Efficiency of Screening and Labor Income Inequality

by

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Abstract

We analyze the importance of information about individual skills for understanding human capital accumulation and income inequality. The paper uses the framework of an OLG economy with endogenous investment in human capital. Agents in each generation differ by random individual ability, or talent, which affects the screening process. The human capital of an agent depends on both his talent and his investment in education. The investment decision is based on a public signal (test outcome) which screens all agents for their talents. We analyze how a better information system, which allows more efficient screening, affects investment in education and, hence, income inequality in equilibrium. As a main result, we find that, typically, less inequality in the distribution of actual incomes can only be achieved at the expense of more inequality in the distribution of income opportunities.

Keywords: Information system, human capital accumulation, income inequality.

JEL classification numbers: D80, J24, J30.

1 Introduction

In recent decades we have witnessed a growing body of research on the role of information in economic models. In particular, the welfare implications of information have been studied extensively. As a main result, these studies have revealed the ambiguous nature of information with regard to economic welfare when risk sharing arrangements are operative (Hirshleifer (1971,1975), Schlee (2001), Campbell (2004), Eckwert and Zilcha (2003)). Surprisingly, the question how information interacts with human capital accumulation and income inequality has received much less attention, even though this question is not unrelated to the welfare problem.

In this paper we focus on the role of information about individual ability. Under the pressure of globalization, many countries in Europe have begun to reform their systems of higher education. In Germany, for instance, new legislation is under consideration that would allow publicly funded universities to select students on the basis of their scores in specificially designed admission tests. This is an example of a reform that will greatly improve the availability of screening information about agents' abilities. In most countries admission to higher education is based on some kind of screening mechanism. Typically, these mechanisms differ with regard to the reliability of the information that is generated.

It is well known that the effects of information in general equilibrium models depend on the scope of risk sharing opportunities (Green (1981), Eckwert and Zilcha (2001)). In this paper we consider an economy where no explicit risk sharing arrangements are operative. Nevertheless, due to imperfect information, in equilibrium some individual risks will be shared across agents: the labor market treats all agents on the basis of the available information, hence it cannot discriminate agents according to their other characteristics. Even with risk sharing markets absent, better information leads to reduced risk sharing. Therefore, information plays a role in the allocation of risks and, hence, it affects investments in human capital formation. In addition, since better information allows more reliable identification of individual characteristics, it also affects in a very natural way investment in education and, hence, the inequality of income distribution. This set up provides a theoretical platform for an analysis of the impact of better information on income inequality, aggregate human capital and their relationship.

Our analytical framework is an OLG economy in which agents live for three periods (youth, working period, retirement). Private investment in education (say, higher education or non-compulsory schooling) takes place while young, and it affects an agent's human capital in his working period. In our model individual human capital depends also on random ability, or talent, which is still unknown when the agent decides how much 'effort' to invest in his/her education and training. However, the investment decision is made after observing a signal (test outcome) which screens agents for their abilities. The signal that an agent receives contains imperfect (public) information about the agent's (random) talent. Since individual abilities are not yet known agents differ only by the signals they have received and their investment in education.

Our framework gives rise to various definitions of 'income inequality', since some information is revealed over time. Individuals' talents are determined at birth even though talent remains unknown until an agent enters his working period. Thus, given the distribution of (unknown) abilities one may be interested in how expected incomes are distributed across agents *before* signals are observed. Making this distribution more equal may be viewed as giving more 'equal opportunities' to the younger generation (see Benabou and Ok (2001)). This notion of inequality has been analyzed in an earlier paper (Eckwert and Zilcha (2007)) which finds that, depending on the agents' attitudes toward risk, the informative power of the test outcomes may either increase inequality of income opportunities or decrease it.

Typically, an agent's conditional ex ante income expectation differs from his actual (ex post) income. Therefore, the distribution of income opportunities within a generation of agents differs from the distribution of actual incomes. In this paper we use an 'ex post' concept of inequality: given the distribution of innate abilities, the information system generates a distribution over signals; based on the signal which influences investment in education, a wage distribution is attained. We shall analyze the inequality of this (ex-post) income distribution. Even though the above mentioned concept of income opportunities has its own economic virtue and its normative implications, we believe that our ex-post income inequality concept (as defined here) is more useful because it is consistent with commonly used empirical inequality measures and with the various remuneration schemes we find in the labor markets. An empirical justification for this approach can be found in the work of Keane and

Wolpin (1997).

In this set up we analyze the effect of better information, i.e., more efficient screening of individual skills, on the (ex-post) intragenerational distribution of income and on the formation of human capital. We demonstrate that better information system always results in higher income inequality. This finding contrasts the result in Eckwert and Zilcha (2007) according to which the impact of information on the distribution of income opportunities is markedly different. It turns out that the implications of information policy for income inequality involves an inherent trade-off: under specifications which are consistent with empirical evidence for developed industrial countries, information policy can achieve less inequality in the distribution of actual incomes only at the expense of more inequality in the distribution of income opportunities. Thus, in dynamic models where information is being revealed over time, the date during the life cycle of agents at which (expected) incomes are compared plays an important role.

The effect of improvement in information on the accumulation of human capital depends on the properties of the individual investment decisions. Since the effort level invested in the education process depends on the degree of intertemporal substitution in consumption we find that: if individual preferences exhibit high elasticity of intertemporal substitution, agents with more favorable signals will choose higher investment levels. Under this constellation more efficient screening leads to higher aggregate human capital stock and, hence, higher growth. By contrast, when the elasticity of intertemporal substitution is small, better information reduces the aggregate stock of human capital.

The paper is organized as follows. In section 2 we describe the OLG economy and define our concept of informativeness. In section 3 we study the effect of information on inequality and human capital formation. All proofs are relegated to a separate Appendix.

2 The Model

We consider an overlapping generations economy with a single commodity and a continuum of individuals in each generation. The commodity can be either consumed or used as an input (physical capital) in a production process. Individuals live for three periods: 'youth' where they obtain education (while still supported by parents), 'middle-age' where they work and consume, and 'retirement' where they only consume. We denote generation t by $G_t, t = 0, 1, \cdots . G_t$ consists of all individuals born at date $t-1$.

In our economy individuals are heterogeneous with regard to their human capital which is affected by innate ability $\varphi \in \Phi := [\underline{\varphi}, \bar{\varphi}] \subset \mathbb{R}_+$. While nature assigns abilities to agents at birth, no agent knows nature's choice when he is still young. Agents learn their abilities only at the beginning of their middle-age period and, therefore, they are exposed to uncertainty in their first period of life. The individual risk about ability is the same for all agents; this risk is described by means of a (time-invariant[1]) probability density $\nu(\varphi)$. In view of a result by Feldman and Gilles (1985, p. 29, Proposition 2) we may assume that there is no risk in the aggregate, i.e., the ex post distribution of ability is exactly ν. Also, for convenience, we normalize the measure of agents in each generation to 1:

$$\int_\Phi \nu(\varphi) \, \mathrm{d}\varphi = 1.$$

Human capital of individual $i \in G_t$ depends on ability $\tilde{\varphi}^i$ (perceived as random and, therefore, marked by a ˜) and on effort $e^i \in \mathbb{R}_+$ invested in education by this individual. Thus we write,

$$\tilde{h}^i = B\tilde{\varphi}^i e^i \tag{1}$$

where i belongs to generation t. $B \in \mathbb{R}_{++}$ represents the impact of publicly provided schooling. Since the level and quality of public schooling is exogenous in our model, we set $B = 1$ without loss of generality.[2]

Before agent i chooses optimal effort in the youth period nature assigns to him a signal $y^i \in Y \subset \mathbb{R}$. This signal, which is publicly observable, might be interpreted as a test outcome.[3] Typically, students receive such signals before they enter higher

[1]This assumption according to which the distribution of abilities across agents is the same in each generation is not needed for our anylysis. Time invariance is assumed just for convenience because it allows us to write the densities without a time index.

[2]According to (1), individual human capital formation in generation G_t is not affected by the level of human capital in generation G_{t-1}. Introducing an intergenerational externality into the accumulation function (1) would make little difference to our analysis of labor income inequality.

[3]Alternatively one could assume that an agent is better informed about his own ability than other

education. Examples include personality tests and matriculation examinations used by universities to screen the field of applicants. The test results are noisy but they are correlated with the characteristics that have been tested.

The signals assigned to agents with ability φ are distributed according to the density $f(\cdot|\varphi)$. Since, by construction, the distributions of signals and of abilities across agents are correlated, the signal y^i contains information about agent i's unknown ability $\tilde{\varphi}^i$. Rational agents use this information for the formation of expectations. Therefore, individual i's choice of effort, e^i, will be based on the conditional distribution of $\tilde{\varphi}^i$ given the signal y^i.

Invoking, once again, the above quoted result in Feldman and Gilles (1985), the distribution of signals received by agents in the same generation has the density

$$\mu(y) = \int_{\Phi} f(y|\varphi)\nu(\varphi)\,\mathrm{d}\varphi. \tag{2}$$

An information system, which will be represented by $f : Y \times \Phi \to \mathrm{I\!R}_{++}$ throughout the paper, specifies for each state of nature φ a conditional probability function over the set of signals. The positive real number $f(y|\varphi)$ defines the conditional probability (density) that if the state of nature is φ, then the signal y will be sent. $F(y|\varphi)$ denotes the c.d.f. for the density $f(y|\varphi)$. Given an information system f, the density function for the updated posterior distribution over Φ is

$$\nu_y(\varphi) = f(y|\varphi)\nu(\varphi)/\mu(y). \tag{3}$$

We assume throughout the paper that the densities $\{f(\cdot|\varphi), \varphi \in \Phi\}$ have the strict monotone likelihood ratio property (MLRP): $y' > y$ implies that for any given (nondegenerate) prior distribution for $\tilde{\varphi}$, the posterior distribution conditional on y' dominates the posterior distribution conditional on y in the first-order stochastic dominance. This implies that higher signal is 'good news' (see Milgrom (1981)). As a consequence, $\int_{\Phi} \vartheta(\varphi)\nu_{y'}(\varphi)\,\mathrm{d}\varphi > \int_{\Phi} \vartheta(\varphi)\nu_y(\varphi)\,\mathrm{d}\varphi$ holds for any strictly increasing function ϑ.

To simplify the modeling of the public signals we may assume that the signals are uniformly distributed on $[0, 1]$, i.e., $\mu(y) = 1\ \forall y \in [0, 1]$. This assumption does not

agents. Asymmetries in information complicate the analysis considerably because in equilibrium information will be endogenously communicated between the individuals.

involve any loss of generality.[4]

Let us denote by $\nu_y(\cdot)$ the density of the conditional distribution of φ given the signal y. Then average ability of all agents who have received the signal y is

$$\bar{\varphi}(y) := E[\tilde{\varphi}|y] = \int_{\Phi} \varphi \nu_y(\varphi) \,\mathrm{d}\varphi. \tag{4}$$

We assume that signals are public information and that the effort employed by the individual is observable.

The agents are expected utility maximizers with von-Neumann Morgenstern lifetime utility function

$$U(e, c_1, c_2) = v(e) + u_1(c_1) + u_2(c_2). \tag{5}$$

Individuals derive negative utility from 'effort' while they are young and positive utility from consumption in the working period, c_1, and from consumption in the retirement period, c_2.

Assumption 1 *The utility functions v and u_j, $j = 1, 2$, have the following properties:*

(i) $v : \mathbb{R}_+ \to \mathbb{R}$ *is decreasing and strictly concave,*

(ii) $u_j : \mathbb{R}_+ \to \mathbb{R}$ *is increasing and strictly concave, $j = 1, 2$.*

In each period, production in our economy, is carried out by competitive firms who use two production factors: physical capital K and human capital H. The process is described by an aggregate production function $F(K, H)$, which exhibits constant returns to scale. If individual i supplies l^i units of labor in his 'working period', his supply of human capital equals $l^i h^i$. We assume inelastic labor supply, i.e., l^i is a constant and it is equal to 1 for all i.

[4]Since the signals are ordered, we can transform any information system into an equivalent one in which signals are uniformly distributed on $[0, 1]$. For a given information system $f : Y \times \Phi \to \mathbb{R}_{++}$ consider the transformation $\tilde{\pi} := F \circ \tilde{y}$, where F is the c.d.f. for the probability density μ defined in (2). Thus for any $y \in Y$, the transformed signal $\pi = F(y)$ represents the probability that, under the information system f, an agent receives a signal less than y. Obviously, $\tilde{\pi}$ is uniformly distributed over $[0, 1]$. Since the transformation is strictly monotonic, the transformed signal $\pi = F(y)$ reveals exactly the same information about the unknown state of nature as the signal y.

Assumption 2 *$F(K, H)$ is concave, homogeneous of degree 1, and satisfies $F_K > 0$, $F_H > 0, F_{KK} < 0, F_{HH} < 0$.*

We assume throughout this paper full international capital mobility, while human capital is assumed to be immobile. Thus the interest rate $\bar{r}_t$ is exogenously given at each date t. This implies that marginal productivity of aggregate physical capital K_t must be equal to $1+\bar{r}_t$ (assuming full depreciation of capital in each period). On the other hand, given the aggregate stock of human capital at date t, H_t, the stock K_t must adjust such that

$$1 + \bar{r}_t = F_K(K_t, H_t) \qquad t = 1, 2, 3, \cdots \tag{6}$$

holds. But this implies, by Assumption 2, that $\frac{K_t}{H_t}$ is determined by the international rate of interest $\bar{r}_t$. Hence the wage rate w_t (price of one unit of human capital), given in equilibrium by the marginal product of aggregate human capital, is also determined once $\bar{r}_t$ is given. Thus we may write

$$w_t = F_L\Big(\frac{K_t}{H_t}, 1\Big) =: \zeta(\bar{r}_t) \qquad t = 1, 2, 3, \cdots . \tag{7}$$

Labor contracts are concluded *after* agents have learned their signals but *before* their abilities become known.

Obviously, the wage income specified in a labor contract cannot be made contingent on individual human capital because individual ability is yet unknown. Therefore agents are unable to appropriate the full marginal product of their human capital. Instead, individuals are grouped according to the signals they have received.[5] And, in the absence of any further information, the market treats all agents in the same group identically. Under these circumstances each individual will receive a wage equal to the marginal product of the mean human capital of those with whom he is grouped.[6]

[5]Such an assumption can be supported by empirical observations. For the case of the USA, Keane and Wolpin (1997) estimated that unobserved endowment heterogeneity, as measured at age of 16, accounts for 90 percent of the variance in lifetime utility; hence time-varying exogenous shocks to skills account for only 10 percent of the variation. Our specification is nevertheless somewhat restrictive because it does not allow an agent's wage income to depend ex post on his true ability. Introducing such dependence would make income uncertain at the time decisions are made. Income uncertainty typically affects savings levels but would probably leave our main results on income inequality and human capital formation unchanged.

[6]This specification requires that employers are able to observe the effort level invested by the agent during his education period as well as his signal. For a similar approach, see Spence (1973).

For all agents who have received the signal y, average ability is given by $\bar{\varphi}(y)$ in equation (4). Therefore, the wage income of agent $i \in G_t$ with signal y is $w_t \bar{h}^i$, where

$$\bar{h}^i = \bar{\varphi}(y) e^i. \tag{8}$$

When young, all agents face the same ability risk. Therefore, any two individuals who receive the same signal choose the same effort level. As a consequence, in equilibrium, aggregate wage income and aggregate human capital in the group of agents with signal y are given by $w_t \bar{h}^i$ and $\bar{h}^i$, respectively. The firm therefore pays the competitive wage in (7), $(w_t \bar{h}^i)/\bar{h}^i = w_t$, for each unit of *aggregate* human capital supplied by agents with signal y.

Now let us consider the optimization problem that each $i \in G_t$ faces, given $\bar{r}_t$ and w_t. At date $t-1$, when 'young', this individual chooses an optimal level of effort employed in obtaining education, e^i, and an optimal level of savings (effected in period t), s^i. These decisions are made under random ability $\tilde{\varphi}$, but after the individual signal y^i has been observed. The timing of events and decisions for agent $i \in G_t$ is as follows (see Figure 1):

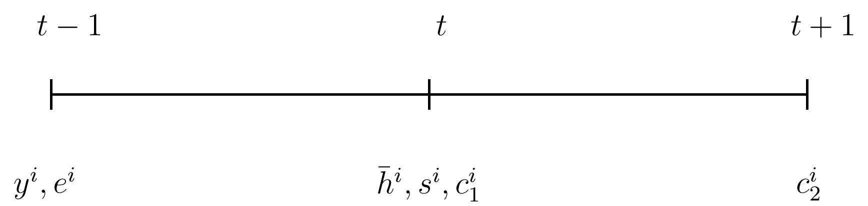

Figure 1

For given levels of w_t and $\bar{r}_t$, the optimal saving and effort decisions of individual $i \in G_t$ are determined by

$$\max_{s^i, e^i} \; v(e^i) + u_1(c_1^i) + u_2(c_2^i) \tag{9}$$
$$\text{s.t. } c_1^i = w_t \bar{h}^i - s^i$$
$$c_2^i = (1 + \bar{r}_t) s^i.$$

Since income is determined by *average ability*, given the signal y^i and the effort level, saving s^i is based on average human capital $\bar{h}^i$ (and not on h^i); as a consequence, period 2 consumption c_2^i is non-random when e^i is chosen.

The first order conditions are necessary and sufficient,

$$-u_1'(w_t\bar{h}^i - s^i) + (1+\bar{r}_t)u_2'((1+\bar{r}_t)s^i) = 0 \tag{10}$$

$$v'(e^i) + w_t\bar{\varphi}(y^i)u_1'(w_t\bar{h}^i - s^i) = 0, \tag{11}$$

where $\bar{h}^i$ is given by equation (8).

Observe that the signal y^i enters the first order conditions only via the term $\bar{\varphi}(y^i)$. Thus we may express the optimal decisions as functions of $\bar{\varphi}(y^i)$ rather than as functions of the signal itself, i.e., $s^i = s_t\big(\bar{\varphi}(y^i)\big)$, $e^i = e_t\big(\bar{\varphi}(y^i)\big)$. Similarly, in equilibrium we have $\bar{h}^i = \bar{h}_t\big(\bar{\varphi}(y^i)\big)$.[7] The functions $s_t : \mathbb{R}_+ \to \mathbb{R}_+$, $e_t : \mathbb{R}_+ \to \mathbb{R}_+$, and $\bar{h}_t : \mathbb{R}_+ \to \mathbb{R}_+$, $\bar{h}_t(\bar{\varphi}) := \bar{\varphi}e_t(\bar{\varphi})$ satisfy (10) and (11), i.e.,

$$-u_1'\big(w_t\bar{h}_t(\bar{\varphi}) - s_t(\bar{\varphi})\big) + (1+\bar{r}_t)u_2'\big((1+\bar{r}_t)s_t(\bar{\varphi})\big) = 0 \tag{12}$$

$$v'\big(e_t(\bar{\varphi})\big) + w_t\bar{\varphi}u_1'\big(w_t\bar{h}_t(\bar{\varphi}) - s_t(\bar{\varphi})\big) = 0, \tag{13}$$

for all $\bar{\varphi} > 0$. In addition, $s_t(0) = e_t(0) = \bar{h}_t(0) = 0$.

Using (2) and (4) the aggregate stock of human capital at date t can be expressed as

$$H_t = E_y\big[\bar{h}_t\big(\bar{\varphi}(y)\big)\big] = \int_0^1 \bar{h}_t\big(\bar{\varphi}(y)\big)\mathrm{d}y, \tag{14}$$

where $\bar{h}_t\big(\bar{\varphi}(y)\big)$ is the average human capital of agents in G_t who have received the signal y.

Definition 1 *Given international interest rates $(\bar{r}_t)_{t=1}^{\infty}$, a competitive equilibrium consists of a sequence $\big\{[e^i(y^i), s^i(y^i)]_{i\in G_t}\big\}_{t=1}^{\infty}$, and a sequence of wages $(w_t)_{t=1}^{\infty}$, such that:*

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[7]This implies that all agents with signal y receive the same labor income $w_t\bar{h}_t\big(\bar{\varphi}(y)\big)$. Note, however, that individual incomes are affected by *both* the signals and the chosen effort levels. Therefore, agents do have incentives to choose positive effort levels. Notwithstanding this observation, *equilibrium labor incomes* depend only on the agents' signals because in this model any two agents with the same signal choose the same effort level.

(i) *At each date t, given* $\bar{r}_t$ *and* w_t*, the optimum for each* $i \in G_t$ *in problem (9) is given by* $[e^i(y^i), s^i(y^i)]$.

(ii) *The aggregate stocks of human capital,* $H_t, t = 1, 2, \cdots$ *, satisfy (14).*

(iii) *Wage rates* $w_t, t = 1, 2, \cdots$*, are determined by (7).*

Next we define our criterion of informativeness. Let $G(\varphi|y)$ be the c.d.f. for the conditional density $\nu_y(\varphi)$ (defined in (3)).

Remark 1: $G(\varphi|y)$ is a decreasing function of y. This follows from MLRP: choose $\hat{\varphi} \in \Phi$ arbitrarily but fixed and define

$$U(\varphi) = \begin{cases} 0 & ; \quad \varphi \leq \hat{\varphi} \\ 1 & ; \quad \varphi > \hat{\varphi} \end{cases} .$$

Since $EU(\tilde{\varphi}|y) = 1 - G(\hat{\varphi}|y)$ is increasing in y by virtue of MLRP, $G(\varphi|y)$ is decreasing in y for all $\varphi \in \Phi$.

Let us denote by $G_y(\varphi|y)$ the partial derivative of the c.d.f. $G(\varphi|y)$ with respect to y. An information system will be regarded as more informative if the observable signal realizations have a uniformly stronger impact on the posterior distribution of states:

Definition 2 (informativeness) *Let* $\bar{f}$ *and* $\hat{f}$ *be two information systems with corresponding c.d.f's* $\bar{G}(\varphi|y)$, $\hat{G}(\varphi|y)$ *for the densities* $\bar{\nu}(\varphi|y)$, $\hat{\nu}(\varphi|y)$. $\bar{f}$ *is more informative than* $\hat{f}$ *(expressed by* $\bar{f} \succ_{\text{inf}} \hat{f}$*), if*

$$\bar{G}_y(\varphi|y) \leq \hat{G}_y(\varphi|y) \tag{15}$$

holds for all $\varphi \in \Phi$ *and* $y \in (0, 1)$.

According to Remark 1, $G(\varphi|y) = \text{prob}\left(\tilde{\varphi} \leq \varphi|y\right)$ is decreasing in the signal y. Inequality (15) says that under a more informative system the posterior distribution over states is more sensitive with respect to signal realizations.[8] Note that this

[8] In a recent paper, Ganuza and Penalva (2006) analyze information problems in auctions. They use an information concept which is also based on differences in posterior distributions and which can be shown to be equivalent to ours.

concept of informativeness does not involve any restrictions on the dispersion of abilities relative to the dispersion of test outcomes. The distribution of abilities is exogenously given by $\nu(\cdot)$ and the test outcomes are uniformly distributed under any information system. Instead, condition (15) involves a restriction on the correlation structure between these two distributions.

In the economics literature various concepts of informativeness have been used, dating back to the seminal work by Blackwell (1951,1953) where the ordering of information has been linked to a statistical sufficiency criterion for signals. More recently, concepts have been developed which represent informativeness as a stochastic dominance order over conditional distributions of signal transformations (Kim (1995), Athey and Levin (1988), Demougin and Fluet (2001)). Some of these partial orderings contain the Blackwell ordering as a subset.[9]

Our concept of information in (15) imposes a restriction on the sensitivities of the posterior state distributions. It has an advantage in terms of tractability over the above mentioned criteria as it involves only signal derivatives of the posteriors rather than more complex measures of stochastic dominance. In dichotomies, i.e., when the number of signals and the number of states are both equal to 2, the restriction in (15) is implied by the Blackwell criterion (Hermelingmeier (2007)). Yet, the Blackwell criterion is itself a restrictive concept that has been generalized in many directions. For a discussion of the relationships between the various extensions of the Blackwell information odering see Jewitt (1997).

3 Formation and Distribution of Human Capital

In the sequel we shall compare equilibria under different information systems with regard to the formation and distribution of human capital. To avoid notational confusion, equilibrium values attained under information system f will be marked by an upper index f.

We begin by analyzing the effects of better information on the formation of human capital. Aggregate human capital of generation t under information system f is

$$H_t^f = \int_0^1 \bar{h}_t\big(\bar{\varphi}^f(y)\big)\,\mathrm{d}y \tag{16}$$

[9]E.g. Kim's criterion can be shown to be strictly weaker than Blackwell's criterion.

where

$$\bar{h}_t(x) := x e_t(x), \qquad x := \bar{\varphi}^f(y).$$

Since $\bar{h}_t \geq 0$ and $\bar{h}_t(0) = 0$, $e_t(x)$ is increasing (decreasing) in x if $\bar{h}_t(x)$ is a convex (concave) function of x.[10]

Depending on the well-known interaction between an income effect and a substitution effect, $\bar{h}_t(\cdot)$ may be convex or concave. The strengths of the income effect and the substitution effect depend on the elasticity of intertemporal substitution between the periods 2 and 3. In Section 3.1 we will consider the special case where preferences exhibit constant elasticity of intertemporal substitution. For such preferences, the income effect is dominant and hence $\bar{h}_t(\cdot)$ is concave, if the elasticity of intertemporal substitution is sufficiently small. As a consequence, a better signal results in lower effort. By contrast, the substitution effect is dominant, if the elasticity of intertemporal substitution is sufficiently high. In that case $\bar{h}_t(\cdot)$ is convex which means that agents step up their efforts when they receive more favorable signals.

The expected marginal product of investment in education, $\bar{\varphi}^f(y)$, is higher for agents with better signals. Thus, convexity of $\bar{h}_t(\cdot)$ (increasing effort function) would be more conducive to the formation of the human capital stock than concavity of $\bar{h}_t(\cdot)$ (which implies a decreasing effort function). We shall therefore call individual behavior *accumulation-inducing* if $\bar{h}_t(\cdot)$ is convex and, hence, good news (higher signal) induces higher investment in education. Similarly, individual investment behavior will be called *accumulation-impeding* if $\bar{h}_t(\cdot)$ is concave, a case where good news results in investing lower effort.

Proposition 1 *Let $\bar{f}$ and $\hat{f}$ be two information systems such that $\bar{f} \succ_{\inf} \hat{f}$. Consider the corresponding competitive equilibria under these two systems.*

(i) *Under accumulation-inducing behavior better information (weakly) enhances human capital formation , i.e., $H_t^{\bar{f}} \geq H_t^{\hat{f}}$ for all $t \geq 1$.*

(ii) *Under accumulation-impeding behavior better information (weakly) reduces human capital formation, i.e., $H_t^{\bar{f}} \leq H_t^{\hat{f}}$ for all $t \geq 1$.*

[10] If $\bar{h}_t(x)$ is convex (concave), the difference quotient $[\bar{h}_t(x) - \bar{h}_t(0)]/x = \bar{h}_t(x)/x = e_t(x)$ is increasing (decreasing) in $x > 0$.

The characterization in Proposition 1 can be interpreted in terms of a simple economic mechanism. Consider part (i), i.e., assume that investment behavior is accumulation-inducing. The implementation of a better information system enhances the reliability of the individual signals. As a consequence, high signals become even better news and induce higher investment in education. Similarly, under a better information system the bad news conveyed by a low signal becomes even worse (because now the news is more reliable). Hence, investment in education declines. Thus, under accumulation-inducing investment behavior, better information tends to increase the efforts of agents with high signals and decrease the efforts of agents with low signals. Since the expected marginal product of effort (in terms of human capital) is higher for agents with higher signals, aggregate human capital increases when the information system becomes more informative. If investment behavior is accumulation-impeding, the same mechanism results in lower aggregate human capital under a more informative system.

Next we look into the effects of better information on income inequality. Our analysis of income inequality focuses on the distribution of labor income within a given generation G_t. Labor income depends both on the information system and on the signal received by an agent,

$$I_t^f(y) = w_t \bar{\varphi}^f(y) e_t\big(\bar{\varphi}^f(y)\big), \tag{17}$$

where

$$\bar{\varphi}^f(y) := E^f[\tilde{\varphi}|y]. \tag{18}$$

To study the impact of information on income inequality we use a concept which is based on the following comparison of distributions:

Definition 3 *Let Y and X be real-valued random variables with zero-mean normalizations $\check{Y} = Y - EY$ and $\check{X} = X - EX$. The distribution of Y is 'more unequal' than the distribution of X if $\check{Y}$ differs from $\check{X}$ by a Mean Preserving Spread (MPS).*

This definition of inequality differs from the requirement that one Lorenz curve is strictly above the other one, which is equivalent to second degree stochastic dominance (see, Atkinson (1970)). Instead, our definition (known as *absolute* Lorenz order) is based on a location-free concept of dispersion. The induced ordering is implied by the Bickel-Lehmann stochastic ordering (see, Landsberger and Meilijson (1994)) which is a concept commonly used in statistics.[11]

The following lemma facilitates the application of our inequality concept in Definition 3:

Lemma 1 *Let $\tilde{y}$ be a random variable which is distributed over the unit interval* $[0, 1]$*. Let $z : [0, 1] \to \mathbb{R}$, $x : [0, 1] \to \mathbb{R}$ be differentiable increasing functions such that*

(i) $\tilde{z} := z \circ \tilde{y}$ differs from $\tilde{x} := x \circ \tilde{y}$ by a MPS,

(ii) $z(y) - x(y)$ is strictly monotone in y.

Let $\vartheta : \mathbb{R} \to \mathbb{R}$ be a differentiable strictly increasing function. The distribution of $\vartheta \circ \tilde{z}$ is more unequal than the distribution of $\vartheta \circ \tilde{x}$, if ϑ is either convex or concave.

Remark 2: The claim in Lemma 1 remains valid if z, x and ϑ are continuous (rather than differentiable) and if $z(y) - x(y)$ is monotone (rather than strictly monotone) in y. This can be shown by means of a straightforward extension of the proof of Lemma 1.

Our concept of income inequality is based on the dominance criterion for normalized distributions in Definition 3.

Definition 4 *Let $\bar{f}$ and $\hat{f}$ be two information systems. Income inequality under $\bar{f}$ is higher than under $\hat{f}$, if the distribution of $I_t^{\bar{f}}(y)$ is more unequal than the distribution of $I_t^{\hat{f}}(y)$ for all $t \geq 1$.*

[11] Denote by F and G the c.d.f's of of X and Y, respectively. The distribution F is less dispersed than G in the Bickel-Lehmann sense, if for any $0 < \alpha < \beta < 1$,

$$F^{-1}(\beta) - F^{-1}(\alpha) \leq G^{-1}(\beta) - G^{-1}(\alpha).$$

Namely, the interval between the α-quantile and the β-quantile of F is less than or equal to that for G. This implies that $G^{-1}(\theta) - F^{-1}(\theta)$ is a non-decreasing function on $(0, 1)$. It is easy to verify that for each constant k, $F(\theta - k)$ and $G(\theta)$ cross at most once, and if they cross then $F(\theta - k)$ lies below $G(\theta)$ to the left of the crossing point (see, Lansberger and Meilijson (1994)). If $-\infty < \int x \, \mathrm{d}G(x) \leq \int x \, \mathrm{d}F(x) < \infty$ holds (in addition to the above inequality) then F dominates G in the sense of SDSD.

Under a better information system individual ability can be assessed more accurately at the time when labor contracts are concluded. We may conjecture, therefore, that firstly the income distribution will be more discriminating with respect to differences in abilities, and secondly that it will be better in line with the distribution of human capital across agents. The following proposition confirms our first conjecture, i.e., the informational mechanism results in higher income inequality.

Proposition 2 *Let $\bar{f}$ and $\hat{f}$ be two information systems such that $\bar{f} \succ_{\text{inf}} \hat{f}$. Under both constellations, i.e., accumulation-inducing behavior as well as accumulation-impeding behavior, the information system $\bar{f}$ results in more income inequality than $\hat{f}$.*

Even though the result in Proposition 2 confirms the above intuitive conjecture, it is not straightforward because the effort decision may be decreasing in the signal y if the elasticity of intertemporal substitution is small. In that case agents who are better talented (on average) invest less in education. This mechanism clearly involves a tendency towards less income inequality for any *given* information system. In view of Proposition 2 it nevertheless remains true that under a better information system the income distribution becomes more unequal in favor of the more talented agents.

The relationship between economic growth and income inequality has been widely debated in the literature in the last decade. Based on empirical evidence, Persson and Tabellini (1994) show that higher growth results in less income inequality – a finding that was challenged by other authors, e.g., Forbes (2000) and Quah (2002). Our study contributes to this controversy with a narrow, information-based focus: we identify the effects of information on indicators for human capital formation and income inequality and obtain the co-movements of both indicators due to changes in information.

From propositions 1 and 2 we obtain, as a corollary, an information-induced link between human capital formation and income inequality:

Corollary 1 *As the result of an improvement of the economy's information system,*

(i) *under accumulation-inducing investment behavior, higher human capital formation is accompanied by more income inequality;*

(ii) *under accumulation-impeding investment behavior, lower human capital formation is accompanied by more income inequality.*

Thus, human capital accumulation (i.e., growth in our framework) and income inequality are positively related if agents respond to better signals with higher investment in education. Yet, the model is also consistent with an inverse relationship between human capital formation and income inequality. Such a pattern arises when better signals induce agents to reduce investment in education.

In our model, an agent's noisy signal provides *imperfect* information about his ability, but *perfect* information about his income. Therefore, in a given signal group, agents differ in their abilities but they do not differ with regard to incomes. The diversity with respect to ability among agents in the same signal group declines as the signals become more reliable under a better information system. Under perfect information, i.e., when the signal reveals the ability level, each signal group consists solely of agents with identical abilities. We have seen that less diversity within signal groups may increase or decrease human capital formation, depending on the shape of the accumulation function $\bar{h}_t(\cdot)$. On the other hand, the agents' incomes depend only on their signals, and the reliability of the signals determines how sensitively incomes vary across signal groups. Therefore, income inequality always increases under a better information system. In particular, under perfect information the economy exhibits a maximum degree of income inequality.

3.1 The Case of CEIS Preferences

To illustrate the critical role of the elasticity of intertemporal substitution for the information-induced link between income inequality and human capital formation we restrict the utility functions $u_1(\cdot)$, $u_2(\cdot)$, and $v(\cdot)$ to be in the family of CEIS (**C**onstant **E**lasticity of **I**ntertemporal **S**ubstitution) :

$$u_1(c_1) = \frac{c_1^{1-\gamma_u}}{1-\gamma_u}; \quad u_2(c_2) = \beta \frac{c_2^{1-\gamma_u}}{1-\gamma_u}; \quad v(e) = -\frac{e^{\gamma_v+1}}{\gamma_v+1}. \tag{19}$$

γ_u and γ_v are strictly positive constants. $1/\gamma_u$ parametrizes the elasticity of intertemporal substitution in consumption.

Using the functional forms of u_j, $j = 1, 2$, in (19), it follows from equation (10) that, given $\bar{r}_t$ and w_t, the saving s^i is proportional to the human capital level h^i. In other words, for each t there is a constant m_t such that for all $i \in G_t$ we have:

$$s^i = m_t h^i, \quad 0 < m_t < w_t, \quad t = 1, 2, \cdots \tag{20}$$

The specifications in (19)and (20) allow us to solve equation (11) for the optimal effort level as a function of average ability $\bar{\varphi}^f(y)$:

$$e_t\big(\bar{\varphi}^f(y)\big) = \delta_t\big(\bar{\varphi}^f(y)\big)^{\rho(1-\gamma_u)} \tag{21}$$

where

$$\delta_t := \left[\frac{w_t}{(w_t - m_t)^{\gamma_u}}\right]^\rho; \quad \rho = \frac{1}{\gamma_v + \gamma_u}.$$

The income of an agent with signal y is

$$I_t^f(y) = w_t \delta_t \big(\bar{\varphi}^f(y)\big)^\tau, \tag{22}$$

and aggregate human capital of generation t is

$$H_t^f = \delta_t \int_0^1 \big(E^f[\tilde{\varphi}|y)]\big)^\tau \, \mathrm{d}y, \tag{23}$$

where

$$\tau := 1 + \rho(1 - \gamma_u) = \frac{1 + \gamma_v}{\gamma_v + \gamma_u} > 0. \tag{24}$$

Corollary 2 *Let $\bar{f}$ and $\hat{f}$ be two information systems such that $\bar{f} \succ_{\text{inf}} \hat{f}$, and assume that the specifications in (19) are valid.*

(i) *High EIS:* *For $1/\gamma_u \geq 1$ better information (weakly) enhances human capital formation, i.e., $H_t^{\bar{f}} \geq H_t^{\hat{f}}$ for all $t \geq 1$.*

(ii) *Moderate EIS:* *For $1/\gamma_u \leq 1$ better information (weakly) reduces human capital formation, i.e., $H_t^{\bar{f}} \leq H_t^{\hat{f}}$ for all $t \geq 1$.*

Proof: Consider the expression in (21) for $x = \bar{\varphi}^f(y)$. Clearly $\bar{h}_t(x)$ is convex if $1+\rho(1-\gamma_u) \geq 0$, namely if $\gamma_u \leq 1$. In that case the effort function $e_t(x)$ is increasing. Similarly, $\bar{h}_t(x)$ is concave, i.e., $e_t(x)$ is decreasing, if $\gamma_u \geq 1$. Thus Corollary 2 follows from Proposition 1. □

From Proposition 2 and Corollary 2 we obtain the following characterization of the information-induced link between human capital formation and income inequality:

Corollary 3 *Assume that the specifications in (19) are valid. As a result of an improvement of the economy's information system,*

(i) *higher human capital formation implies more income inequality, if the elasticity of intertemporal substitution in consumption is high, i.e.,* $1/\gamma_u \geq 1$*,*

(ii) *lower human capital formation implies more income inequality, if the elasticity of intertemporal substitution in consumption is small, i.e.,* $1/\gamma_u \leq 1$*.*

3.2 Policy Implications

Due to the noise in the signals received by the agents, the screening process in the education sector involves a certain amount of misperception on the part of the individuals. By investing into activities that would improve the validity of the tests the government can reduce the extent to which agents misperceive their abilities. Our analysis suggests that such policies may produce unwelcome effects with regard to human capital formation and the distribution of income. These effects critically depend on the elasticity of intertemporal substitution. More precisely, for small values of the elasticity of intertemporal substitution better information slows human capital formation and, at the same time, leads to more income inequality.

Empirical evidence suggests that in developed industrial countries the elasticity of intertemporal substitution is in fact small, and possibly close to zero (Hall (1988)). For developed countries our model therefore predicts that more efficient screening produces less human capital formation and more income inequality. At the same time, the distribution of income opportunities becomes less unequal (Eckwert and Zilcha (2007)).[12] Thus, abstracting from the impact on human capital formation, any information policy involves an inherent trade-off: less inequality in the distribution

[12] In the work by Eckwert and Zilcha (2007) the definition of income opportunities is based on the individuals' expectations about their random future incomes conditional on their true abilities. This special concept of 'income opportunities' is critical for the established link between income inequality and inequality of income opportunities.

of actual incomes can only be achieved at the expense of more inequality in the distribution of income opportunities.

Of course, ultimately it is a matter of value judgement whether the distribution of actual incomes or of income opportunities should be an issue of political concern. An important lesson to learn from our analysis is, therefore, to recognize the existence of a trade-off between these two goals: less inequality in the distribution of income opportunities comes at the expense of more inequality in the distribution of actual incomes.

4 Conclusion

We presented a framework in which we have studied the effects of information on income inequality and human capital formation. Our analysis has demonstrated the role played by a monotonicity property (in 'favorableness of information') of individual investment in education. If consumer preferences exhibit high intertemporal substitution in consumption, agents with better test results and, hence, higher ability prospects, choose higher investments in education. In this case both income inequality and the stock of human capital increase when the information system is improved. Similarly, if consumer preferences exhibit low intertemporal substitution in consumption, agents with more favorable signals invest less. This means that higher inequality is accompanied by less human capital accumulation (hence less growth). In addition, under specifications which are consistent with empirical evidence for developed industrial countries, information policy can achieve less inequality in the distribution of actual incomes only at the expense of more inequality in the distribution of income opportunities.

Due to the simple intertemporal structure of our dynamic model, the competitive equilibrium always exists and it is unique. Convergence to a steady state is possible but not inevitable. The welfare properties of the model, which have not been discussed here, depend on the economy's information system. It is well known, however, that in models with risk sharing arrangements economic welfare does not necessarily increase with better information (Schlee (2001)).

In the economic literature various notions of what constitutes 'more information' have been suggested. Blackwell's prominent sufficiency criterion has been widely used, but this concept is understood to be quite demanding and, in fact, stronger than needed for many economic applications. In recent years other concepts based on the sensitivity of the posterior state distribution with regard to signals have been developed and successfully applied to economic problems (e.g., Kim (1995), Athey, Levin (1998), and Persico (2000)). Our notion of informativeness belongs to this class of extensions, i.e., the informativeness order emerges from a restriction on the distribution of state posteriors. While this information concept works well in our theoretical context, it admits some scope of generalization that we have not fully taken advantage of. In our model, only the ex-ante distribution over the conditional expectations of a worker's ability induced by a given information system has any real effects. Thus, the results in this paper could be derived from restrictions put only on the ex-ante distribution over conditional expectations of abilities, rather than on the full distribution of abilities conditional on signals. In particular, the properties that are established in Lemma 3 in the appendix might directly be used as the criterion for a partial ordering of information systems. Such a partial ordering would be more permissive (although less intuitive) than the one we have adopted here.

The analysis in this paper has been simplified by the assumption of a small open economy and full international capital mobility. Endogenous interest rates would increase the model's complexity and could modify some of the derived results. While the main mechanisms on which our analysis has concentrated would still be operative, more stringent assumptions might be necessary to preserve the robustness of our findings.

The time structure of our model implies that agents receive wage payments which are based on the agents' signals and investment in education rather than on their true (ex post) abilities. Thus individual wage incomes are based on assessments of each agent's 'potential' rather than on the human capital that is actually contributed in the production process.

However, let us briefly discuss the case where wage contracts are contingent on ex post individual human capital rather than on the signals and investment in education. In such a setting each agent i is characterized by a pair (y^i, A^i), but his economic decisions are based solely on the signal y^i (while A^i is still random). In this case information no longer plays a role in the process of (partial) risk sharing across agents. Therefore, the impact of information on income inequality and human capital formation will presumably be weaker than in our model. This conjecture requires further analysis and we intend to examine it in some future work.

Appendix

In this appendix we prove Lemma 1 and the two propositions.

Proof of Lemma 1: Assume that ϑ is convex, i.e., ϑ' is increasing (we deal with the case where ϑ is concave in step 5).

Step 1: Since $z(y) - x(y)$ is strictly monotone and $z \circ \tilde{y}$ is obtained from $x \circ \tilde{y}$ by a MPS, there exists y^* such that $z(y^*) = x(y^*)$ and

$$\begin{aligned} z(y) &< x(y) \quad \text{for } y < y^* \\ z(y) &> x(y) \quad \text{for } y > y^*. \end{aligned}$$

Clearly the above inequalities are preserved under the strictly monotone increasing transformation ϑ, i.e.,

$$\vartheta \circ z(y) < \vartheta \circ x(y) \quad \text{for } y < y^*, \tag{25}$$

$$\vartheta \circ z(y) > \vartheta \circ x(y) \quad \text{for } y > y^*. \tag{26}$$

Step 2: We show that for $y \geqslant y^*$, $\vartheta \circ z(y) - \vartheta \circ x(y)$ is strictly monotone increasing.

$$\frac{\partial}{\partial y}\Big[\vartheta \circ z(y) - \vartheta \circ x(y)\Big] = \vartheta'\big(z(y)\big)z'(y) - \vartheta'\big(x(y)\big)x'(y) > 0. \tag{27}$$

The inequality is satisfied since $z'(y) > x'(y)$ holds by assumption; and since $\vartheta'\big(z(y)\big) \geqslant \vartheta'\big(x(y)\big)$ holds due to the convexity of ϑ and due to the fact that $z(y) \geqslant x(y)$ for $y \geqslant y^*$.

Step 3: We show that the normalized random variables $\check{z}(y) := \vartheta \circ z(y) - E\big[\vartheta \circ z(\tilde{y})\big]$ and $\check{x}(y) := \vartheta \circ x(y) - E\big[\vartheta \circ x(\tilde{y})\big]$ satisfy

$$\check{z}(y) \neq \check{x}(y) \quad \text{for } y < y^*. \tag{28}$$

Since $z(y)$ is a MPS of $x(y)$ and ϑ is convex,

$$E\big[\vartheta \circ z(\tilde{y})\big] \geqslant E\big[\vartheta \circ x(\tilde{y})\big] \tag{29}$$

holds. Equations (25) and (29) then imply

$$\check{z}(y) < \check{x}(y) \quad \text{for } y < y^*.$$

Step 4: Since $\check{z}(\tilde{y})$ and $\check{x}(\tilde{y})$ have the same mean, there exists y^{**} such that $\check{z}(y^{**}) = \check{x}(y^{**})$. In view of (28), $y^{**} \geqslant y^*$ holds. The intersection point y^{**} is unique since $\check{z}(y) - \check{x}(y)$ is strictly monotone increasing for $y \geqslant y^*$ according to step 2. Thus we have shown that

$$\check{z}(y) \overset{(>)}{<} \check{x}(y) \quad \text{for } y \overset{(>)}{<} y^{**}. \tag{30}$$

These inequalities imply that $\check{z} = \vartheta \circ \tilde{z} - E\big[\vartheta(\tilde{z})\big]$ is obtained from $\check{x} = \vartheta \circ \tilde{x} - E\big[\vartheta(\tilde{x})\big]$ by a MPS and, hence, the distribution of $\vartheta \circ \tilde{z}$ is more unequal than the distribution of $\vartheta \circ \tilde{x}$.

Step 5: If ϑ is concave, then in step 2 inequality (27) holds for $y \leq y^*$ and, consequently, in step 3 we get $\check{z}(y) \neq \check{x}(y)$ for $y > y^*$. This implies $y^{**} < y^*$ in step 4 from which, once again, the inequalities in (30) follow. □

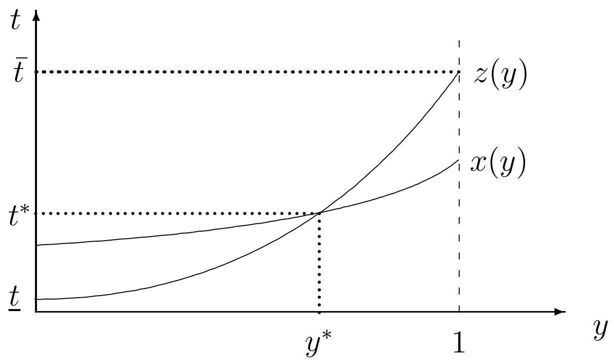
We prove two preliminary results before we proceed with the proofs of the propositions.

Lemma 2 (MPS) *Let $\tilde{y}$ be a random variable which is distributed over $[0,1]$ according to the Lebesgue density ϕ. Let $z : [0,1] \to [\underline{t}, \bar{t}]$ and $x : [0,1] \to [\underline{t}, \bar{t}]$ be differentiable strictly increasing functions such that $E[z \circ \tilde{y}] = E[x \circ \tilde{y}]$, i.e.,*

$$\int_0^1 z(y)\phi(y)\,\mathrm{d}y = \int_0^1 x(y)\phi(y)\,\mathrm{d}y. \tag{31}$$

Assume further that $z(y)$ and $x(y)$ have the single crossing property with $z(y^*) = x(y^*) =: t^*$ and $z(y) \overset{(\geq)}{\leq} x(y)$ for $y \overset{(\geq)}{\leq} y^*$. Then $Z := z \circ \tilde{y}$ differs from $X = x \circ \tilde{y}$ by a MPS.

Remark 3: If $z(y)$ and $x(y)$ are strictly decreasing and the other conditions in Lemma 1 are satisfied, then $X = x \circ \tilde{y}$ differs from $Z = z \circ \tilde{y}$ by a MPS.



<u>Proof of Lemma 2:</u> Let G and F be the c.d.f.'s for Z and X. Denote by g and f the (Lebesgue) densities of G and F, and define $S := G - F$. From $z(y) \overset{(\geq)}{\leq} x(y)$ for $y \overset{(\geq)}{\leq} y^*$ we conclude $S(t) \overset{(\leq)}{\geq} 0$ for $t \overset{(\geq)}{\leq} t^*$ and, hence,[13]

$$\begin{aligned} \int_{\underline{t}}^{\bar{t}} S(t)\,\mathrm{d}t &= \underbrace{tS(t)\Big|_{\underline{t}}^{\bar{t}}}_{=0} - \int_{\underline{t}}^{\bar{t}} t[g(t) - f(t)]\,\mathrm{d}t \\ &= \int_0^1 [z(y) - x(y)]\phi(y)\,\mathrm{d}y = 0. \end{aligned} \tag{32}$$

$$\int_{\underline{t}}^{\hat{t}} S(t)\,\mathrm{d}t = \int_{\underline{t}}^{t^*} S(t)\,\mathrm{d}t + \int_{t^*}^{\hat{t}} S(t)\,\mathrm{d}t \geq 0. \tag{33}$$

The inequality in (33) follows from (32) and the fact that $S(t) \overset{(\leq)}{\geq} 0$ for $t \overset{(\geq)}{\leq} t^*$. (32) and (33) together imply that Z differs from X by a MPS. □

Lemma 3 *Let $\bar{f}$ and $\hat{f}$ be two information systems with $\bar{f} \succ_{\text{inf}} \hat{f}$. For any increasing differentiable function $\vartheta : \Phi \to \mathbb{R}_+$ the random variable $\bar{\theta}(y) := E^{\bar{f}}[\vartheta(\varphi)|y]$ differs from $\hat{\theta}(y) := E^{\hat{f}}[\vartheta(\varphi)|y]$ by a MPS. Also, $\bar{\theta}(y) - \hat{\theta}(y)$ is monotone increasing in y.*

[13] The first equality in the second line of (32) follows from

$$\int_{\underline{t}}^{\bar{t}} tg(t)\,\mathrm{d}t = \int_{z^{-1}(\underline{t})}^{z^{-1}(\bar{t})} z(y)\underbrace{g(z(y))z'(y)}_{=\phi(y)}\,\mathrm{d}y = \int_0^1 z(y)\phi(y)\,\mathrm{d}y.$$

Remark 4: If ϑ is a decreasing function, $\hat{\theta}(y)$ differs from $\bar{\theta}(y)$ by a MPS.

Proof of Lemma 3: By the law of iterated expectations, $\int_0^1 \bar{\theta}(y)\,\mathrm{d}y = \int_0^1 \hat{\theta}(y)\,\mathrm{d}y$. Therefore, in view of Lemma 2, it suffices to show that $\bar{\theta}(y) - \hat{\theta}(y)$ is increasing in y.

$$\begin{aligned}
\bar{\theta}'(y) - \hat{\theta}'(y) &= \int_\Phi \vartheta(\varphi) \frac{\partial}{\partial y}\Big[\bar{\nu}(\varphi|y) - \hat{\nu}(\varphi|y)\Big]\,\mathrm{d}\varphi \\
&= -\int_\Phi \vartheta'(\varphi) \frac{\partial}{\partial y}\Big[\int_{\underline{\varphi}}^{\varphi} \Big(\bar{\nu}(\varphi'|y) - \hat{\nu}(\varphi'|y)\Big)\,\mathrm{d}\varphi'\Big]\,\mathrm{d}\varphi \\
&= -\int_\Phi \vartheta'(\varphi)\Big[\bar{G}_y(\varphi|y) - \hat{G}_y(\varphi|y)\Big]\,\mathrm{d}\varphi \geq 0.
\end{aligned}$$

The last inequality follows from (15), since $\vartheta' \geq 0$ has been assumed. □

Proof of Proposition 1: According to Lemma 3, $\bar{\varphi}^{\bar{f}}(y)$ differs from $\bar{\varphi}^{\hat{f}}(y)$ by a MPS. In addition, if the investment behavior is efficiency-inducing (inefficiency-inducing), $\bar{h}_t(\cdot)$ is a convex (concave) function. Therefore,

$$\int_0^1 \bar{h}_t\big(\bar{\varphi}^{\bar{f}}(y)\big)\,\mathrm{d}y \overset{(\leq)}{\geq} \int_0^1 \bar{h}_t\big(\bar{\varphi}^{\hat{f}}(y)\big)\,\mathrm{d}y$$

holds (see Rothschild/Stiglitz, 1970) and, hence, $H_t^{\bar{f}}$ in (17) is larger (smaller) than $H_t^{\hat{f}}$. □

Proof of Proposition 2: Incomes under the two information systems are given by

$$I_t^{\bar{f}}(y) = w_t \bar{\varphi}^{\bar{f}}(y) e_t\big(\bar{\varphi}^{\bar{f}}(y)\big), \quad I_t^{\hat{f}}(y) = w_t \bar{\varphi}^{\hat{f}}(y) e_t\big(\bar{\varphi}^{\hat{f}}(y)\big),$$

where

$$\bar{\varphi}^{\bar{f}}(y) := E^{\bar{f}}[\tilde{\varphi}|y], \quad \bar{\varphi}^{\hat{f}}(y) := E^{\hat{f}}[\tilde{\varphi}|y].$$

According to Lemma 3, $\bar{\varphi}^{\bar{f}}(y)$ and $\bar{\varphi}^{\hat{f}}(y)$ differ by a MPS and $\bar{\varphi}^{\bar{f}}(y) - \bar{\varphi}^{\hat{f}}(y)$ is monotone in y. Below we show that $\bar{h}_t(\bar{\varphi}) = \bar{\varphi} e_t(\bar{\varphi})$ is monotone increasing in $\bar{\varphi}$. Lemma 1 (and Remark 2 following Lemma 1) then implies that the income distribution is more unequal under $\bar{f}$ than under $\hat{f}$.

First observe that $s_t\big(\bar{\varphi}(y)\big)$ and $w_t \bar{h}_t\big(\bar{\varphi}(y)\big) - s_t\big(\bar{\varphi}(y)\big)$ are both co-monotone with $\bar{h}_t\big(\bar{\varphi}(y)\big)$. This observation is immediate from (10) since u_1' is a decreasing function.

Now assume, by contradiction, that as $\bar{\varphi}$ increases $\bar{h}_t(\cdot)$ declines. By co-monotonicity, $w_t\bar{h}_t(\cdot) - s_t(\cdot)$ declines as well. As a consequence, $\bar{\varphi}u_1'\big(w_t\bar{h}_t(\cdot) - s_t(\cdot)\big)$ increases and, according to (11), $e_t(\cdot)$ increases. However, in view of (8), an increase in $e_t(\cdot)$ contradicts our assumption that $\bar{h}_t(\cdot)$ declines as $\bar{\varphi}$ increases. $\square$

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