$

Differentiating (10) and using (14) we find that optimal saving,  $s_t(A, \bar{A}_y)$ , is strictly increasing in both arguments:

$$\frac{\partial s_t(\cdot)}{\partial A} = \frac{u_1'' \overbrace{\left[ w_t g(x_t) - x_t \frac{R_t}{\bar{A}_y} \right]}^{> c_t^1 > 0}}{u_1'' + R_{t+1}^2 u_2''} > 0 \quad (16)$$

$$\frac{\partial s_t(\cdot)}{\partial \bar{A}_y} = \frac{u_1'' x_t R_t A / (\bar{A}_y)^2}{u_1'' + R_{t+1}^2 u_2''} > 0 \quad (17)$$

Similarly, it can be verified that consumption in the second period of life,  $c_t^1(A, \bar{A}_y)$ , and in the third period of life,  $c_t^2(A, \bar{A}_y)$ , are both strictly increasing in  $\bar{A}_y$ .

Aggregate investment in education,  $X_t$ , and the aggregate stock of human capital at date  $t$ ,  $H_t$ , can be represented as

$$X_t = E[x_t(\bar{A}_y)] = \int_Y x_t(\bar{A}_y) \mu(y) dy \quad (18)$$

$$H_t = E[\bar{A}_y g(x_t(\bar{A}_y))] = \int_Y \bar{A}_y g(x_t(\bar{A}_y)) \mu(y) dy. \quad (19)$$

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<sup>5</sup>Note from equation (14) that optimal investment depends on the signal only via  $\bar{A}_y$ .

Our economy starts at date 0 with given initial stocks of physical capital,  $K_0$ , and human capital,  $H_0$ . The dynamic equilibrium describes the time path of factor prices, savings, investments, and consumption profiles.

**Definition 1** *Given the international interest rates  $(\bar{r}_t)$  and the initial stocks of human and physical capital  $H_0$  and  $K_0$ , a competitive equilibrium consists of a sequence  $\{(c_1^i, c_2^i, s^i, x^i)_{i \in G_t}\}_{t=1}^\infty$ , and a sequence of wages  $(w_t)_{t=1}^\infty$ , such that: At each date  $t$ ,  $t = 1, 2, \dots$ ,*

- (i) *given  $\bar{r}_t$  and  $w_t$ , the solution to problems (7)-(9) and (11)-(13) is given by  $(c_1^i, c_2^i, s^i, x^i)$  for each  $i \in G_t$ ,*
- (ii) *the aggregate stocks of human capital,  $H_t$ , satisfy (19),*
- (iii) *the factor prices satisfy  $w_t = F_L(K_t/H_t, 1)$  and  $1 + \bar{r}_t = F_K(K_t/H_t, 1)$ .*

Our analysis compares the allocations of economies with different information systems along these dynamic equilibrium paths period by period. The initial stocks,  $K_0, H_0$ , are the same for all economies.

When young, individuals are ignorant about the innate abilities that were assigned randomly to them according to the density  $\nu$ . Yet, since the distributions of signals and of abilities across individuals in the same generation are correlated, each agent uses his signal,  $y$ , to update the prior distribution,  $\nu$ , of his ability. The updated distribution has density

$$\nu_y(A) = \nu_A(y)\nu(A)/\mu(y). \quad (20)$$

An *information system*, which will be represented by  $\nu_A$  throughout the paper, specifies for each level of ability  $A \in \mathcal{A}$  a conditional density function over the set of signals. The positive real number  $\nu_A(y)$  is the conditional density of all agents with ability  $A$  to whom nature has assigned the signal  $y$ .

Hence, the positive real number  $\nu_A(y)$  defines the perceived conditional probability (density) that if ability is  $A$ , then the signal  $y$  will be sent. We assume that the densities  $\{\nu_A(\cdot), A \in \mathcal{A}\}$  have the strict monotone likelihood ratio property (MLRP):  $y' > y$  implies that for any given (nondegenerate) prior distribution

for  $A$ , the posterior distribution conditional on  $y'$  dominates the posterior distribution conditional on  $y$  in the first-order stochastic dominance.<sup>6</sup> As a consequence,  $\int_{\mathcal{A}} \varphi(A) \nu(A|y') dA > \int_{\mathcal{A}} \varphi(A) \nu(A|y) dA$  holds for any strictly increasing function  $\varphi$ .

Following Blackwell (1953) we rank different information systems by their informational contents. Let  $\bar{\nu}_A$  and  $\hat{\nu}_A$  be two information systems with associated density functions  $\bar{\nu}_y, \hat{\nu}_y, \bar{\mu}, \hat{\mu}$ .

**Definition 2** (*informativeness*.)  $\bar{\nu}_A$  is said to be more informative than  $\hat{\nu}_A$  (expressed by  $\bar{\nu}_A \succ_{\text{inf}} \hat{\nu}_A$ ), if there exists an integrable function  $\lambda : Y^2 \rightarrow \mathbb{R}_+$  such that

$$\int_Y \lambda(y', y) dy' = 1 \quad (21)$$

holds for all  $y$ , and

$$\hat{\nu}_A(y') = \int_Y \bar{\nu}_A(y) \lambda(y', y) dy \quad (22)$$

holds for all  $A \in \mathcal{A}$ .

The concept of informativeness is based on a simple intuitive idea: consider a stochastic mechanism, compatible with equation (21), that transforms a signal  $y$  into another signal  $y'$  according to the probability density  $\lambda(y', y)$ . If the  $y'$ -values are generated in this way, the information system  $\hat{\nu}_A$  can be interpreted as being obtained from the information system  $\bar{\nu}_A$  by adding some random noise. The following criterion turns out to be a useful tool for our analysis.

**Lemma 1** Let  $\hat{\nu}_A$  be an information system and let  $F$  be a real-valued function defined on the set of density functions over  $\mathcal{A}$ . If  $F$  is convex (concave) on the set  $\text{Conv}\{\hat{\nu}_y|y \in Y\}$ , then

$$\int_Y F(\bar{\nu}_y) \bar{\mu}(y) dy \stackrel{(\leq)}{\geq} \int_Y F(\hat{\nu}_y) \hat{\mu}(y) dy$$

holds for any information system  $\bar{\nu}_A \succ_{\text{inf}} \hat{\nu}_A$ .

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<sup>6</sup>For details see Milgrom (1981).

Remark: Conv stands for the convex hull. Note that in Lemma 1 the prior distribution  $\nu$  and the system  $\hat{\nu}_A$ , relative to which information improves, are fixed.

Lemma 1 can be proved by slightly modifying the line of argument in Kihlstrom (1984). Note that  $\bar{\nu}_y$  and  $\hat{\nu}_y$  are the posterior beliefs under the two information systems. Thus, Lemma 1 implies that an improvement of the information system  $\hat{\nu}_A$  (weakly) raises the expectation of any function,  $F$ , which is convex in the posterior beliefs under  $\hat{\nu}_A$ . For concave functions, the inequality is reversed, and for linear functions it holds with equality.

### 3 Investment and Human Capital Formation

In this section we analyze the implications of the information system for the level of aggregate investment in education and for aggregate human capital formation. Since agents differ with regard to their abilities they realize different ex post returns on their investments in education. For the economy as a whole, the transformation process of aggregate investment into aggregate human capital can be expected to be more efficient the better the investments of individuals are aligned with their true abilities. We now analyze whether this goal can be achieved through more efficient screening via a better information system. As it turns out, the curvature of the accumulation function,  $g(x)$ , is of critical importance. Define

$$\hat{K}(x) := -g''(x)/(g'(x))^2 \quad [ = K(x)/g'(x) ].$$

$K(\cdot)$  and  $\hat{K}(\cdot)$  are (different) measures of concavity which allow us to define the concepts of ‘moderately decreasing concavity’ and ‘strongly decreasing concavity’.

**Definition 3** *Given the restrictions formulated in Assumption 1, the accumulation function  $g(x)$  exhibits*

- (i) *moderately decreasing concavity, if  $\hat{K}(x)$  is increasing in  $x$ .*
- (ii) *strongly decreasing concavity, if  $\hat{K}(x)$  is decreasing in  $x$ .*

Note that the properties in (i) and (ii) of Definition 3 are mutually exclusive. Depending on the curvature of the accumulation function, aggregate investment in education may increase or decline under a better information system:

**Proposition 1** *Aggregate investment in education increases (decreases) with better information, if the accumulation function  $g(\cdot)$  exhibits strongly (moderately) decreasing concavity.*

Proof: See Appendix.

The link between uncertainty and investment decisions has been studied extensively in the literature. It is well known that under standard assumptions a reduction of uncertainty raises the level of investment (Sandmo, 1971). At first sight, our result in Proposition 1 may appear inconsistent with this literature. Better information reduces the risk exposure of the decision makers and, therefore, should unambiguously raise investment. Note, however, that the mechanism which links information to investment in our model is quite different from the above line of argument. In particular, this mechanism is not based on an individual's residual ability uncertainty: conditional on the signal, individual ability risks are pooled; hence, they do not affect investment decisions. According to (14), investment in education depends only on the conditional mean, but not on the variability, of random ability.

To understand the critical role of the curvature of  $g(\cdot)$  in Proposition 1 note that, according to (14), agents choose investment in education so as to equate marginal return (per unit of ability),  $w_t g'(x)$ , and marginal cost (per unit of ability),  $R_t / \bar{A}_y$ . This implies that a change in marginal cost has a bigger impact on investment when (locally) the accumulation function is flatter, i.e., less concave. Therefore, since  $g(\cdot)$  exhibits decreasing concavity, a cost change induces large adjustment in investment when the received signal is high, and small adjustment in investment when the received signal is low. Both effects work in opposite directions since under a better information system investment costs (per unit of ability) decrease for agents with high signals and increase for agents with low signals. The faster (local) concavity of  $g(\cdot)$  declines the stronger is the first effect relative to the second

effect. Proposition 1 tells us that, in the aggregate, the first effect is dominant if  $g(\cdot)$  exhibits strongly decreasing concavity, while the second effect is dominant if  $g(\cdot)$  exhibits only moderately decreasing concavity.

Surprisingly, while aggregate investment in education may either be higher or lower under a better information system, aggregate human capital always increases.

**Proposition 2** *Aggregate human capital increases under a better information system.*

Proof: See Appendix.

Under a better information system, the economy-wide process which transforms investments in education into aggregate human capital becomes more efficient because individual investments and individual abilities are better aligned: highly talented agents (with high returns) invest more, and poorly talented agents (with low returns) invest less. As a consequence of this ‘efficiency-effect’, aggregate human capital accumulates faster in our economy. On the other hand, aggregate investment in education may increase or decrease with better information according to Proposition 1. This ‘investment-effect’ on the formation of aggregate human capital may therefore be positive or negative. Both effects work in the same direction and stimulate aggregate human capital formation, if the accumulation function exhibits strongly decreasing concavity. If  $g(\cdot)$  exhibits moderately decreasing concavity, then the two effects counteract because aggregate investment in education declines. Nevertheless, the net effect of better information on aggregate human capital formation is positive (Proposition 2); hence, the positive efficiency-effect always outweighs the negative investment-effect.

## 4 Welfare implications of better information

It is well-known that in our economy public information, which resolves (part of) the uncertainty, may destroy risk sharing opportunities and thereby impose welfare costs on risk-averse agents (Hirshleifer, 1971, Schlee, 2001). On the other hand, we have seen in the previous section that a better information system enhances the

efficiency of the aggregate human capital formation process by providing incentives for better talented individuals to invest more, and for poorly talented individuals to invest less. Better information may therefore raise the welfare of the economy even if it results in less risk sharing opportunities. Let us analyze now the interaction of these two information-induced welfare effects.

In this economy all agents in the same generation are identical *ex-ante*, i.e., before the individual signals have been observed. Therefore, the welfare of generation  $G_t$  can be defined in a natural way as the ex-ante expected utility of each member in  $G_t$ . A welfare improvement for the economy implies that the welfare levels of all generations increase.

Welfare of generation  $G_t$  is defined by

$$W_t(\nu_A) = E[V_t(\nu_{\bar{y}})] = \int_Y V_t(\nu_y) \mu(y) dy, \quad (23)$$

where

$$\begin{aligned} V_t(\nu_y) = & \int_{\mathcal{A}} \left[ u_1 \left( w_t A g(x_t(\bar{A}_y)) - x_t(\bar{A}_y) \frac{R_t A}{\bar{A}_y} - s_t(A, \bar{A}_y) \right) \right. \\ & \left. + u_2(R_{t+1} s_t(A, \bar{A}_y)) \right] \nu_y(A) dA. \end{aligned} \quad (24)$$

$V_t(\nu_y)$ , the value function for generation  $G_t$ , represents the conditional expected utility of a member of  $G_t$  who has received the signal  $y$ .

In the sequel we will analyze how the welfare of generation  $G_t$  is affected by the informativeness of the information system. In our model only part of the diversifiable educational investment risk is insured. Nevertheless, the overall investment process is free from distortions: all individuals make socially optimal investment decisions, i.e., they choose investment profiles which are consistent with maximization of ex ante welfare in (23). To see this, note that at a social optimum aggregate income

$$\int_Y \left[ w_t \bar{A}_y g(x_t(y)) - x_t(y) R_t \right] \mu(y) dy$$

is maximized. Thus, the socially optimal investment level of agents with signal  $y$  satisfies

$$w_t g'(x_t(y)) = R_t / \bar{A}_y$$

and, hence, coincides with the individually optimal investment choice (cf. (14)).

In this economic setting, there are two channels through which better information can affect welfare: (i) better information enhances the efficiency of the human capital accumulation process, and (ii) better information may adversely affect risk sharing from an ex ante perspective. Our next proposition confirms the intuition that the second transmission channel becomes negligible if the agents are (almost) risk neutral in their youth period.

**Proposition 3** *Let  $\bar{v}_A$  and  $\hat{v}_A$  be two information systems satisfying  $\bar{v}_A \succ_{\text{inf}} \hat{v}_A$ . If  $u_1'' = 0$ , i.e., agents are risk-neutral in the first period of life, then all agents are better-off (or at least nobody is worse-off) under  $\bar{v}_A$  than  $\hat{v}_A$ .*

Remark: The claim in Proposition 3 remains valid, if risk aversion in the first period of life is positive but sufficiently small, i.e.,  $|u_1''(\cdot)| \leq \varepsilon$  (uniformly) for sufficiently small  $\varepsilon > 0$ .

Proof: See Appendix.

Thus, in the absence of risk aversion in the first period of life, welfare is higher under better information. This is not too surprising because the implications for risk sharing under a more informative system do not affect welfare if the agents are risk neutral. Yet, if the agents' preferences exhibit strong risk aversion, then welfare may be higher under a less informative system. We demonstrate this possibility below for an economy with CRRA preferences. Our further analysis will be based on the following functional forms:

$$u_1(c_1) = \frac{c_1^{1-\gamma}}{1-\gamma}; \quad u_2(c_2) = \beta \frac{c_2^{1-\gamma}}{1-\gamma}; \quad g(x) = \frac{1}{1-\alpha} (x^{1-\alpha} - \underline{x}^{1-\alpha}), \quad (25)$$

where  $0 \leq \gamma \neq 1 \neq \alpha > 0$  and  $\underline{x}$  is a lower bound for investment in education which satisfies

$$0 < \underline{x} < \alpha^{\frac{1}{1-\alpha}} \left( \frac{w_t A}{R_t} \right)^{\frac{1}{\alpha}}, \quad \text{for all } t. \quad (26)$$

The restriction in (26) makes sure that optimal savings are strictly positive and that optimal investment is strictly larger than  $\underline{x}$ . We think of  $\underline{x}$  as a positive number which is close to zero. Note that  $g(\cdot)$  exhibits moderately decreasing concavity if  $\alpha > 1$ , and strongly decreasing concavity if  $\alpha \in (0, 1)$ .

Solving the first-order-conditions for  $s$ ,  $x$ ,  $c_1$  and  $c_2$  yields

$$x(\bar{A}_y) = \left( \frac{R_t}{w_t \bar{A}_y} \right)^{-\frac{1}{\alpha}} \quad (27)$$

$$s(A, \bar{A}_y) = \frac{w_t A \left[ \alpha \left( \frac{R_t}{w_t \bar{A}_y} \right)^{\frac{\alpha-1}{\alpha}} - \underline{x}^{1-\alpha} \right]}{(1-\alpha) \left[ 1 + \beta^{-1/\gamma} R_{t+1}^{(\gamma-1)/\gamma} \right]} \quad (28)$$

$$c_1(A, \bar{A}_y) = \beta^{-1/\gamma} R_{t+1}^{(\gamma-1)/\gamma} \frac{w_t A \left[ \alpha \left( \frac{R_t}{w_t \bar{A}_y} \right)^{\frac{\alpha-1}{\alpha}} - \underline{x}^{1-\alpha} \right]}{(1-\alpha) \left[ 1 + \beta^{-1/\gamma} R_{t+1}^{(\gamma-1)/\gamma} \right]} \quad (29)$$

$$c_2(A, \bar{A}_y) = R_{t+1} \frac{w_t A \left[ \alpha \left( \frac{R_t}{w_t \bar{A}_y} \right)^{\frac{\alpha-1}{\alpha}} - \underline{x}^{1-\alpha} \right]}{(1-\alpha) \left[ 1 + \beta^{-1/\gamma} R_{t+1}^{(\gamma-1)/\gamma} \right]}. \quad (30)$$

The value function can then be written as

$$V(\nu_y) = \left( 1 + \beta^{-1/\gamma} R_{t+1}^{(\gamma-1)/\gamma} \right)^\gamma \frac{\beta (w_t R_{t+1})^{1-\gamma}}{(1-\gamma)} \rho(\bar{A}_y) \int_{\mathcal{A}} A^{1-\gamma} \nu_y(A) dA, \quad (31)$$

where

$$\rho(\bar{A}_y) := \left[ \frac{\alpha \left( \frac{w_t \bar{A}_y}{R_t} \right)^{\frac{1-\alpha}{\alpha}} - \underline{x}^{1-\alpha}}{1-\alpha} \right]^{1-\gamma}. \quad (32)$$

Note that the term in brackets on the RHS of (32) is positive and increasing in  $\bar{A}_y$  for all  $\alpha \neq 1$ .

**Lemma 2** *Let  $\hat{\rho} : \mathcal{A} \rightarrow \mathbb{R}_+$ . The function*

$$\hat{V}(\nu_y) := \hat{\rho}(\bar{A}_y) \int_{\mathcal{A}} A^{1-\gamma} \nu_y(A) dA$$

*is convex in the posterior belief  $\nu_y$  under each one of the following conditions:*

- (i)  $\gamma \leq 1$  and  $\hat{\rho}(\cdot)$  is increasing and convex;
- (ii)  $\gamma \geq 1$  and  $\hat{\rho}(\cdot)$  is decreasing and convex.

Proof: See Appendix.

Remark 1: From Lemma 2 and (32) we conclude that  $V(\nu_y)$  in (31) is convex in the posterior belief  $\nu_y$  for  $\gamma < 1$  and concave for  $\gamma > 1$ , as long as  $\rho$  is a convex function.

Combining Remark 1 with Lemma 1 we easily derive

**Proposition 4** *Assume that  $\gamma < 1$ . Then, if  $\rho(\cdot)$  is a convex function, an improvement of the information system results in higher economic welfare.*

Remark 2: For  $\alpha \in (0, \frac{1}{2})$  (which implies strongly decreasing concavity) and  $0 \leq \gamma < 1 - \frac{\alpha}{1-\alpha}$  the convexity condition in Proposition 4 is satisfied if the lower bound  $\underline{x} \in \mathbb{R}_{++}$  is sufficiently small. More precisely, if the following condition holds:

$$\underline{x} \leq \left( \frac{w_t \underline{A}}{R_t} \right)^{\frac{1}{\alpha}} \left[ \alpha \left( 1 - \gamma \frac{1-\alpha}{1-2\alpha} \right) \right]^{\frac{1}{1-\alpha}}$$

In our model, better information adversely affects risk sharing from an ex ante point of view. At the same time, better information improves the efficiency of the process which transforms aggregate investment in education into aggregate human capital. Therefore, the information-induced welfare gains (or losses) depend on both the risk aversion parameter  $\gamma$  and the technological parameter  $\alpha$ . If  $\underline{x}$  tends to zero,

then the sign of  $\frac{(1-\alpha)(1-\gamma)}{\alpha} - 1$  determines whether  $\rho(\cdot)$  is convex or concave: with higher  $\alpha$ , a smaller degree of risk aversion is needed in order for  $\rho(\cdot)$  to be convex.

In high risk aversion economies better information reduces economic welfare:

**Proposition 5** *If the economy is highly risk-averse in the sense that  $\gamma > 1$  is satisfied, then an improvement of the information system results in lower economic welfare.*

Proof: See Appendix.

In our parametrized economy, the concavity of the accumulation function,  $g(\cdot)$ , declines more rapidly<sup>7</sup> if  $\alpha$  assumes smaller values. Therefore, the information-induced efficiency gains in the transformation process are larger for smaller values of  $\alpha$ . Yet, even for  $\alpha$  close to zero, the efficiency gains are not big enough to outweigh the deterioration of the risk allocation, if the measure of relative risk aversion,  $\gamma$ , is larger than 1. As a consequence, economic welfare declines under a better information system.

In this model we have considered a loan market for educational investment where individual payback obligations differ across signal groups: agents with better signals are able to sign loan contracts on more favorable terms. As a consequence, risk sharing is incomplete because individual ability risks are pooled only among agents in the same signal group. Yet, our model mimicks the case of unrestricted risk pooling, if it is endowed with the null-information system: if the signals are uninformative, then  $\bar{A}_{y^i} = \bar{A}_{y^j}$  for all  $i, j \in G_t$  and, hence, the terms of repayment do not differ across signal groups. Thus, all agents in  $G_t$  sign the same contract and ability risks are pooled across the entire generation. We may, therefore, conclude from Proposition 2 that aggregate human capital accumulates faster if risk pooling is restricted to signal groups; i.e., unrestricted risk pooling, if it were to be implemented through some government regulation, slows down the human capital formation process. On the other hand, in view of propositions 4 and 5, unrestricted risk pooling leads to higher economic welfare in a high-risk-aversion-economy, but may reduce economic welfare if the economy is moderately risk-averse.

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<sup>7</sup>in the sense that  $\hat{K}(x)$  decreases at a higher rate.

## 5 Conclusion

Friedman's suggestion for the equity financing of investment in higher education has been used in various countries as the basis of operating programs of income-contingent loan repayment plans. Yet, since rating human beings with respect to their potential future incomes is extremely difficult and costly, these plans use certain types of screening information in order to group individuals in rather broadly defined repayment cohorts. Our paper has analyzed how the precision of such screening information affects investment in education, human capital formation, and economic welfare.

To date, the nature of the educational transformation process is not well understood. In this area, more empirical and theoretical work is necessary since our results clearly demonstrate that the curvature of the human capital formation function is of critical importance for the efficiency of the transformation process as well as for the contribution to overall welfare created in the higher education sector.

We have not addressed the question whether, and if so, to what extent, the higher education sector should be subsidized by the government. Any answer to that question would have to take into account the externalities of higher education, many of which are of a generally noneconomic character and, therefore, difficult to measure. In fact, very little is known about the quantitative importance of external effects in higher education. It is well possible, though, that the information system affects the externalities and, hence, the size of a subsidy that could be justified on these grounds.

We have also excluded moral hazard and adverse selection problems from our analysis. In a more general setting which allows for informational asymmetries, individual decisions may be subject to moral hazard phenomena if the returns to human capital accumulation contain noneconomic components. Such components cannot be captured in a loan repayment scheme and, therefore, may reduce the agents' ex post incentives to choose monetarily remunerative jobs. In addition, the cohort of agents who are willing to participate in an income-contingent loan repayment scheme can be adversely selected. This is because individuals with poor income prospects are more likely to borrow under such a program than are those

with favorable income prospects. An extension of our approach that would allow for the existence of informational asymmetries between the students and the financial institution might yield further insights into the role of information for the performance of the higher education sector. This is left for future research.

## Appendix

In this Appendix we prove propositions 1-5 and Lemma 2 in the main text.

**Proof of Proposition 1:** Using (14), equation (15) can be written as

$$\frac{\partial x_t(\bar{A}_y)}{\partial \bar{A}_y} = \frac{w_t}{\hat{K}(x_t(\cdot))R_t}. \quad (33)$$

Since  $x_t(\cdot)$  is increasing in  $\bar{A}_y$  according to (14),  $x_t(\cdot)$  is convex (concave) in  $\bar{A}_y$  if  $g(\cdot)$  exhibits strongly (moderately) decreasing concavity. The claim then follows from (18) in combination with Lemma 1.  $\square$

**Proof of Proposition 2:** Aggregate human capital can be written as

$$H_t = \int_Y \bar{h}_t(\bar{A}_y) \mu(y) dy,$$

where

$$\bar{h}_t(z) := zg(x_t(z)), \quad z \in \mathcal{A}.$$

$\bar{A}_y$  is linear in the posterior probabilities. Therefore, in view of Lemma 1, we need to show that  $\bar{h}_t(\cdot)$  is convex. Differentiating  $\bar{h}_t(\cdot)$  and using equation (14) we get

$$\bar{h}_t''(z) = \frac{Rx_t'(z)}{w_t z} \left[ 1 + \frac{x_t''(z)z}{x_t'(z)} \right]. \quad (34)$$

Differentiating (15) and rearranging yields

$$\frac{x_t''(z)z}{x_t'(z)} = - \left( 1 + \frac{K'(x_t(z))}{[K(x_t(z))]^2} \right). \quad (35)$$

Combining (34) and (35) we obtain

$$\bar{h}_t''(z) = - \frac{K'(x_t(z))/z}{[K(x_t(z))]^2 \hat{K}(x_t(z))}.$$

By Assumption 1,  $K'(\cdot)$  is non-positive and, hence,  $\bar{h}_t(\cdot)$  is a convex function.  $\square$

**Proof of Proposition 3:** With  $u_1'' = 0$  the value function can be written as

$$V(\nu_y) = \int_{\mathcal{A}} U(A, \bar{A}_y) \nu_y(A) dA, \quad (36)$$

where

$$U(A, \bar{A}_y) := u_1(w_t A g(x_t(\bar{A}_y)) - R_t x_t(\bar{A}_y) - s_t(A, \bar{A}_y)) + u_2(R_{t+1} s_t(A, \bar{A}_y)). \quad (37)$$

We show that the value function in (36) is convex in the posterior belief  $\nu_y$ . Assume  $\nu_y = \alpha \bar{\nu}_y + (1 - \alpha) \hat{\nu}_y$ ,  $\alpha \in [0, 1]$ , and denote average ability under the beliefs  $\bar{\nu}_y$  and  $\hat{\nu}_y$  by  $\bar{A}_y$  and  $\hat{A}_y$ .

$$\begin{aligned} V(\nu_y) &= \int_{\mathcal{A}} U(A, \bar{A}_y) [\alpha \bar{\nu}_y(A) + (1 - \alpha) \hat{\nu}_y(A)] dA \\ &= \alpha \left[ \int_{\mathcal{A}} U(A, \bar{A}_y) \bar{\nu}_y(A) dA \right] + (1 - \alpha) \left[ \int_{\mathcal{A}} U(A, \bar{A}_y) \hat{\nu}_y(A) dA \right] \\ &\leq \alpha \left[ \int_{\mathcal{A}} U(A, \bar{A}_y) \bar{\nu}_y(A) dA \right] + (1 - \alpha) \left[ \int_{\mathcal{A}} U(A, \hat{A}_y) \hat{\nu}_y(A) dA \right] \\ &= \alpha V(\bar{\nu}_y) + (1 - \alpha) V(\hat{\nu}_y). \end{aligned}$$

The inequality holds because  $[x_t(\bar{A}_y), s_t(A, \bar{A}_y)]$  and  $[x_t(\hat{A}_y), s_t(A, \hat{A}_y)]$  maximize expected utility, if the posterior belief is given by  $\bar{\nu}_y$  and  $\hat{\nu}_y$ , respectively. In view of Lemma 1, convexity of the value function implies the claim in Proposition 3.  $\square$

**Proof of Lemma 2:** (i) Let  $\bar{y}$  and  $\hat{y}$  be two signals with  $\bar{y} \geq \hat{y}$ , and choose  $\lambda \in [0, 1]$

arbitrarily.

$$\begin{aligned}
& \hat{V}(\lambda\nu_{\bar{y}} + (1 - \lambda)\nu_{\hat{y}}) \\
&= \hat{\rho}(\lambda\bar{A}_{\bar{y}} + (1 - \lambda)\bar{A}_{\hat{y}}) \left[ \lambda \int_{\mathcal{A}} A^{1-\gamma}\nu_{\bar{y}}(A) \, dA + (1 - \lambda) \int_{\mathcal{A}} A^{1-\gamma}\nu_{\hat{y}}(A) \, dA \right] \\
&\stackrel{(\hat{\rho} \text{ convex})}{\leq} \lambda\hat{\rho}(\bar{A}_{\bar{y}}) \left[ \lambda \int_{\mathcal{A}} A^{1-\gamma}\nu_{\bar{y}}(A) \, dA + (1 - \lambda) \int_{\mathcal{A}} A^{1-\gamma}\nu_{\hat{y}}(A) \, dA \right] \\
&+ (1 - \lambda)\hat{\rho}(\bar{A}_{\hat{y}}) \left[ \lambda \int_{\mathcal{A}} A^{1-\gamma}\nu_{\bar{y}}(A) \, dA + (1 - \lambda) \int_{\mathcal{A}} A^{1-\gamma}\nu_{\hat{y}}(A) \, dA \right] \\
&= \lambda\hat{V}(\nu_{\bar{y}}) + (1 - \lambda)\hat{V}(\nu_{\hat{y}}) \\
&- \lambda(1 - \lambda) \left[ \int_{\mathcal{A}} A^{1-\gamma}\nu_{\bar{y}}(A) \, dA - \int_{\mathcal{A}} A^{1-\gamma}\nu_{\hat{y}}(A) \, dA \right] \left[ \hat{\rho}(\bar{A}_{\bar{y}}) - \hat{\rho}(\bar{A}_{\hat{y}}) \right] \quad (38) \\
&\leq \lambda\hat{V}(\nu_{\bar{y}}) + (1 - \lambda)\hat{V}(\nu_{\hat{y}}).
\end{aligned}$$

The second inequality follows from  $\bar{y} \geq \hat{y}$  and the MLRP, which imply that the two terms in brackets are both non-negative.

(ii) Since  $\hat{\rho}$  is convex, the first inequality in (ii) remains valid. The second inequality also remains intact: Since  $\bar{A}_{\bar{y}} \geq \bar{A}_{\hat{y}}$  and  $\hat{\rho}(\cdot)$  is decreasing, the term in the last bracket in (38) is non-positive. The term in the first bracket is also non-positive, because  $\gamma \geq 1$  and MLRP hold. This proves the convexity of  $\hat{V}(\cdot)$  in the posterior belief  $\nu_{\bar{y}}$ .  $\square$

The proof of Proposition 5 makes use of the following two lemmas:

**Lemma 3** Let  $\hat{x} \in \mathbb{R}_+$ ,  $z > (\hat{x})^{1/\alpha}$ ,  $h : [z, \infty) \rightarrow \mathbb{R}$ ,

$$h(x) = (x^a - \hat{x})^b, \quad b < 0; \quad a > 0. \quad (39)$$

$h(\cdot)$  is a convex function.

**Proof:**

$$h''(x) = ab[x^a - \hat{x}]^{b-2} x^{2a-2} \left[ (ab - 1) + (1 - a) \frac{\hat{x}}{x^a} \right] \quad (40)$$

Thus  $h(\cdot)$  is convex if

$$(ab - 1) + (1 - a)\frac{\hat{x}}{x^a} \leq 0 \quad \forall x \geq z,$$

or,

$$(1 - a)\frac{\hat{x}}{x^a} \leq (1 - ab) \quad \forall x \geq z.$$

Since  $\hat{x}/x^a < 1$ , the above inequality is always satisfied.  $\square$

**Lemma 4** Let  $\hat{x} \in \mathbb{R}_+$ ,  $z > (\hat{x})^{1/\alpha}$ ,  $\theta : [z, \infty) \rightarrow \mathbb{R}$ ,

$$\theta(x) = (\hat{x} - x^a)^b, \quad b < 0; \quad a < 0. \quad (41)$$

$\theta(\cdot)$  is convex for any  $\hat{x} > 0$ .

**Proof:**

$$\theta'(x) = -b[\hat{x} - x^a]^{b-1} ax^{a-1} \quad (42)$$

Obviously,  $\theta'(x)$  is increasing in  $x$ .  $\square$

**Proof of Proposition 5:** In view of Remark 1 and Lemma 1, we need to show that for  $\gamma > 1$  the function  $\rho(\cdot)$  in (32) is convex for all  $\alpha \neq 1$ .

(i) For  $\alpha > 1$ ,  $\rho(\bar{A}_y)$  can be written as

$$\rho(\bar{A}_y) = \left(\frac{1}{\alpha - 1}\right)^{1-\gamma} \left[\alpha \left(\frac{w_t}{R_t}\right)^{\frac{1-\alpha}{\alpha}}\right]^{1-\gamma} \left\{ \frac{x^{1-\alpha}}{\alpha \left(\frac{w_t}{R_t}\right)^{\frac{1-\alpha}{\alpha}}} - \bar{A}_y^{\frac{1-\alpha}{\alpha}} \right\}^{1-\gamma}$$

According to Lemma 4,  $\rho(\cdot)$  is a convex function.

(ii) For  $\alpha < 1$  the convexity of  $\rho(\cdot)$  follows from Lemma 3.  $\square$

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