Trash it or sell it? A strategic analysis of the market introduction of product innovations[*]

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Abstract

In this paper a quantity-setting duopoly is considered where one firm develops a new product which is horizontally differentiated from the existing product. The main question examined is which strategically important effects occur if the decision to develop the innovation and the decision to introduce the new product in the market are separated. In our multi-stage game the firm's launch decision is explicitly taken into account and we find an equilibrium where the competitor of the potential innovator strategically over-invests in process innovation. In this equilibrium the competitor over-invests in order to push the potential innovator to introduce the new product since this reduces the competition for the existing product. It is shown that this effect has positive welfare implications in comparison with the case where the innovator commits ex ante to launching the new product.

1 Introduction

Until a firm introduces a new product in the market, a sequence of decisions have to be made. This multi-stage nature of the R&D process is captured by the now widely used Stage-Gate model (see Cooper 2001), which incorporates the following typical decisions: Screening and Scoping, Building of Business Cases, Development, Testing and Validation, and finally the Product Launch. As empirical evidence suggests, firms typically launch only a small fraction of the innovative products they develop. In a seminal study Mansfield et al. (1977) use data of 16 companies in the chemical, drug, petroleum and electronics industries to estimate the probability of commercialization of R&D projects given technical completion. The average probability in the sample is 65%, where values differ significantly between firms ranging from 12% to more than 90% [p. 24]. More recently, Astebro (2003) and Astebro and Simons (2003) employ data from the Canadian Innovation Centre to show that only 7% of the inventions recorded from independent inventors lead to a successful commercialization. Hence, there is a significant gap between the number of product innovation projects firms undertake and the number of product innovations actually introduced in the market. In order to analyze a firm's decision leading to the introduction of new products, it is therefore important to consider not only the incentives to invest in product innovation projects but also the firms *incentives to launch* a developed product.

Starting with the seminal analysis of Arrow (1962), a vast literature in economics and management has analyzed the incentives of firms to *invest* in innovative activities under different market environments. A large part of this literature has focused either on process or on product innovations, and only recently authors have considered the interplay between the two types of innovative activities and the resulting incentive effects. Athey and Schmutzler (1995) show in a monopoly setting that these two types of innovative activities are complementary and that this induces also complementarities with respect to investments increasing product and process flexibility. Using a duopoly model Lin and Saggi (2002) confirm the existence of complementarities between product and process innovation. They also examine the effect of the type of market competition on innovation incentives and demonstrate that firms are inclined to do more product R&D under price competition whereas firms invest more in process R&D under quantitiy competition. The effect of intensity of competition on incentives for product and process innovation has also been studied in Bonanno and Haworth (1998), Boone (2000) and Symeonidis (2003). Yin and Zuscovitch (1998) and Rosenkranz (2003) analyze the effect of firm and market size on the balance between product and process innovation, where the latter also reconsiders the analysis of R&D cartels (see also e.g. Kamien et al. (1992)) under the additional aspect that firms invest in product *and* process innovation. Unfortunately, none of these studies take the multi-phase structure of the decision making process leading to the actual launch of the new product into account.

Other streams of literature do consider the multi-stage structure of R&D projects, however here the interplay between the incentives to invest in process and product innovation in the presence of strategic interaction is neglected. First, in several recent papers the value of R&D projects has been analyzed using a real options approach. The focus of this work is either on the value of flexibility in the R&D process under uncertainty (e.g. Huchzermeier and Loch (2001), Jägle (1999), Lint and Pennings (1998)) or on the trade-off between flexibility and commitment under oligopolistic competition (see, in particular, Smit and Trigeorgis (2004)). Second, the work on patent-races and innovation timing games takes into account the dynamic nature of R&D projects and provides insights into the resulting strategic effects, but the focus is on the adoption of new technology, technological competition, and the optimal timing of bringing a new product or process to market (see e.g. Hoppe and Lehmann-Grube (2005), Doraszelski (2003), Judd (2003)).

The goal of this paper is different from the work above. We try to initiate a rigorous analysis of the implications of the multi-stage nature of R&D projects in a market environment where firms are acting strategically and are active in product *and* process innovation. A step in this direction has been recently taken by Lukach et al. (2006) who study the role of sequential investment decisions in process innovation in a market setting with potential competition. Our analysis differs from this paper in two important aspects. First, our emphasis is on the interplay of process *and product* innovation. Second, we consider a scenario with actual rather than potential competition.

We study a duopoly market with Cournot competition. Ex-ante the two producers are able to offer identical products at identical costs. Firm 1 is in the process of developing a new product, which is horizontally differentiated from the existing product. At this stage it is uncertain, however, how consumers will perceive the degree of differentiation between the new product and the old product. Both firms can invest in process innovation which reduces production costs for the existing product. After finishing the product innovation project, firm 1 obtains information about the perceived degree of differentiation and, based on this, the firm decides whether to enter the competition with the existing product or to introduce the new product on the market. If firm 1 decides to launch the new product, competition is less strong due to the differentiation effect. However, it has to take into account that e.g. due to lost learning curve effects the average production costs of the new product are higher than for the existing product and that firm 1's investments in process innovation are lost.[1]

We find that three different types of equilibria with quite distinct interpretations can occur in this game. Which type of equilibrium exists depends crucially on the higher production costs faced by firm 1 if it decides to launch the new product. If the difference in production costs between the existing and the new product is small, then firm 1 will introduce the differentiated product. If this cost difference is high, firm 1 will introduce the existing product. More interestingly, we discover that there is an intermediate range for the cost difference, where firm 2 strategically over-invests in process innovation in order to push its competitor to launch the developed product. As it turns out, to obtain this insight it is crucial to consider the multi-stage structure of firm 1's product innovation project. Once firm 1 has started the project, firm 2 has an incentive to influence the continuation decision of its competitor (i.e. the launch decision for the new product). In our framework, where the new product developed by firm 1 is horizontally differentiated, firm 2 has incentives to push firm 1 to introduce the developed innovation, thereby leaving the market segment for the existing product to firm 2 alone. By choosing a high level of process innovation, firm 2 reduces its own production costs to a level where the market for the existing product becomes unattractive for firm 1. Hence, in this type of equilibrium firm 2 is indeed able to

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[1] Clearly, the assumption that process innovations are completely product specific is very strong, but the main effects do not change if we allow for a positive but diminished cost reducing effect of stage one investments on production costs of the new product. Also, one could allow firm 1 to make process innovation investments specific to the new product in stage one. For reasons of tractability we have not done so, see Section 5 for a discussion of this issue.

successfully influence the outcome of the subsequent launch decision of firm 1. However, in order to reach this goal firm 2 has to overinvest, i.e. it has to choose an investment level which is above the level that would be optimal ex-post given that firm 1 launches the new product. A welfare comparison between the different types of equilibria shows that such limit R&D behavior of firm 2 reduces the profits of firm 1 but actually is welfare-improving.

The paper is organized as follows. We introduce our model in section 2 and characterize the subgame perfect equilibria of the game in section 3. The analytical findings are illustrated with a numerical example in section 4 where we also compare the different types of equilibria with respect to firm profits and welfare. We discuss the robustness of our findings with respect to changes in the model structure in section 5 and concluding remarks are given in section 6. The formal equilibrium analysis and all proofs are given in Appendix A.

2 The Model

We consider a duopoly with quantity competition. There are three decision stages which we call the innovation stage, the product selection stage, and the production stage[2].

Innovation Stage: It is assumed that both firms have the ability to produce an identical product variant which we refer to as the 'old product'. Additionally, firm 1 is in the process of developing a different product variant ('new product'), where the investments in product development are sunk. The outcome of the development process, i.e. the degree of perceived differentiation, is uncertain. For reasons of simplicity it is assumed that only two outcomes are possible: high differentiation with probability p or low differentiation with probability $1-p$. While the new product development project of firm 1 is still going on, both firms decide simultaneously how much to invest in process innovation for the existing product. Hence, the process innovation decisions are made before the outcome of the product innovation process is known and before firm 1 has decided whether to introduce the new product or the old product. Without any process innovation, future marginal production costs of the old product would be at a level $c_o > 0$. Reducing these costs by x requires an investment of $k(x) = \alpha x + \beta x^2, \alpha, \beta > 0$. Both the initial cost level and the efficiency of

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[2] We adopt the usual sequence product innovation - process innovation - market competition from other game-theoretic studies focusing on sequential decisions (see e.g. Lin and Saggi (2002)), but add the launch decision as an additional decision between the process innovation stage and the market competition stage. We abstain from adding another process innovation stage after the launch decision, since such a more complex game structure would be hardly tractable and distract attention form the main point of the paper.

process innovation investments are assumed to be identical for both firms. We denote the cost reductions due to process innovation investments of firm i by x_i.

Product Selection Stage: Firms observe the decisions their respective competitor has made in the innovation phase. Furthermore, between the innovation stage and the product selection stage the outcome of firm 1's product innovation project is revealed to both parties,[3] i.e. both firms observe the realized degree of differentiation. Firm 1 then has the choice either to continue producing the old (homogeneous) product or to introduce the new (differentiated) product in the market. If firm 1 decides to introduce the new product, it stops producing the old product. This assumption can be motivated by the existence of limited capacities of the producer or by additional fixed costs arising if the number of products offered on the market is increased. Ruling out the case where firm 1 simultaneously offers both products helps to keep the analysis as simple as possible. We discuss in Section 5 in how far our results would change if firm 1 also had the option to offer both products simultaneously. Marginal costs of production for the new product are c_n where it is assumed that $c_o < c_n < 2c_o$.[4] Firm 1's launch decision of the developed product is represented by the binary variable P_1, where $P_1 = N$ means that the new product is introduced whereas $P_1 = O$ if firm 1 introduces the old product.

Production Stage: Both firms know the competitor's cost level and the degree of product differentiation. All investments in process and product innovation are sunk at this point. The firms then simultaneously choose their profit maximizing output quantities.

The demand for the firm's product depends on the degree of differentiation. The inverse demand function is assumed to have the linear form

$$p_i = a - q_i - \gamma q_j, \;\; i, j \in \{1, 2\}, i \neq j. \tag{1}$$

The variables q_1, q_2 denote the quantities produced by the two firms and p_1, p_2 the corresponding prices. The parameter γ reflects the degree of product differentiation.[5] In

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[3] Actually, it would be sufficient to assume that firm 2 learns about the value of the degree of differentiation (γ) only in cases where firm 1 has decided to introduce the new product in the market.

[4] Note that firm 1 can realize the benefits from process innovation in the innovation phase only if it decides to introduce the old product.

[5] This demand structure can be derived from the utility optimization problem of a representative consumer with utility function $U(q_1, q_2; \gamma) = a(q_1 + q_2) - (q_1^2 + 2\gamma q_1 q_2 + q_2^2)/2 + m$ choosing quantities q_1 of good 1, q_2 of good 2 and m of a numeraire good (see Spence (1976), Dixit and Stiglitz (1977)).

particular, γ takes the value γ_h if a product with high degree of differentiation or γ_l if a product with low degree of differentiation is offered, where $0 < \gamma_h < \gamma_l < 1$. If firm 1 offers the old product, we have $\gamma = 1$, i.e products are perfect substitutes.

The profit in the production phase is then

$$\pi_i(\gamma, q_i, q_j) = ([a - q_i - \gamma q_j] - c_i(x_i))\, q_i, \tag{2}$$

where $a > c_i$. For the marginal cost functions we have

$$\begin{aligned} c_1(x_1) &= \begin{cases} c_o - x_1 & \text{for } P_1 = O \\ c_n & \text{for } P_1 = N \end{cases} \\ c_2(x_2) &= c_o - x_2 \end{aligned} \tag{3}$$

We will characterize the equilibria of this game and discuss the implications of the players' strategic behavior on investments in process innovation and on the likelihood that the new product is actually launched. We will show that different types of equilibria with quite distinct properties may be observed in our setup depending on the values of market and cost parameters.

As discussed in the introduction, our main motivation for considering this kind of a model is to study strategic implications of the multi-stage structure of innovation projects. However, as far as relevant strategic effects go, the crucial assumption is that when both firms decide on their process innovation efforts for the old product they know that firm 1 will have the option to introduce a new product in the following stage. Accordingly, the model also captures strategic aspects of interactions where the innovation project is assumed to have only one stage but firm 1 has publicly announced that it will develop a new product.

In what follows we make several assumptions in order to exclude trivial cases and parameter constellations which induce counter-intuitive effects of a new product introduction on the profits of the firms:

(A1) Throughout the analysis it is assumed that if the outcome of the product innovation process is favorable, it is optimal for firm 1 to introduce the product regardless of the previous choices of process innovation investments.

(A2) Regardless of the realization of γ and the levels of process innovation investments firm 2's profits are larger if firm 1 introduces the new product than if it continues to offer the old product.

Although we deal with the potential introduction of a horizontally differentiated product without quality advantages, in principle this introduction might still have negative effects for the competitor of the innovator. This is in particular true if the competitor has large cost advantages for the existing product, the new product is highly differentiated and the market is relatively small. Here we restrict attention to the case where the softening of competition in the market for the old product which results from the introduction of the horizontally differentiated new product leads to increased profits for the competitor.

Direct calculations show that (A1) and (A2) always hold for $a > 8c_o$ and γ_h sufficiently small.

(A3) Optimal process investments of firm 1 are positive for sufficiently small expected x_2 if the firm stays in the old market for $\gamma = \gamma_l$. Optimal process investments of firm 2 are positive for sufficiently small expected x_1[6].

Note that these assumptions also guarantee the positivity of quantities and profits of both firms. In addition we will assume $\beta > 1$ to ensure concavity of the two firms payoff functions with respect to process innovation investments.

3 Equilibrium Analysis

We consider subgame-perfect equilibria of the game and hence analyze the game by backward induction. The difference $c_n - c_o$ is interpreted as the loss of specific production know-how which firm 1 encounters when it decides to introduce the new product. This number can be seen as a measure of the technological differences between the old and the new product. In our analysis we will characterize how the value of this variable influences the emergence of equilibria of the game.

The details of the backward induction analysis of the game are rather technical and given in Appendix A. In this Appendix we calculate the equilibrium quantities and resulting profits at the production stage on the one hand, for the case where both firms offer the old product and, on the other hand, for the case where products are differentiated with parameter $\gamma \in \{\gamma_l, \gamma_h\}$. Furthermore, we show that there exists a threshold $x_2^{post}(x_1)$ such that it is optimal for firm 1 to launch the new product at the product selection stage if and only if $x_2 \geq x_2^{post}$. This threshold is a linear increasing function of x_1. This is quite

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[6] A sufficient condition for this to hold is

$$0 \leq \alpha < \min\left[\frac{27}{64}, \frac{4(1-p)}{9}(a - c_o)\right].$$

intuitive since the more firm 1 has invested in process innovation for the old product the more difficult it is to make the firm launch the new product.

Here we will concentrate on the first stage and discuss the best response correspondences of the two firms at the innovation stage, where simultaneously $x_1, x_2 \in [0, c_o]$ are chosen. Since we consider subgame perfect equilibria, it is assumed that both firms follow the described equilibrium strategies at the subsequent product selection and production stage. After our description of the best response correspondences, we will characterize the different types of equilibria that are occurring in our model.

3.1 Best Response Correspondences

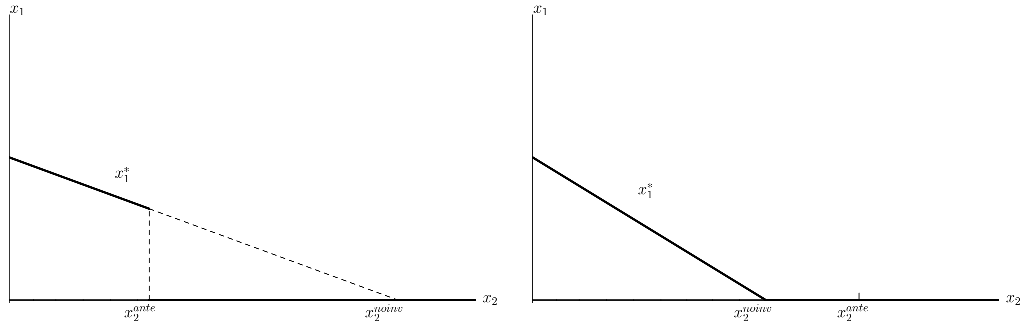
Let us first consider the choice of process innovation investment x_1 of firm 1 at the process innovation stage. Note that the intensity of the incentives for process innovation for firm 1 depends on whether it expects to launch the new product at the product selection stage or to offer the old product. In the former case process innovation effects will be lost for firm 1 and therefore there are no investment incentives. In the latter case firm 1 will in general have positive incentives to invest in process innovation, where incentives decrease if the process innovation efforts of firm 2 go up (process innovation efforts are strategic substitutes). The more firm 1 invests in process innovation for the old product the more attractive it becomes for the firm to decide to introduce the old product at the product selection stage.

We define $x_1^*(x_2)$ as the best response of firm 1 under the assumption that it does not launch the new product and x_2^{noinv} as the minimum level of x_2 needed to make it optimal for firm 1 to invest zero for process innovation even if it plans to offer the old product for $\gamma = \gamma_l$. Furthermore, denote by x_2^{ante} the minimum level of x_2 needed to make it **ex-ante** (i.e. before the process innovation decision) optimal for firm 1 to decide to launch the new product also if $\gamma = \gamma_l$. Existence and uniqueness and monotonicity properties of these thresholds are established in Appendix A. The best response correspondence of firm 1 at the innovation stage can be characterized as follows:

Proposition 1 *The best reply correspondence of firm 1 at the innovation stage has the form*

$$BR_1(x_2) = \begin{cases} x_1^*(x_2) & x_2 < \min[x_2^{noinv}, x_2^{ante}] \\ \{0, x_1^*(x_2)\} & x_2 = \min[x_2^{noinv}, x_2^{ante}] \\ 0 & x_2 > \min[x_2^{noinv}, x_2^{ante}] \end{cases}$$

for $x_2 \in [0, c_o]$.



(a) (b)

Figure 1: Best response correspondence BR_1 for player 1: (a) $x_2^{ante} < x_2^{noinv}$ and (b) $x_2^{ante} \geq x_2^{noinv}$.

Note that BR_1 is continuous if $x_2^{ante} \geq x_2^{noinv}$ but has one downward jump if $x_2^{ante} < x_2^{noinv}$. We provide an illustration of the typical form of BR_1 for the cases $x_2^{ante} < x_2^{noinv}$ and $x_2^{ante} \geq x_2^{noinv}$ in Figure 1. Note that the discrete nature of the choice of firm 1 at the product selection stage is responsible for the jump in the best response correspondence depicted in figure 1(a). Along the part of the best response correspondence that is to the left of x_2^{ante}, firm 1 will decide to introduce the old product at the product selection stage. As x_2 becomes larger than x_2^{ante} the maximal profit to be earned by firm 1 if it introduces the old product is smaller than the profit it will make by spending zero on process innovation and then launching the new product at the product selection stage. Accordingly, for such values of x_2 the optimal action of firm 1 becomes $x_1 = 0$ combined with $P_1 = N$ at the product selection stage.

Let us now turn to the best response correspondence of firm 2 at the innovation stage. Also for firm 2 there exist important relations between firm 2's investment in process innovation and the expected decision of firm 1 at the product selection stage. Two main effects have to be taken into account. On the one hand, the incentives for firm 2 to reduce its production costs through process innovation depend on whether firm 1 will introduce the old product or launch the new differentiated product. We define $x_2^{*N}(x_1)$ as the profit maximizing process innovation effort of firm 2 under the assumption that firm 1 invests x_1 and then launches the new product also in the case of low differentiation, i.e. $\gamma = \gamma_l$. Note that x_2^{*N} is constant in x_1 because if firm 1 launches the new product in any case its process innovation investments have no influence on its production costs. Analogously $x_2^{*O}(x_1)$ is the best response of firm 2 under the assumption that firm 1 introduces the old product. Due to the fact that x_1 and x_2 are strategic substitutes, the function $x_2^{*O}(x_1)$ is decreasing in x_1. One the other hand, by making high process innovation investments firm 2 makes it less attractive for firm 1 to introduce the old product and thereby increases incentives to launch the new product. Formally this is realized by the existence of a threshold $x_2^{post}(x_1)$ which gives the minimal process innovation investment by firm 2 that, given firm 1 has invested x_1, makes it optimal for firm 1 to launch the new product at the product selection stage. The optimal response correspondence of firm 2 results from a mix of the two effects described above.

Proposition 2 *There exist thresholds* $0 \leq x_1^{T1} \leq x_1^{T2} \leq c_o$ *such that the best reply correspondence of firm 2 at the innovation stage is given by*

$$BR_2(x_1) = \begin{cases} x_2^{*N} & x_1 < x_1^{T1} \\ x_2^{post}(x_1) & x_1^{T1} \leq x_1 < x_1^{T2} \\ \{x_2^{post}(x_1), x_2^{*O}(x_1)\} & x_1 = x_1^{T2} \\ x_2^{*O}(x_1) & x_1 > x_1^{T2} \end{cases}$$

for $x_1 \in [0, c_o]$.

The rather lengthy expressions for the two investment levels x_2^{*N} and $x_2^{*O}(x_1)$ are given in Appendix A, where we also provide formal definitions and a characterization of the thresholds x_1^{T1} and x_1^{T2}. In Figure 2 we illustrate the typical form of BR_2. Note that when the best response of firm 2 is given by x_2^{post} this will induce firm 1 to launch the new product. Therefore, firm 2 invests more than would be ex-post optimal given firm 1's

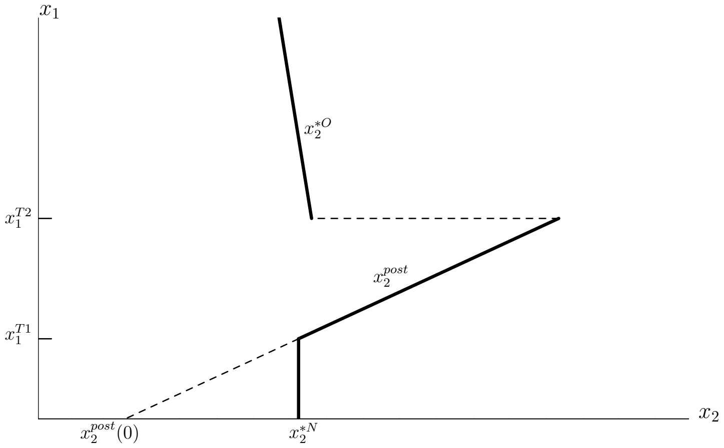


Figure 2: Best response correspondence for player 2.

decision to launch the new product at the product selection stage ($x_2^{post} > x_2^{*N}$). There is a trade-off between investing 'too much' in process innovation and the reduced profit opportunities that would result if firm 1 introduces the old product. The best reply of firm 2 is not everywhere monotonic decreasing, as in the standard Cournot duopoly models with process innovation, but there is an increasing branch which is caused by the additional strategic incentives to induce the competitor to launch the new product. Put differently, the standard strategic substitutes property for process innovation models partly disappears if the launch decision is explicitly taken into account.

To provide additional insights into the structure of firm 2's best response BR_2 we give in Figure 7 in Appendix B a graphical illustration of the decision problem of firm 2 for different values of x_1. Furthermore, in Lemma 4 (Appendix A) we show that if the introduction of the new product does not generate increases in variable production costs ($c_n = c_o$), then the threshold x_1^{T1} is strictly positive. Accordingly – at least for small investments x_1 – firm 1 launches the new product anyway and there is no need for firm 2 to invest more in process innovation than would be optimal ex post. The interval $[0, x_1^{T1}]$ of x_1-values where this holds true shrinks as the cost differential $c_n - c_o$ increases. Numerical evidence suggests that also x_1^{T2} decreases with c_n, however obtaining analytical conditions which guarantee this property seems to be very involved and we abstain from presenting any such conditions.

3.2 Equilibria

We are now in a position to give a characterization of the different types of subgame-perfect-equilibria which might occur in the model for different scenarios. In particular, we will discuss the evolution of equilibria as the unit production costs for the new product c_n increases, starting from $c_n = c_o$. Recall that $c_n - c_o$ can be interpreted as the loss in production know-how if firm 1 introduces the new product. We distinguish three different types of equilibria:[7]

- **Determined Innovator Equilibrium (D.I.E.)**: Firm 1 does not invest in process innovation and introduces the new product regardless of the degree of product differentiation which results from product innovation. Firm 2 chooses the level of process innovation which is optimal given that firm 1 launches the new product in any case.

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[7] We will carry out the analysis under the assumption that β is sufficiently large. This assumption is needed for the proof of a technical lemma (Lemma 5 in Appendix A) which we use in the further analysis. However, numerical evidence suggests that the needed properties also hold for values of β only slightly larger than 1.

Put formally[8], $x_1^e = 0, x_2^e = x_2^{*N}$ and firm 1 chooses $P_1^e = N$ after observing $\gamma = \gamma_l$ at the product selection stage.

- **Pushed Innovator Equilibrium (P.I.E.)**: Firm 1 does not invest in process innovation and introduces the new product regardless of the degree of differentiation which results from product innovation. Firm 2's investment in process innovation is just sufficiently high to make firm 1 indifferent between producing the old product or launching the new product if $\gamma = \gamma_l$. The level of investment in process innovation of firm 2 is above the level which would be optimal ex post given that firm 1 launches the new product. Put formally, $x_1^e = 0, x_2^e = x_2^{post}(0)$ and firm 1 chooses $P_1^e = N$ after observing $\gamma = \gamma_l$ at the product selection stage.

- **Cautious Innovator Equilibrium (C.I.E.)**: Firm 1 introduces the new product only if $\gamma = \gamma_h$ and invests the corresponding optimal amount for process innovation. Firm 2 chooses the optimal level of process innovation given that firm 1 produces the old product for $\gamma = \gamma_l$. Put formally, $x_1^e = \hat{x}_1, x_2^e = \hat{x}_2$ and $P_1^e = O$ at the product selection stage for $\gamma = \gamma_l$, where $(\hat{x}_1, \hat{x}_2)$ is the unique solution[9] to

$$\begin{aligned} x_2 &= x_2^{*O}(x_1) \\ x_1 &= x_1^*(x_2) \end{aligned} \tag{4}$$

  in $[0, c_o]^2$.

In Figure 3 we depict the typical form of the individual best replies leading to each of the three types of equilibria. We also present a scenario where no equilibrium in pure strategies exists.

Our analysis starts with two results dealing with the first two types of equilibria (Propositions 3 and 4). Conditions for the third type of equilibrium are given in Proposition 5. We will focus on situations where all three types of equilibria can exist. A necessary condition for the existence of a pushed innovator equilibrium is that $x_2^{noinv} \leq c_o$, and since this type of equilibrium is the most interesting from a strategic perspective, we derive the following propositions under the assumption that this inequality holds.

We know that x_1^{T1} is positive for $c_n = c_o$ and that there exists a unique $c_n^T > c_o$ such that $x_1^{T1} = 0$ for $c_n = c_n^T$ (see Lemma 4 (a) and (c) in Appendix A). Note that for $c_n = c_n^T$

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[8] Superscripts 'e' denote equilibrium values.

[9] It can be easily checked that $x_2^{*O}(x_1^*(0)) > 0$. Taking into account the shape of x_1^* and x_2^{*O} this implies that there exists a unique solution to the considered pair of equations.

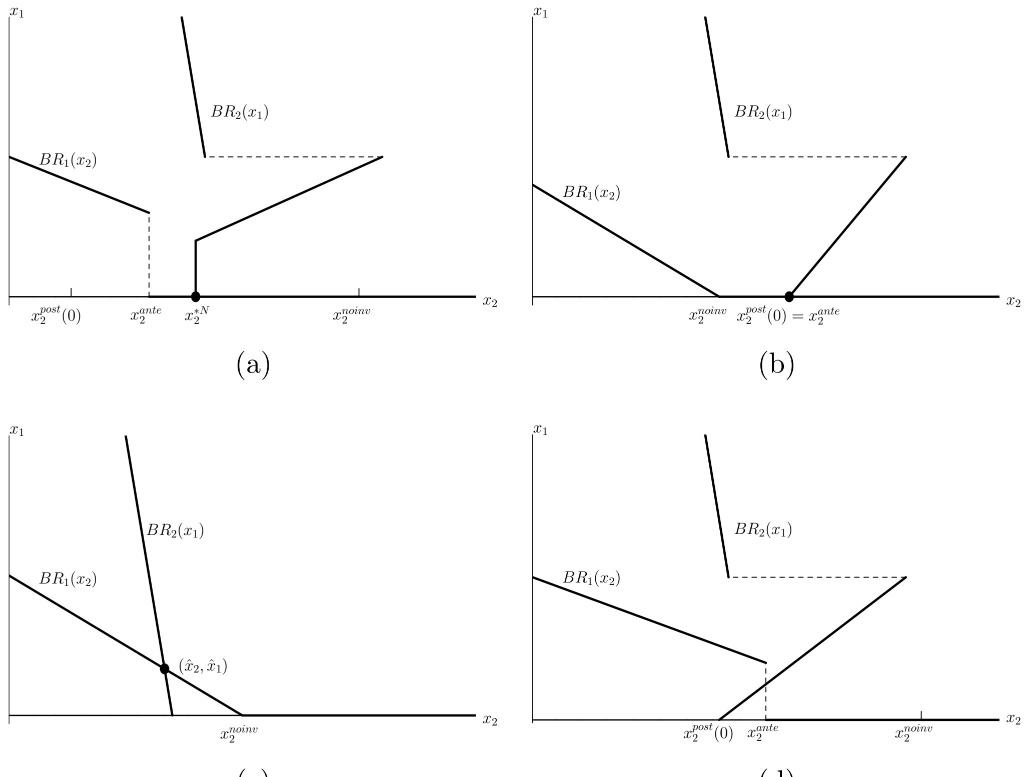


Figure 3: Typical forms of the best replies inducing (a) a determined innovator equilibrium, (b) a pushed innovator equilibrium, (c) a cautious innovator equilibrium, (d) no equilibrium in pure strategies.

we must also have $x_2^{post}(0) = x_2^{*N}$. Intuitively, for $c_n = c_n^T$ the level of process innovation which is optimal for firm 2, given that firm 1 always introduces the new product, is just sufficient to make firm 1 indifferent between introducing the new product and offering the old product for $\gamma = \gamma_l$ and $x_1 = 0$. For values of c_n higher than this threshold, firm 2 has to invest extra amounts in order to induce firm 1 to launch the new product if the degree of differentiation is low ($\gamma = \gamma_l$). We distinguish between two scenarios: (i) given that $c_n = c_n^T$ and $x_2 = x_2^{*N}$ firm 1 has an incentive to choose a positive x_1 if it offers the old product for $\gamma = \gamma_l$; (ii) $x_1 = 0$ is optimal for $c_n = c_n^T$ and $x_2 = x_2^{*N}$ even if firm 1 offers the old product for $\gamma = \gamma_l$. In the latter case we say that process innovation incentives for firm 1 are weak.

Definition 1 *Process innovation incentives for firm 1 are called weak if* $x_2^{noinv} \leq x_2^{post}(0)$ *for* $c_n = c_n^T$. *If* $x_2^{noinv} > x_2^{post}(0)$ *for* $c_n = c_n^T$ *we say that process innovation incentives for firm 1 are strong.*

Intuitively, weak (strong) process innovation incentives correspond to scenarios where the probability p for a good outcome of firm 1's product innovation project is high (low). This follows from the fact that for $\gamma = \gamma_h$ firm 1 always introduces the new product and therefore looses the positive effect of its process innovation investments.

A subgame perfect equilibrium in our game is a profile of the form

$$\sigma = ((x_1^e, P_1^e(x_1, x_2), q_1^e(\gamma, c_1, c_2)), (x_2^e, q_2^e(\gamma, c_1, c_2))).$$

We will characterize the equilibria by the investments of the two firms in process innovation (x_1^e, x_2^e) and the resulting action of firm 1 at the product selection stage: $P_1^e(x_1^e, x_2^e)$.

Proposition 3 *If firm 1 has weak process innovation incentives then the conditions for the existence of an equilibrium where the new product is introduced can be characterized as follows.*

(a) For $c_n \in [c_o, c_n^T]$ there exists a subgame-perfect determined innovator equilibrium.

(b) Let $C = \{c_n \in (c_n^T, 2c_o] | x_1^{T2} > 0\}$. For $c_n \in C$ there exists a subgame-perfect pushed innovator equilibrium.

(c) For $c_n \in (c_n^T, 2c_o] \backslash C$ there exists no (pure-strategy) subgame-perfect equilibrium where firm 1 chooses $P_1^e = N$ after observing $\gamma = \gamma_l$ at the product selection stage.

The results obtained in this proposition are quite intuitive. If c_n is close to c_o, then the resulting loss of specific production know-how if firm 1 launches the new product is small. In this case, there is an equilibrium where firm 1 will introduce the new product even if firm 2 chooses the level of process innovation which is optimal ex post. On the other hand, for large values of c_n, there exists no equilibrium where firm 1 introduces the new product also for $\gamma = \gamma_l$. Finally, the most interesting situation occurs for intermediate ranges of c_n. In this case there is an equilibrium where firm 1 always introduces the new product, but firm 2's investments in process innovation are above the level which would be optimal given that firm 1 launches the new product. Obviously, the incentive for firm 2 to overinvest stems from the insight that it can successfully push the competitor out of the own market segment. The rationale of this behavior is similar to the well known limit-pricing results (see e.g. Spence (1977)). In this sense firm 2's behavior can be seen as 'limit R&D expenditures'.

A characterization of the equilibria occurring in the case of strong process innovation incentives is given in Proposition 4.

Proposition 4 *If firm 1 has strong process innovation incentives then there exists a unique* $\underline{c}_n \in [c_o, c_n^T]$ *such that* $x_2^{ante} = x_2^{*N}$ *and a unique* $\bar{c}_n > c_n^T$ *such that* $x_2^{ante} = x_2^{noinv}$ *for* $c_n = \bar{c}_n$*. We have:*

(a) For $c_n \in [c_o, \underline{c}_n]$ *there exists a subgame-perfect determined innovator equilibrium.*

(b) Let $D = \{c_n \in (\bar{c}_n, 2c_o] | x_1^{T2} > 0\}$*. For* $c_n \in D$ *there exists a subgame-perfect pushed innovator equilibrium.*

(c) For $c_n \in (\underline{c}_n, 2c_o] \setminus D$ *there exists no (pure-strategy) subgame-perfect equilibrium where firm 1 chooses* $P_1^e = N$ *after observing* $\gamma = \gamma_l$ *at the product selection stage.*

Note that combination of parts (b) and (c) of this proposition shows that for $c_n \in (\underline{c}_n, \bar{c}_n)$ there exists no pure strategy equilibrium where the new product is introduced although for larger values of c_n such an equilibrium does exist. The reason is that in such scenarios the game has a structure similar to the well known 'Chicken' game. Under the assumption that firm 1 launches the new product even if $\gamma = \gamma_l$, the optimal investment of firm 2 is so small that for such a value of x_2 firm 1 prefers to offer the old product for $\gamma = \gamma_l$. However, given that firm 1 decides to introduce the old product, firm 2 should invest a higher amount and such a high x_2 would make it optimal for firm 1 to trash the old product and introduce the new one. Hence, there is no pure strategy equilibrium. There is a possibility that mixed equilibria exist where the strategies of both players have a continuum as support, but we do not investigate these types of equilibria in detail. Such a scenario cannot occur if process innovation incentives are weak, but this is the only qualitative difference between the characteristics of equilibria under weak and strong process innovation incentives.

In the next section we will illustrate the evolution of equilibrium constellations when c_n is increased, i.e. when the situation is getting worse in terms of loss of product specific know-how. We will study the change in equilibria for the case of weak process innovation incentives (see Proposition 3, $x_2^{noinv} \leq x_2^{post}(0)$) and for the case of strong process innovation incentives (see Proposition 4, $x_2^{noinv} > x_2^{post}(0)$).

Finally, we turn to the third type of equilibrium, where firm 1 only launches the new product if the degree of differentiation is high. As discussed above, at such an equilibrium the levels of investment have to satisfy (4). Therefore, a subgame-perfect cautious innovator equilibrium exists if at the levels of investment $(\hat{x}_1, \hat{x}_2)$ it is on the one hand (ex-ante) optimal for firm 1 to stick with the old product and on the other hand firm 2 cannot gain by pushing firm 1 to introduce the new product. Taking that into account the proof of our next proposition follows directly from the characterizations of the two best replies BR_1 and BR_2 given above.

Proposition 5 *(a)* $\hat{x}_1 = 0$*: A subgame-perfect cautious innovator equilibrium exists if and only if* $x_1^{T2} \leq 0$*.*

(b) $\hat{x}_1 > 0$*: A subgame-perfect cautious innovator equilibrium exists if and only if* $x_1^{T2} \leq \hat{x}_1$ *and* $x_2^{ante} \geq \hat{x}_2$*.*

It should further be noted that no other types of equilibria are possible. In particular, it is not possible to have solutions of $x_1 = x_1^*(x_2), x_2 = x_2^{*N}$ or $x_1 = x_1^*(x_2), x_2 = x_2^{post}(x_1)$ with $x_1 > 0$. This is easy to see. We can only have $x_1 > 0$ in equilibrium if firm 1 offers the old product for $\gamma = \gamma_l$. But in this case the optimal response of firm 2 should be $x_2 = x_2^{*O}(x_1)$ rather than x_2^{*N} or $x_2^{post}(x_1)$. Hence, the three propositions above provide a complete characterization of the possible subgame-perfect equilibria of the game.

The following corollary, which gives a simple sufficient condition for the existence of a cautious innovator equilibrium, follows directly from Proposition 5.

Corollary 1 *If* $x_1^{T2} \leq 0$ *then there exists a cautious innovator equilibrium.*

In particular, there is always a cautious innovator equilibrium if case (c) of Proposition 3 applies, and therefore we obtain

Corollary 2 *If firm 1 has weak process innovation incentives there exists for each admissible value of* c_n *at least one pure strategy subgame-perfect equilibrium.*

The following discussion of a numerical example will further show that co-existence of different types of equilibria is possible.

4 Comparison of Equilibria Types

Having characterized the potential equilibrium constellations of the game, several questions arise. How does the probability p for a successful product innovation influence the equilibrium constellation? How does it determine whether the scenarios described in Proposition 3 or 4 arise? How do the different types of equilibria compare with respect to profits and welfare? In particular, what is the welfare effect of the strategic 'over-investment' in process innovation by firm 2 in a Pushed-Innovator-Equilibrium? Due to the complexity of the expressions involved in the characterization of the equilibria of the game, it is impossible to provide a rigorous analytical treatment of these issues. Therefore, in this section we will provide some insights using a numerical example.

4.1 Equilibrium Investment Levels

We choose the following values for the market and cost parameters and the degree of differentiation, $a = 10, \beta = 5, \alpha = 0.77, \gamma_l = 0.75, \gamma_h = 0.2, c_o = 1$, and examine the effects of changes in p and c_n. For $p = 0.8$ we get $c_n^T = 1.5388$ and for $c_n = c_n^T = 1.5388$ we have $x_2^{noinv} < x_2^{post}(0)$. Accordingly, there are *weak* process innovation incentives for firm 1 and Proposition 3 applies. In Figure 4 we depict the equilibrium investment levels of both players for c_n in the range $[1.5, 1.65]$.

The results of Proposition 3 are nicely illustrated. For small values of c_n there is a determined innovator equilibrium (D.I.E.), for an intermediate range there is a pushed innovator equilibrium (P.I.E.) and for large c_n we have a cautious innovator equilibrium (C.I.E.). We know from the discussion in the previous section that if firm 1 has weak process innovation incentives there always exists at least one equilibrium in pure strategies. In our numerical example we have exactly one equilibrium for each value of c_n. Although co-existence of a C.I.E. with a D.I.E. or a P.I.E. can not be ruled out, in all the numerical examples we have considered the equilibrium was unique whenever process innovation incentives of firm 1 were weak. Observe that for the entire range of c_n values firm 1 does not invest in process innovation. On the other hand, the equilibrium investment of firm 2 is always positive. As long as there is a determined innovator equilibrium, the equilibrium investments increase slightly as c_n goes up. The increase becomes much larger as soon as the equilibrium becomes a pushed innovator equilibrium. At the transition from the pushed innovator to the cautious innovator equilibrium there is a significant drop of firm 2's investment in process innovation. In Figure 4 (b) we also show the socially optimal level of x_2. It is interesting to note that the distance between the investment in equilibrium and the socially optimal level is smallest if there is a pushed innovator equilibrium.

If we slightly decrease the probability of a successful product innovation to $p = 0.795$ we have $c_n^T = 1.5387$ and $x_2^{noinv} > x_2^{post}(0)$. Hence, this is a case where firm 1 has strong process innovation incentives and Proposition 4 applies. As can be seen in Figure 5, there is a range of c_n values with no equilibrium in pure strategies and also an interval where the pushed innovator equilibrium and the cautious innovator equilibrium co-exist (indicated by the dashed vertical lines). As before, firm 2's investments are highest in a pushed innovator equilibrium, whereas investments of firm 1 are highest in the cautious innovator equilibrium.

If the P.I.E. and the C.I.E. co-exist, a typical equilibrium selection problem arises and it depends on the type of equilibrium selected whether firm 1 launches the new product even if the degree of differentiation of the new product is low, or not. So, in this case neither the

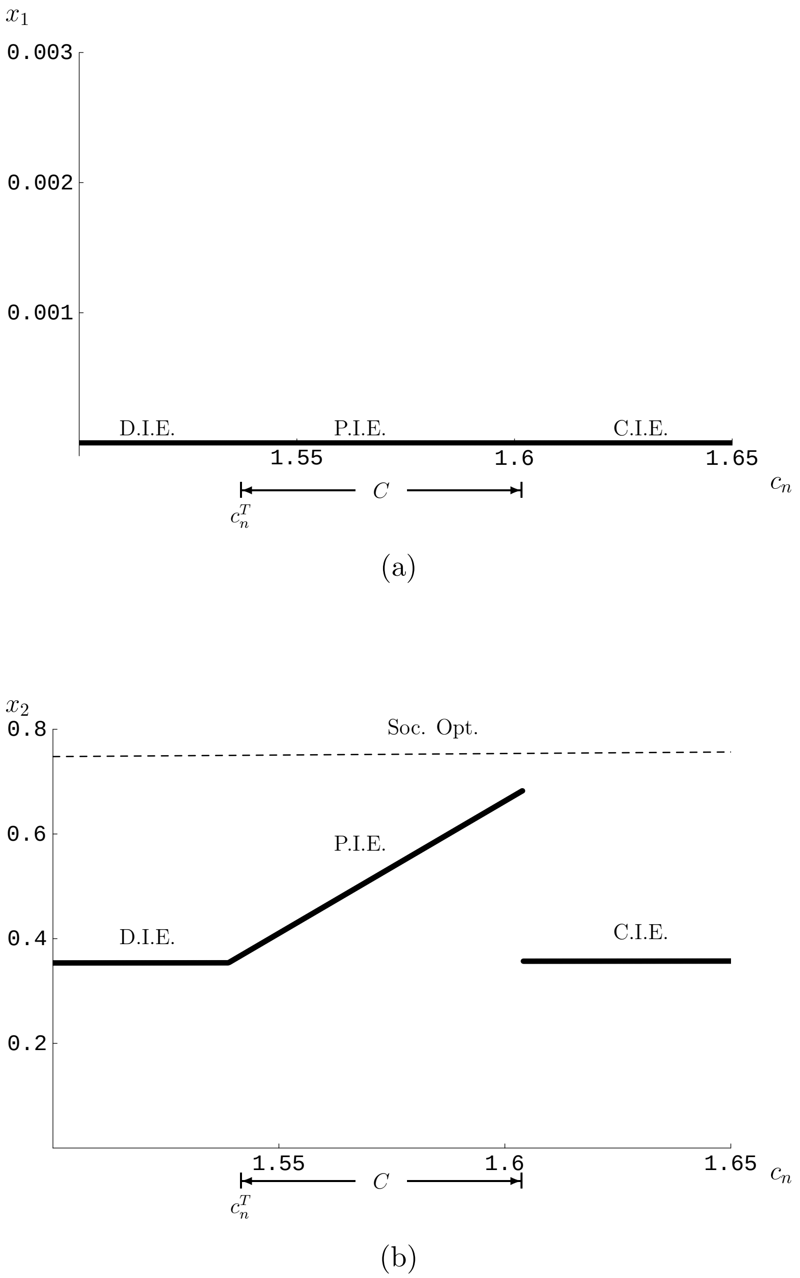
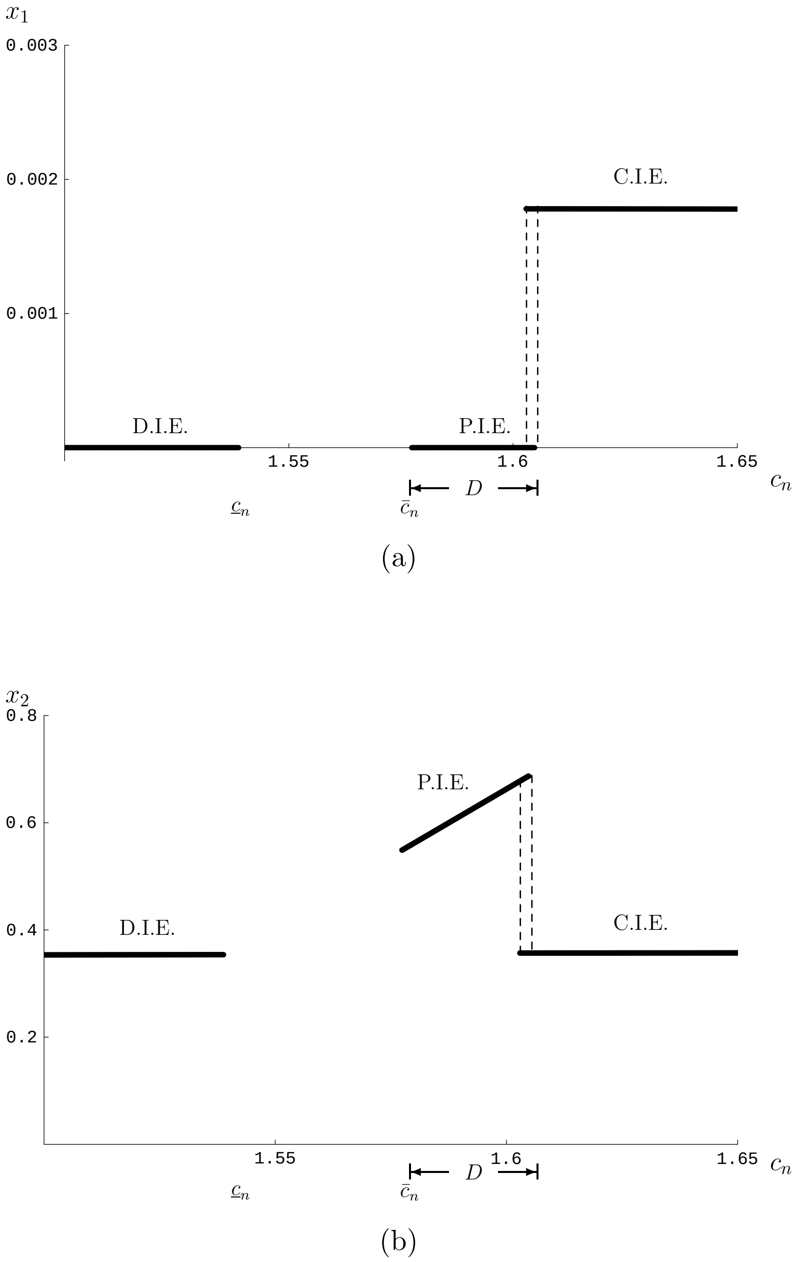


Figure 4: Equilibrium investment levels of both players for $p = 0.8$ and c_n in the range $[1.5, 1.65]$.



(a)

(b)

Figure 5: Equilibrium investment levels of both players for $p = 0.795$ and c_n in the range $[1.5, 1.65]$.

levels of process investments nor the likelihood that the new product is actually introduced in the market can be uniquely predicted based on an equilibrium analysis.

4.2 Firm Profits and Welfare

We now return to the question how the different types of equilibria compare with respect to the expected profits of the two firms – we denote these profits by Π_i^* – and the expected overall welfare denoted by W. For reasons of simplicity we restrict our attention here to the case where firm 1 has weak process innovation incentives. The extension of our insights to the case with strong process innovation incentives is straightforward. Figure 6 shows the profits of both firms and welfare.[10]

Several interesting observations can be made. In the range of c_n where we have a determined innovator equilibrium or a cautious innovator equilibrium, profits of firm 1 and welfare decrease with increasing c_n, whereas profits of firm 2 increase. Since c_n influences only the production costs of firm 1 these effects are as anticipated. In the range where a pushed innovator equilibrium arises, profits of firm 2 however decrease with increasing c_n. Furthermore, the profits of firm 1 decrease more sharply with increasing c_n compared to the scenarios of D.I.E. or C.I.E.. This has the implication that at the transition from P.I.E. to C.I.E. a further increase in c_n leads to an upward jump of the profits of firm 1. Hence, in equilibrium an increase in production costs for the new product has positive effects on the profits of firm 1.

Social welfare *increases* for increasing costs c_n in a subinterval of the range where a pushed innovator equilibrium occurs. This is due to the fact that firm 2 extends its process innovation investments beyond its ex-post optimal level, which is below the socially efficient level, and thereby gets closer to the social optimum. Hence, the strategic implication of explicitly taking firm 1's decision to launch the new product into account, at least to some extent weakens the result that equilibrium process innovation investments are below the socially optimal level, an observation which has been frequently reported in the literature (see e.g. Dasgupta and Stiglitz (1980), D'Aspremont and Jaquemin (1988), Qiu (1997)). Figure 6b also nicely illustrates the rationale of firm 2 in the P.I.E.. By pushing the competitor to a different market segment through higher investments in process innovation it smoothes the gap between its profit if firm 1 launches the new product and the profit it would obtain if firm 1 produces the old product, which is a perfect substitute for firm 2's product.

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[10] Expected welfare is calculated in the standard way, see Appendix C for details.

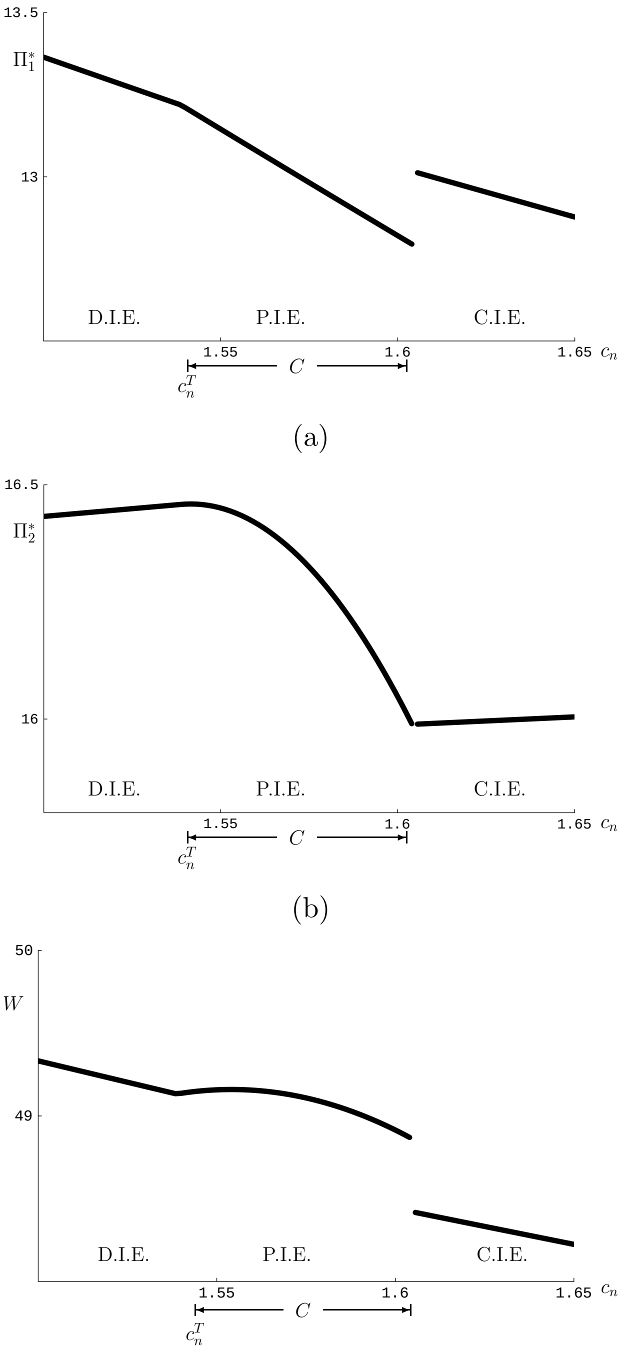

Figure 6: Expected equilibrium profits for both firms (panels (a) and (b)) and expected social welfare (panel (c)) for $p = 0.8$ and c_n in the range $[1.5, 1.65]$.

5 Robustness of Results

The starting point of this paper is the question which kind of strategic incentives are created by the fact that a firm's decision to launch a new product is separated from the decision to develop an innovation. Our analysis shows that an explicit consideration of the launch decision indeed has effects on process innovation incentives, new product introduction and welfare. The main finding in this respect is the existence of a Pushed Innovator Equilibrium, where firm 2 strategically over-invests in order to induce firm 1 to introduce the new product in the market. For reasons of tractability of the model we have made several simplifying assumptions concerning the structure of the interaction between the two firms and one might wonder how crucial these simplifications are for our findings. In order to discuss the issue of robustness, it is important to realize that there are two main effects driving our results. First, the assumption that the new product is only horizontally differentiated implies that the profits of firm 2 go up if firm 1 introduces the new product and leaves the market segment for the old (homogeneous) product. Second, the negative marginal effect of the process innovation efforts of firm 2 on profits of firm 1 are stronger if firm 1 offers the old rather than the new product. The combination of these two effects is responsible for the existence of a Pushed Innovator Equilibrium.

One simplifying assumption of our analysis is that firm 1 trashes the old product whenever the new product is introduced. Without such an assumption firm 1 would have three options at the product selection stage, namely (i) offer the old product, (ii) offer the new and the old product, (iii) replace the old with the new product. Obviously dealing with all possible scenarios that arise in that framework would make the analysis substantially more complex but would not alter the direction of the main forces at work. Calculating the equilibria at the production stage shows that the profit of firm 2 in case (ii) coincides with its profit in case (i). Furthermore, the marginal effect of the process innovation effort of firm 2 on the profits of firm 1 is larger in cases (i) and (ii) compared to case (iii). Therefore, ceteris paribus firm 2 prefers firm 1 to choose option (iii) and can provide incentives to do so by choosing high process innovation incentives. Accordingly, the strategic reasons for the existence of a Pushed Innovator Equilibrium would also be present in such a setup. If we assume that simultaneously offering the new and the old product generates larger fixed costs than producing just one of the two (e.g. due to double advertising, product management, etc.), the option of adding the new to the old product might actually be irrelevant for firm 1. For the parameter setting considered in subsection 4.1 additional fixed costs of $FC = 2$ for adding the second product – which is about 15% of the variable profit of firm 1 – is sufficient to make option (ii) suboptimal for firm 1 regardless of the values of x_1 and x_2.

Another simplifying assumption in our analysis is that firm 1 is not allowed to invest in process innovation for the new product. Again, it is quite obvious that adding such an option would not qualitatively alter the findings obtained here. If firm 1 had that option, it would in equilibrium invest positive amounts since the new product is always introduced when $\gamma = \gamma_h$. The optimal amount will depend on whether the old or the new product is introduced for $\gamma = \gamma_l$. Similar to the process innovation choice for the old product there is no strategic reason to overinvest or underinvest with respect to the ex-post optimal level. Therefore, adding this option would increase the range of c_n values where the product is launched in equilibrium but no additional qualitative insights would be obtained.

Finally, we have assumed quantity competition in this analysis. In several papers differences between quantity competition and price competition with respect to innovation incentives in differentiated product duopolies have been pointed out (e.g. Qiu (1997), Symeonidis (2003)). It turns out however that our findings are robust with respect to changes in the mode of competition. In a version of the model with price competition the qualitative features of the equilibria exactly match our findings with quantity competition.

6 Discussion and Conclusion

In order to put our findings into perspective it is interesting to compare them with a scenario where firm 1 commits ex ante to introduce the new product, which is implicitly assumed in the majority of the literature in this field. Two cases might be considered. First, if firm 1 commits ex ante to introduce the new product regardless of γ, it is easy to realize that the equilibrium values of process innovation investments would be given by $(0, x_2^{*N})$. Second, if firm 1 commits ex ante to introduce the new product only if $\gamma = \gamma_h$ (an assumption which is consistent with studies where it is assumed that product innovation efforts are successful only with a certain probability and only then lead to the introduction of a new product; see e.g. Yin and Zuscovitch (1998)) then there would always be a unique equilibrium with process innovation efforts $(\hat{x}_1, \hat{x}_2)$. Basically, these two cases would correspond to an ex ante commitment to a determined innovator equilibrium (D.I.E.) or a cautious innovator equilibrium (C.I.E.). Considering Figure 6 we can therefore easily see that, at least in the range of c_n where a P.I.E. exists, any such commitment would actually reduce welfare. If firm 1 would ex ante commit to introduction regardless of γ this could on the other hand increase profits of both firms compared to the pushed innovator equilibrium. In order to maximize its own profit for $c_n \in C$ firm 1 would however ex ante want to commit to launch the product only if $\gamma = \gamma_h$.

Early commitments of firms to new product launches like the ones described above are however very hard and not common. A strong commitment of the first type (launch regardless of γ) would to some degree be possible through early pre-announcements of a new product to be introduced. It could be argued that for real world firms typically the costs of launching a new product are so high that ex ante full commitment to launch the new product even if it is only slightly differentiated from the existing product is not optimal. In terms of our model this means that firms operate in a range of $c_n \geq c_n^T$ (respectively $c_n \geq \underline{c}_n$ in the case of strong process innovation incentives for firm 1). An ex ante commitment to launch conditional on the perceived degree of product differentiation would in such a case be optimal but a credible announcement of this type seems infeasible since the value of γ is not verifiable. Given this, at least for the range of values of c_n where a determined or pushed innovator equilibrium exists, unconditional commitment to launch is the best available option for firm 1 and therefore in these cases our results provide an additional rationale for product pre-announcements (see e.g. Lilly and Walters (1997) for a discussion of motives for product pre-announcements). On the other hand, it has been pointed out in the literature that firms in many instances do not abide to their pre-announcements (e.g. Bayus et al. (2001)). Accordingly, the strategic effects discussed in this paper will be of relevance even if pre-announcements have been made. In principle, firm 1 could also try to commit to a launch decision by reducing capacities for process innovation for the old product, but there are again credibility problems. Furthermore, our analysis shows that in a pushed innovator equilibrium firm 2 engages in limit R&D although firm 1 does not make any process innovation investments.

A number of interesting extensions that might add qualitative insights compared to the present model come to mind. Here we deal exclusively with horizontal differentiation. It would be important to examine the implications if the newly developed product of firm one is also vertically differentiated. Furthermore, this paper does not deal with the decision of firm one to start the product development project in the first place and also does not allow to carry out process innovations for the new product prior to its launch. Incorporating these aspects is left for future research.

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Appendix A: Equilibrium Analysis

Following the backward induction approach we start by characterizing the equilibrium strategies for the production stage.

A.1 Production Stage

In the production stage the two firms compete in a Cournot duopoly with differentiated products. Standard analysis gives the following equilibrium quantities and profits:

$$\begin{aligned} q_i^e(\gamma, c_1, c_2) &= \frac{(2-\gamma)a + \gamma c_j - 2c_i}{4-\gamma^2}, \ i,j \in \{1,2\}, \ i \neq j \\ \pi_i^e(\gamma, c_1, c_2) &= \frac{((2-\gamma)a + \gamma c_j - 2c_i)^2}{(4-\gamma^2)^2}, \ i,j \in \{1,2\}, \ i \neq j \end{aligned}$$

A.2 Product Selection Stage

At the product selection stage the new product development project has been finished and firm 1 knows x_1, x_2 and γ. It is optimal to choose $P_1 = N$ iff

$$\pi_1^e(\gamma, c_n, c_o - x_2) \geq \pi_1^e(1, c_o - x_1, c_o - x_2).$$

We consider only equilibria where this inequality holds true for $\gamma = \gamma_h$. For $\gamma = \gamma_l$ we get that firm 1 chooses $P_1 = N$ iff

$$x_2 \geq x_2^{post}(x_1) := \frac{6c_n - (2 - 3\gamma_l + \gamma_l^2)a - (4 + 3\gamma_l - \gamma_l^2)c_o}{4 - 3\gamma_l - \gamma_l^2} + \frac{2(4-\gamma_l^2)}{4 - 3\gamma_l - \gamma_l^2}x_1. \tag{5}$$

Note that $x_2^{post}(x_1)$ is an increasing function of x_1. Also, the value of $x_2^{post}(x_1)$ increases with c_n for all x_1.

In game-theoretic terms each combination (x_1, x_2) is the root of a subgame and the equilibrium strategy in these subgames induces

$$P_1^e(x_1, x_2) = \begin{cases} N & x_2 > x_2^{post}(x_1) \\ \{N, O\} & x_2 = x_2^{post}(x_1) \\ O & x_2 < x_2^{post}(x_1) \end{cases}$$

at the product selection stage.

A.3 Process Innovation Stage

At this stage the two firms simultaneously choose $x_i \in [0, c_o]$. If $P_1 = N$ is chosen for $\gamma = \gamma_l$ the expected profit of firm i reads

$$\Pi_i^N(x_1, x_2) = p\pi_i^e(\gamma_h, c_n, c_o - x_2) + (1-p)\pi_i^e(\gamma_l, c_n, c_o - x_2) - \alpha x_i - \beta x_i^2.$$

For $P_1 = O$ we have

$$\Pi_i^O(x_1, x_2) = p\pi_i^e(\gamma_h, c_n, c_o - x_2) + (1-p)\pi_i^e(1, c_o - x_1, c_o - x_2) - \alpha x_i - \beta x_i^2,$$

where the choice of P_1 depends on (x_1, x_2) as described in subsection A.2. In order to characterize the equilibrium choices of (x_1, x_2) we separately consider both firms best-reply correspondences.

A.3.1 Best Reply of Firm 1

To derive the characterization of the best reply correspondence of firm 1 we give formal definitions of the functions and thresholds introduced in subsection 3.1 of the main text. We define

$$x_1^*(x_2) = \text{argmax}_{x_1 \in [0, c_o]} \Pi_1^O(x_1, x_2)$$

and

$$x_2^{noinv} = \min(x_2 \in [0, c_o] | x_1^*(x_2) = 0).$$

Simple calculations give

$$\begin{aligned} x_1^*(x_2) &= \min\left[\max\left[\frac{4(1-p)(a-c_o) - 9\alpha}{18\beta - 8(1-p)} - \frac{4(1-p)}{18\beta - 8(1-p)} x_2, 0\right], c_o\right] \\ x_2^{noinv} &= \min\left[\max\left[a - c_o - \frac{9\alpha}{4(1-p)}, 0\right], c_o\right]. \end{aligned}$$

Note that the second order conditions are satisfied due to our assumption that $\beta > 1$.

Finally, we define x_2^{ante} by

$$x_2^{ante} = \begin{cases} \tilde{x}_2, & \text{if } \exists \tilde{x}_2 \in [0, c_o] \text{ with } \Pi_1^N(0, \tilde{x}_2) = \Pi_1^O(x_1^*(\tilde{x}_2), \tilde{x}_2) \\ -\epsilon & \text{if } \Pi_1^N(0, x_2) > \Pi_1^O(x_1^*(x_2), x_2) \ \forall x_2 \in [0, c_o] \\ c_o + \epsilon & \text{if } \Pi_1^N(0, x_2) < \Pi_1^O(x_1^*(x_2), x_2) \ \forall x_2 \in [0, c_o] \end{cases},$$

where $\epsilon > 0$ is an arbitrary positive parameter.[11] If $x_2^{ante} \in [0, c_o]$ firm 1 is indifferent between the two options for $x_2 = x_2^{ante}$. In Lemma 1 we show that $\Pi_1^N(0, x_2) - \Pi_1^O(x_1^*(x_2), x_2)$ is monotonic with respect to $x_2 \in [0, c_o]$ and therefore x_2^{ante} is unique and well-defined.

---

[11] The parameter ϵ is introduced for technical reasons in order to guarantee that x_2^{ante} is well-defined for any parameter constellation.

Lemma 1 *The function* $f1(x_2) = \Pi_1^N(0, x_2) - \Pi_1^O(x_1^*(x_2), x_2)$ *is strictly monotone increasing for* $x_2 \in [0, c_o]$.

Proof.

$$\begin{aligned}
& f1'(x_2) \\
= \; & \frac{\partial \Pi_1^N(0, x_2)}{\partial x_2} - \frac{\partial \Pi_1^O(x_1^*(x_2), x_2)}{\partial x_2} - \frac{\partial \Pi_1^O(x_1^*(x_2), x_2)}{\partial x_1}\frac{\partial x_1^*(x_2)}{\partial x_2} \\
= \; & \frac{\partial \Pi_1^N(0, x_2)}{\partial x_2} - \frac{\partial \Pi_1^O(x_1^*(x_2), x_2)}{\partial x_2} \\
= \; & (1-p)\frac{\partial \pi_1(\gamma_l, c_n, c_o - x_2)}{\partial x_2} - (1-p)\frac{\partial \pi_1(1, c_o - x_1^*(x_2), c_o - x_2)}{\partial x_2}.
\end{aligned}$$

The third term in the second line is zero due to the envelope theorem for $x_1^*(x_2) \in (0, c_o)$ and due to $\frac{\partial x_1^*(x_2)}{\partial x_2} = 0$ for $x_1^* \in \{0, c_o\}$. Inserting the expressions for π_1 and straightforward transformations show that the last line is positive if and only if

$$(16 - 18\gamma_l + \gamma_l^2 + \gamma_l^4)a + (16 - 17\gamma_l^2 + \gamma_l^4)(c_o - x_2) + 18\gamma_l c_n - 2(4 - \gamma_l^2)^2(c_o - x_1^*(x_2)) > 0.$$

The coefficients of a, c_n, and x_1 are positive and the coefficient of x_2 is negative (for $\gamma_l \in (0, 1)$), therefore, setting $a = \frac{2+\gamma_l}{4-\gamma_l}$, $c_n = c_o$,$x_1 = 0$, and $x_2 = c_o$ gives a lower bound and we get

$$\begin{aligned}
& (16 - 18\gamma_l + \gamma_l^2 + \gamma_l^4)a + (16 - 17\gamma_l^2 + \gamma_l^4)(c_o - x_2) + 18\gamma_l c_n \\
& -2(4 - \gamma_l^2)^2(c_o - x_1^*(x_2)) \\
> \; & (16 - 18\gamma_l + \gamma_l^2 + \gamma_l^4)\frac{4 + \gamma_l}{2 - \gamma_l}c_o + 18\gamma_l c_o - (32 - 16\gamma_l^2 + 2\gamma_l^4)c_o \\
= \; & 3\gamma_l(2 + \gamma_l - 2\gamma_l^2 - \gamma_l^3)c_o \\
> \; & 0.
\end{aligned}$$

■

It is quite intuitive that both, the ex-ante threshold x_2^{ante} and the ex-post threshold $x_2^{post}(0)$ are increasing in c_n. A proof of this claim is given in Lemma 2 together with characterizations of other properties of the thresholds which are useful for the further analysis.

Lemma 2 *(a) If* $x_2^{post}(0) \in (0, c_o)$ *it is strictly monotonous increasing in* c_n.

(b) If $x_2^{ante} < x_2^{noinv}$ *then* $x_2^{post}(0) < x_2^{ante}$. *If* $x_2^{ante} \geq x_2^{noinv}$ *then* $x_2^{post}(0) = x_2^{ante}$

(c) If $x_2^{ante} \in (0, c_o)$, *then* x_2^{ante} *is strictly monotonous increasing in* c_n.

(d) For $c_n = c_o$ we have $x_2^{ante} < x_2^{noinv}$.

Proof. Claims (a) and (b) follow directly from the definitions of the thresholds.
(c): For $x_2^{ante} \geq x_2^{noinv}$ we know from (b) that $x_2^{ante} = x_2^{post}(0)$ and the claim follows from (a). Hence we only have to deal with scenarios where $x_2^{ante} \in (0, x_2^{noinv})$. We define

$$f2(x_1, x_2) = \Pi_1^N(0, x_2) - \Pi_1^O(x_1, x_2)$$

and note that this function is a quadratic polynomial in x_1. Simple calculations show that $f2(x_1, x_2) = \frac{9(4-\gamma_l^2)^2}{1-p}\left[K_1 x_1^2 + K_2(x_2)x_1 + K_3(x_2)\right]$, where

$$\begin{aligned}
K_1 &= (4-\gamma_l^2)^2\left(\frac{9\beta}{1-p} - 4\right) \\
K_2 &= -4(4-\gamma_l^2)^2(a-c_o) + \frac{9(4-\gamma_l^2)^2\alpha}{1-p} + 4(4-\gamma_l^2)^2 x_2 \\
K_3 &= 20a^2 + 16(2a-c_o)c_o - 36(2a-c_n)c_n - 36\gamma_l(a-c_o)(a-c_n) + 17\gamma_l^2(a-c_o)^2 \\
&\quad -\gamma_l^4(a-c_o)^2 \\
&\quad +[32(a-c_o) - 36\gamma_l(a-c_n) + 2\gamma_l^2(a-c_o) + 2\gamma_l^4(a-c_o)]x_2 + [-16 + 17\gamma_l^2 - \gamma_l^4]x_2^2
\end{aligned}$$

Due to $\beta > 1$ we have $K_1 > 0$. If $x_2^{ante} \in (0, x_2^{noinv})$ we have $x_1^*(x_2^{ante}) \in (0, c_o)$. Since $x_1^*(x_2^{ante})$ is in the interior of $[0, c_o]$, and $\Pi_1^O(x_1, x_2^{ante})$ is a quadratic function in x_1, the global maximum of $\Pi_1^O(x_1, x_2^{ante})$ is obtained at $x_1 = x_1^*(x_2^{ante})$. Accordingly, the global minimum of $f2(x_1, x_2^{ante})$ is reached for $x_1 = x_1^*(x_2^{ante})$. By definition $f2(x_1^*(x_2^{ante}), x_2^{ante}) = f1(x_2^{ante}) = 0$. Hence, the two solutions of $f2(x_1, x_2^{ante}) = 0$ have to coincide which is equivalent to the condition that the two roots of $K_1 x_1^2 + K_2(x_2^{ante})x_1 + K_3(x_2^{ante}) = 0$ coincide. Therefore, we must have

$$K_2(x_2^{ante})^2 - 4K_1K_3(x_2^{ante}) = 0.$$

Calculating the left hand side shows that it is a quadratic polynomial in x_2. We write

$$f3(x_2) := K_2(x_2)^2 - 4K_1K_3(x_2) = M_1x_2^2 + M_2x_2 + M_3,$$

where

$$\begin{aligned}
M_1 &= 144\gamma_l^2(4-\gamma_l^2) + \frac{36\beta}{1-p}(4-\gamma_l^2)^2(16 - 17\gamma_l^2 + \gamma_l^4) > 0 \\
M_2 &= 72(4-\gamma_l^2)^2\left[4\gamma_l^2(a-c_o) - 8\gamma_l(a-c_n) + [18\gamma_l(a-c_n) - (16+\gamma_l^2+\gamma_l^4)(a-c_o)]\frac{\beta}{1-p}\right. \\
&\quad \left. +(4-\gamma_l^2)^2\frac{\alpha}{1-p}\right]
\end{aligned}$$

$$\begin{aligned} M_3 &= 9(4-\gamma_l^2)^2 \Big[9(\gamma_l(a-c_o)-2(a-c_n))^2 \\ &\quad -[4(6(a-c_n)-(4-\gamma_l)(1+\gamma_l)(a-c_o))(6(a-c_n)+(4+\gamma_l)(1-\gamma_l)(a-c_o))]\frac{\beta}{1-p} \\ &\quad -[8(4-\gamma_l^2)^2(a-c_o)]\frac{\alpha}{1-p}+9(4-\gamma_l^2)^2\frac{\alpha^2}{(1-p)^2}\Big]. \end{aligned}$$

From $f1'(x_2^{ante})>0$ we conclude that for $x_2=\tilde{x}_2$ slightly larger than x_2^{ante} the global minimum of $f2(x_1,\tilde{x}_2)$ is positive and there exists no real solution of $f2(x_1,\tilde{x}_2)=0$. Accordingly, we must have $f3(\tilde{x}_2)<0$ and we conclude that $f3'(x_2^{ante})<0$. Implicit differentiation of

$$f3(x_2^{ante};c_n)=0$$

with respect to c_n gives

$$\frac{\partial x_2^{ante}}{\partial c_n}=-\frac{1}{f3'(x_2^{ante})}\frac{\partial f3(x_2^{ante})}{\partial c_n}.$$

In order to prove claim (c), we still have to show that $\frac{\partial f3(x_2^{ante})}{\partial c_n}>0$. Differentiating the coefficients $M_i, i=1,\dots,3$ with respect to C_N gives:

$$\begin{aligned} \frac{\partial M_1}{\partial c_n} &= 0 \\ \frac{\partial M_2}{\partial c_n} &= -72(4-\gamma_l^2)^2\gamma_l\left(18\frac{\beta}{1-p}-8\right)<0 \\ \frac{\partial M_3}{\partial c_n} &= 144(4-\gamma_l^2)^2\left(\frac{9\beta}{1-p}-4\right)(2(a-c_n)-\gamma_l(a-c_o)) \end{aligned}$$

Because of $\frac{\partial M_2}{\partial c_n}<0$ we get

$$\begin{aligned} &\frac{\partial f3(x_2^{ante})}{\partial c_n} \\ &= \frac{\partial M_2}{\partial c_n}x_2^{ante}+\frac{\partial M_3}{\partial c_n} \\ &> \frac{\partial M_2}{\partial c_n}c_o+\frac{\partial M_3}{\partial c_n} \\ &= 144(4-\gamma_l^2)^2\left[\left(\frac{9\beta}{1-p}-4\right)((2-\gamma_l)a-2c_n)\right] \\ &> 0. \end{aligned}$$

The last inequality follows from assumptions (A2) and (A4) which imply $2(2-\gamma_l)a\geq(4+\gamma_l)c_n$. This proves claim (c).

(d): For $c_n=c_o$ we have

$$f2(0,x_2)=(1-p)[\pi_1(\gamma_l,c_o,c_o-x_2)-\pi_1(1,c_o,c_o-x_2)]$$

It is easy to check that $\pi_1(\gamma_l, c_o, c_o - x_2) - \pi_1(1, c_o, c_o - x_2) > 0$ for all $x_2 \in [0, c_o]$. For $x_2 \geq x_2^{noinv}$ we have $x_1^*(x_2) = 0$ and therefore $f1(x_2) = f2(x_1^*(x_2), x_2) = f2(0, x_2) > 0$. This implies that $x_2^{ante} < x_2^{noinv}$.

For further reference we also note that $M_2 < 0$. To see this, note that the coefficients of c_n in M_2 is negative. Therefore we get an upper bound for M_2 by setting $c_n = c_o$. Doing this yields

$$\alpha < (a - c_o)\frac{\beta(1-\gamma_l)(8 + \gamma_l(3+\gamma_l)) + 4(1-p)\gamma_l}{(2-\gamma_l)(2+\gamma_l)^2}$$

as a sufficient condition for $M_2 < 0$. Since the right hand side is decreasing in γ_l, it is minimized for $\gamma_l = 1$. Accordingly,

$$\alpha < (a - c_o)\frac{4(1-p)}{9}$$

is a sufficient condition for $M_2 < 0$. Due to assumption (A2) this condition is fulfilled.

■

The characterization of the best response correspondence of firm 1 given in proposition 1 is a direct implication of the discussion provided above.

A.3.2 Best Reply of Firm 2

From the analysis of the decisions made at the product selection stage we infer that the expected profit function for firm 2 is given by $\Pi_2^N(x_1, x_2)$ for $x_2 \geq x_2^{post}(x_1)$ and by $\Pi_2^O(x_1, x_2)$ for $x_2 < x_2^{post}(x_1)$. We define

$$x_2^{*N}(x_1) = \text{argmax}_{x_2 \in [0, c_o]} \Pi_2^N(x_1, x_2)$$

and analogously $x_2^{*O}(x_1)$. Direct calculations give the following rather lengthy full expressions for these two optimal investment levels

$$\begin{aligned} x_2^{*N} &= \min[c_o, \max[0, N_{Na}a + N_{Nn}c_n + N_{No}c_o + N_{N\alpha}\alpha]] \\ x_2^{*O}(x_1) &= \min[c_o, \max[0, N_{Oa}a + N_{On}c_n + N_{Oo}c_o + N_{O\alpha} + N_{Ox}x_1]], \end{aligned}$$

with coefficients

$$\begin{aligned} \tilde{N} &= 2(4-\gamma_l^2)^2(4-\gamma_h^2)^2\beta - 8(4-\gamma_l^2)^2 p - 8(4-\gamma_h^2)^2(1-p) > 0 \\ N_{Na} &= \frac{1}{\tilde{N}}[4(4-\gamma_l^2)^2(2-\gamma_h)p + 4(4-\gamma_h^2)^2(2-\gamma_l)(1-p)] > 0 \\ N_{Nn} &= \frac{1}{\tilde{N}}[4(4-\gamma_l^2)^2\gamma_h p + 4(4-\gamma_h^2)^2\gamma_l(1-p)] > 0 \end{aligned}$$

$$\begin{aligned}
N_{No} &= -\frac{1}{\tilde{N}}[8(4-\gamma_l^2)^2 p + 8(4-\gamma_h^2)^2(1-p)] < 0 \\
N_{N\alpha} &= -\frac{1}{\tilde{N}}(4-\gamma_l^2)^2(4-\gamma_h^2)^2 < 0 \\
\tilde{\tilde{N}} &= 18(4-\gamma_h^2)^2\beta - 72p - 8(4-\gamma_h^2)^2(1-p) > 0 \\
N_{Oa} &= \frac{1}{\tilde{\tilde{N}}}[36(2-\gamma_h)p + 4(4-\gamma_h^2)^2(1-p)] > 0 \\
N_{On} &= \frac{1}{\tilde{\tilde{N}}}36\gamma_h p > 0 \\
N_{Oo} &= -\frac{1}{\tilde{\tilde{N}}}[72p + 4(4-\gamma_h^2)^2(1-p)] < 0 \\
N_{O\alpha} &= -\frac{1}{\tilde{\tilde{N}}}9(4-\gamma_h^2)^2 < 0 \\
N_{Ox} &= -\frac{1}{\tilde{\tilde{N}}}4(4-\gamma_h^2)^2(1-p) < 0
\end{aligned}$$

These expressions show that $x_2^{*O}(x_1)$ is strictly decreasing and linear in x_1.

Due to assumption (A2) the inequality $\Pi_2^N(x_1,x_2) - \Pi_2^O(x_1,x_2) \geq 0$ holds for all $(x_1,x_2) \in [0,c_o]^2$. Hence, it is obvious that the optimal choice for firm 2 is x_2^{*N} whenever such investment induces firm 1 to introduce the new product even if $\gamma = \gamma_l$, i.e. if $x_2^{*N} \geq x_2^{post}(x_1)$. If this inequality is violated, the optimal choice of firm 2 either has to be at $x_2^{post}(x_1)$ or at $x_2^{*O}(x_1)$. In the former case firm 1 is induced to introduce the new product regardless of γ, but in the latter case firm 1 trashes the new product and sells the old product for $\gamma = \gamma_l$. The best reply of firm 2 is therefore a piece-wise linear function which might jump between the candidates x_2^{*N}, $x_2^{post}(x_1)$, and $x_2^{*O}(x_1)$.

We denote the level of x_1 where the optimal choice of firm 2 switches from x_2^{*N} to x_2^{post} by x_1^{T1}:

$$x_1^{T1} = \begin{cases} \tilde{x}_1, & \text{if } \exists \tilde{x}_1 \in [0,c_o] \text{ with } x_2^{post}(\tilde{x}_1) = x_2^{*N} \\ -\epsilon & \text{if } x_2^{post}(x_1) > x_2^{*N}\ \forall x_1 \in [0,c_o] \\ c_o + \epsilon & \text{if } x_2^{post}(x_1) < x_2^{*N}\ \forall x_1 \in [0,c_o]. \end{cases}$$

Furthermore, we define x_1^{T2} as the maximal level of x_1 where firm 2 prefers investing a high amount in process innovation, thereby inducing firm 1 to introduce the new product in any case, to accepting that firm 1 introduces the new product only for $\gamma = \gamma_h$. We denote the difference in maximal profits for firm 2 under the two scenarios by

$$g(x_1) = \max_{x_2 \in [x_2^{post}(x_1), c_o]} \Pi_2^N(x_1,x_2) - \max_{x_2 \in [0, x_2^{post}(x_1))} \Pi_2^O(x_1,x_2).$$

Using this notation we can write x_1^{T2} as

$$x_1^{T2} = \begin{cases} \tilde{x}_1, & \text{if } \exists \tilde{x}_1 \in (0, c_o] \text{ with } g(\tilde{x}_1) = 0 \\ -\epsilon & \text{if } g(x_1) < 0 \ \forall x_1 \in [0, c_o] \\ c_o + \epsilon & \text{if} g(x_1) > 0 \ \forall x_1 \in [x_1^{T1}, c_o]. \end{cases},$$

It is shown in Lemma 3 that both thresholds are unique and well defined and that $x_1^{T1} \leq x_1^{T2}$ with strict inequality if at least one of the two thresholds is in $[0, c_o]$.

Lemma 3 *(a) There exists at most one solution of* $x_2^{post}(x_1) = x_2^{*N}$ *in* $[0, c_o]$.

(b) There exists at most one solution of $g(x_1) = 0$ *in* $[0, c_o]$.

(c) $x_1^{T1} \leq x_1^{T2}$ *with strict inequality if at least one of the two thresholds is in* $[0, c_o]$.

Proof. (a) The expression $x_2^{post}(x_1) - x_2^{*N}$ is monotonously increasing in x_1. Accordingly, this expression has at most one root in $[0, c_o]$.

(b) Because of $\Pi_2^N(x_1, x_2) > \Pi_2^O(x_1, x_2) \ \forall (x_1, x_2) \in [0, c_o]^2$ we have $\Pi_2^N(x_1, x_2^{*N}(x_1)) > \max_{x_2 \in [0, x_2^{post}(x_1)]} \Pi_2^O(x_1, x_2)$. Therefore $g(x_1) = 0$ can only hold if $x_2^{post}(x_1) > x_2^{*N}(x_1)$ which, due to the monotonicity of $x_2^{post}(x_1) - x_2^{*N}$, is equivalent to $x_1 > x_1^{T1}$. Furthermore, if $x_2^{post}(x_1) < x_2^{*O}(x_1)$ we have

$$\max_{x_2 \in [0, x_2^{post}(x_1)]} \Pi_2^O(x_1, x_2) = \Pi_2^O(x_1, x_2^{post}(x_1)) < \Pi_2^N(x_1, x_2^{post}(x_1)) \leq \max_{x_2 \in [x_2^{post}(x_1), c_o]} \Pi_2^N(x_1, x_2).$$

Accordingly, $x_2^{post}(x_1) > x_2^{*O}(x_1)$ is a necessary condition for $g(x_1) = 0$ to hold. Under this condition we have $\max_{x_2 \in [0, x_2^{post}(x_1)]} \Pi_2^O(x_1, x_2) = \Pi_2^O(x_1, x_2^{*O}(x_1))$.

Given this, it is easy to see that $g(x_1) = 0$ holds if and only if $f4(x_1) = 0$, with

$$f4(x_1) = \Pi_2^N(x_1, x_2^{post}(x_1)) - \Pi_2^O(x_1, x_2^{*O}(x_1)),$$

where only $x_1 \in [x_1^{T1}, c_o]$ has to be considered.

In the remainder of the proof we show that there exists at most one solution of $f4(x_1) = 0$ in $[x_1^{T1}, c_o]$. We first show that $f4'' < 0$. Taking into account $\frac{\partial \Pi_2^N}{\partial x_1} = 0$ and the envelope theorem we get

$$f4'(x_1) = \frac{\partial \Pi_2^N(x_1, x_2^{post}(x_1))}{\partial x_2} (x_2^{post})'(x_1) - \frac{\partial \Pi_2^O(x_1, x_2^{*O}(x_1))}{\partial x_1}. \tag{6}$$

We know that x_2^{post} is linear in x_1, therefore

$$f4''(x_1)$$

$$\begin{aligned}
&= \frac{\partial^2 \Pi_2^N(x_1, x_2^{post}(x_1))}{\partial x_2^2}[(x_2^{post})'(x_1)]^2 - \frac{\partial^2 \Pi_2^O(x_1, x_2^{*O}(x_1))}{\partial x_1^2} \\
&= \left[\frac{8p}{(4-\gamma_h^2)^2} + \frac{8(1-p)}{(4-\gamma_l^2)^2} - 2\beta\right]\left[\frac{2(4-\gamma_l^2)}{4-3\gamma_l-\gamma_l^2}\right]^2 - \frac{2(1-p)}{9} \\
&< \frac{8p}{(4-\gamma_h^2)^2} + \frac{8(1-p)}{(4-\gamma_l^2)^2} - 2\beta - \frac{2(1-p)}{9} \\
&< 0,
\end{aligned}$$

where the last two inequalities follow from $\beta > 1$. Furthermore define $\tilde{x}_1$ as the unique solution of $x_2^{post}(x_1) = x_2^{*N}$ in $\mathbb{R}$. Obviously $f4(\tilde{x}_1) > 0$, and, taking into account $f4'' < 0$, this implies that there exists a unique root of $f4(x_1)$ in $(\tilde{x}_1, \infty)$. For all $x_1^{T1} < c_o$ we must have $x_1^{T1} \geq \tilde{x}_1$ and therefore there can be at most one root of $f4(x_1)$ in $(x_1^{T1}, c_o]$.

(c): It follows directly from the arguments in the proof of (b) that if $g(x_1^{T2}) = 0$ then $x_1^{T2} > x_1^{T1}$. Furthermore, it is obvious that $x_1^{T2} = -\epsilon$ can only hold if $x_2^{post}(0) > x_2^{*N}(0)$ which implies $x_1^{T1} = -\epsilon$. ■

Put together these arguments directly imply the characterization of the best reply of firm 2 that is given in Proposition 2.

A.4 Equilibria

The following two lemmas give some results on how the different threshold values change as c_n is altered. We will use these Lemmas in the proofs of the propositions 3 and 4.

Lemma 4 *(a) For $c_n = c_o$ we have $x_1^{T1} > 0$.*

(b) If $x_1^{T1} \in (0, c_o)$, then an increase in c_n induces a decrease in x_1^{T1}.

(c) There exists a unique $c_n^T > c_o$ such that $x_1^{T1} = 0$ for $c_n = c_n^T$.

Proof. (a): For $c_n = c_o$ we have $x_2^{post}(0) = -\frac{2-3\gamma_l+\gamma_l^2}{4-3\gamma_l-\gamma_l^2}(a - c_o) < 0$. It follows from assumption (A3) that $x_2^{*N} > 0$. Taking into account that x_2^{post} increases with x_1 we conclude that $x_1^{T1} > 0$ for $c_n = c_o$.

(b): Implicit differentiation gives

$$\frac{\partial x_1^{T1}}{\partial c_n} = \left.\frac{\partial\left(x_2^{*N} - x_2^{post}(x_1)\right)}{\partial c_n}\right|_{x_1 = x_1^{T1}} / [x_2^{post}(x_1^{T1})]'.$$

Since $[x_2^{post}(x_1)]' > 0$ we have to show that

$$\frac{\partial x_1^{T1}}{\partial c_n} = \frac{\partial(x_2^{*N} - x_2^{post}(x_1)}{\partial c_n} < 0. \tag{7}$$

After inserting the expressions for x_2^{*N} and x_2^{post} and a few transformation we obtain the following inequality equivalent to (7):

$$\beta > \left(\frac{(4-3\gamma_l-\gamma_l^2)\gamma_h}{3(4-\gamma_h^2)^2} + \frac{4}{(4-\gamma_h^2)^2} \right) p + \left(\frac{(4-3\gamma_l-\gamma_l^2)\gamma_l}{3(4-\gamma_l^2)^2} + \frac{4}{(4-\gamma_l^2)^2} \right) (1-p).$$

It can be easily seen that the right hand side is bounded above by 1, so we have proven the claim.

(c): From (a) we know that $x_1^{T1} > 0$ for $c_n = c_o$. The arguments in the proof of (b) show that the slope of x_1^{T1} with respect to c_n is not only negative but also cannot converge to zero. Accordingly, there exists a $c_n^T > 0$ such that $x_1^{T1} = 0$ for $c_n = c_n^T$. ■

The following lemma shows that for sufficiently large β there can be at most one value of c_n where x_2^{ante} and x_2^{*N} coincide. The assumption of a large β is needed for the proof of the lemma but numerical evidence suggests that this property also holds for small value β. Actually, in all our numerical studies we found $x_2^{ante} - x_2^{*N}$ strictly monotonously increasing with c_n as long as they stay in the interior of $[0, c_n]$. Also the second claim of the lemma, which is that firm 1 never stays in the old market if $c_n = c_o$ was numerically verified also for small values of β. In what follows we will always assume that β is sufficiently large such that Lemma 5 holds.

Lemma 5 *(a) For sufficiently large β, keeping all other parameters fixed, there exists at most one value of c_n where $x_2^{ante} = x_2^{*N} \in (0, c_o)$.*

(b) For sufficiently large β and $c_n = c_o$ we have $x_2^{ante} = 0$.

Proof. (a): We show that $\frac{df3(x_2^{*N})}{dc_n} > 0 \ \forall x_2^{*N} \in (0, c_o)$, where $f3$ is defined as in the proof of Lemma 2. Thus, there can be at most one value of c_n with $f3(x_2^{*N}) = 0$. This will prove our claim because x_2^{ante} is defined as the smaller root of $f3$. To show that $\frac{df3(x_2^*)}{dc_n} > 0$ we observe that

$$\begin{aligned} &\frac{df3(x_2^{*N})}{dc_n} \\ &= \quad f3'(x_2^{*N}) \frac{\partial x_2^{*N}}{\partial c_n} + \frac{\partial f3(x_2^{*N})}{\partial c_n}. \end{aligned}$$

Using calculations carried out in the proof of Lemma 2 and taking into account that $(2-\gamma_l)a > \frac{4+\gamma_l}{2} c_n$ (this follows from assumptions (A2) and (A4)) we get

$$\frac{\partial f3(x_2^{*N})}{\partial c_n}$$

$$
\begin{aligned}
&> \; 144(4-\gamma_l^2)^2\left[\left(\frac{9\beta}{1-p}-4\right)((2-\gamma_l)a-2c_n)\right]\\
&> \; 144(4-\gamma_l^2)^2\left[\left(\frac{9\beta}{1-p}-4\right)\frac{\gamma_l c_n}{2}\right].
\end{aligned}
$$

For $f3'(x_2^{*N})$ we have

$$
\begin{aligned}
&f3'(x_2^{*N})\\
&= \; 2M_1x_2^{*N}+M_2\\
&> \; M_2\\
&> \; 36(4-\gamma_L^2)^2\left[-8\gamma_L(2-\gamma_L)(a-c_o)+[36\gamma_L(a-c_N)-2(16+\gamma_L^2+\gamma_L^4)(a-c_o)]\frac{\beta}{1-p}\right].
\end{aligned}
$$

From the expression for x_2^{*N} derived above we get immediately

$$
\begin{aligned}
&\frac{\partial x_2^{*N}}{\partial c_n}\\
&= \; N_{IN}\\
&= \; 2\left[\frac{\gamma_h}{(4-\gamma_h^2)^2}p+\frac{\gamma_l}{(4-\gamma_l^2)^2}(1-p)\right]\Big/\left[\beta-\frac{4}{(4-\gamma_h^2)^2}p-\frac{4}{(4-\gamma_l^2)^2}(1-p)\right]\\
&< \; \frac{2\gamma_l}{9\beta-4}.
\end{aligned}
$$

Taking into account that $M_2<0$ this gives all-together

$$
\begin{aligned}
&\frac{df3(x_2^*)}{dc_n}\\
&> \; \frac{36(4-\gamma_l^2)^2}{9\beta-4}\left[4(9\beta-4)\left(\frac{9\beta}{1-p}-4\right)\frac{\gamma_l c_n}{2}\right.\\
&\quad \left.+\left(-16\gamma_l^2(2-\gamma_l)(a-c_o)+[72\gamma_l^2(a-c_n)-4\gamma_l(16+\gamma_l^2+\gamma_l^4)(a-c_o)]\frac{\beta}{1-p}\right)\right]
\end{aligned}
$$

.

The expression is square brackets is a quadratic polynomial in β where the coefficient of β^2 is positive. Accordingly we have $\frac{df3(x_2^*)}{dc_n}>0$ for sufficiently large β.
(b): Straight forward calculations show that for $c_n=c_o$ the coefficient of β in $M3$ is negative. Hence, for sufficiently large β we have $M3<0$, which implies that the smaller root of $f4(x_2)$ is negative. Therefore $x_2^{ante}=0$. ■

Proof of Proposition 3:

Proof. (a): From Lemmas 4 and 5 (b) we know that $x_2^{ante}=0$ and $x_2^{*N}>0$ for $c_n=c_o$. It follows from Lemma 2 and continuity considerations that there must be a value $\tilde{c}_n \in [c_o, c_n^T]$ such that $x_2^{ante} = x_2^{noinv} = x_2^{post}(0)$ holds for $c_n = \tilde{c}_n$. Furthermore, we have

$$\frac{\partial}{\partial c_n}(x_2^{post}(0) - x_2^{*N}) > \frac{6}{4 - 3\gamma_l - \gamma_l^2} - \frac{2\gamma_l}{9\beta - 4} > 0.$$

Accordingly, we must have $x_2^{*N} > x_2^{post}(0) = x_2^{ante}$ for $c_n = \tilde{c}_n$. Together with Lemma 5 (a) this shows that $x_2^{*N} \geq x_2^{ante}$ for all $c_n \in [c_o, \min[\tilde{c}_n, 2c_o]]$. Therefore, $BR_1(x_2^{*N}) = 0$ for all $c_n \in [c_o, \min[\tilde{c}_n, 2c_o]]$. For $c_n > \tilde{c}_n$ we have $x_2^{*N} > x_2^{noinv}$ and therefore $BR_1(x_2^{*N}) = 0$ as well. Since x_1^{T1} is monotonously decreasing in c_n (Lemma 4 (b)), $x_1^{T1} > 0$ has to hold for all $c_n \in [c_o, \min[c_n^T, 2c_o]]$. Therefore $BR_2(0) = x_2^{*N}$. This shows that $x_1 = 0, x_2 = x_2^{*N}$ are indeed equilibrium actions on the process innovation stage. Furthermore, it follows from $x_2^{post}(0) = x_2^{post}(0) < x_2^{*N}$ $\forall c_n \in [c_o, \min[c_n^T, 2c_o]]$ that in equilibrium firm 1 introduces the new product at the product selection stage even if $\gamma = \gamma_l$.

(b): For $c_n > c_n^T$ we have $x_1^{T1} = 0$, therefore $BR_2(0) = x_2^{post}(0) = x_2^{post}(0)$ if $x_1^{T2} > 0$. Furthermore, $x_2^{post}(0) > x_2^{noinv}$, so $BR_1(x_2^{post}(0)) = 0$.

(c): Obviously, in any equilibrium where firm 1 introduces the new product regardless of γ we must have $x_1 = 0$. For $c_n > c_n^T$ and $x_1^{T2} = 0$, we have $BR_2(0) = x_2^{*O}(0) < x_2^{post}(0)$. Therefore, it is optimal for firm 1 to choose $P_1 = O$ having observed $\gamma = \gamma_l$ and $x_1 = 0, x_2 = x_2^{*O}(0)$. Accordingly, there is no equilibrium where $P_1 = N$ is optimal for firm 1. ■

Proof of Proposition 4:

Proof. The existence and uniqueness of $\underline{c}_n$ follows by continuity considerations from $x_2^{ante} < x_2^{*N}$ for $c_n = c_o$, $x_2^{ante} > x_2^{post}(0) = x_2^{*N}$ for $c_n = c_n^T$ and Lemma 5. Existence and uniqueness of $\bar{c}_n$ follows from monotonicity of $x_2^{noinv} - x_2^{post}(0)$ with respect to c_n.

(a) and (b): analogous to (a) and (b) of Proposition 3.

(c): For $c_n \in [\underline{c}_n, \bar{c}_n]$ we have $BR_2(0) \leq x_2^{post}(0) < x_2^{ante}$. Therefore, $BR_1(BR_2(0)) > 0$ and there is no (pure-strategy) equilibrium where $x_1 = 0$. Accordingly, there is no pure-strategy equilibrium where $P_1 = N$ is chosen in equilibrium. For $c_n \in [\bar{c}, c_o] \setminus D$ the proof is analogous to (c) in Proposition 3. ■

Appendix B

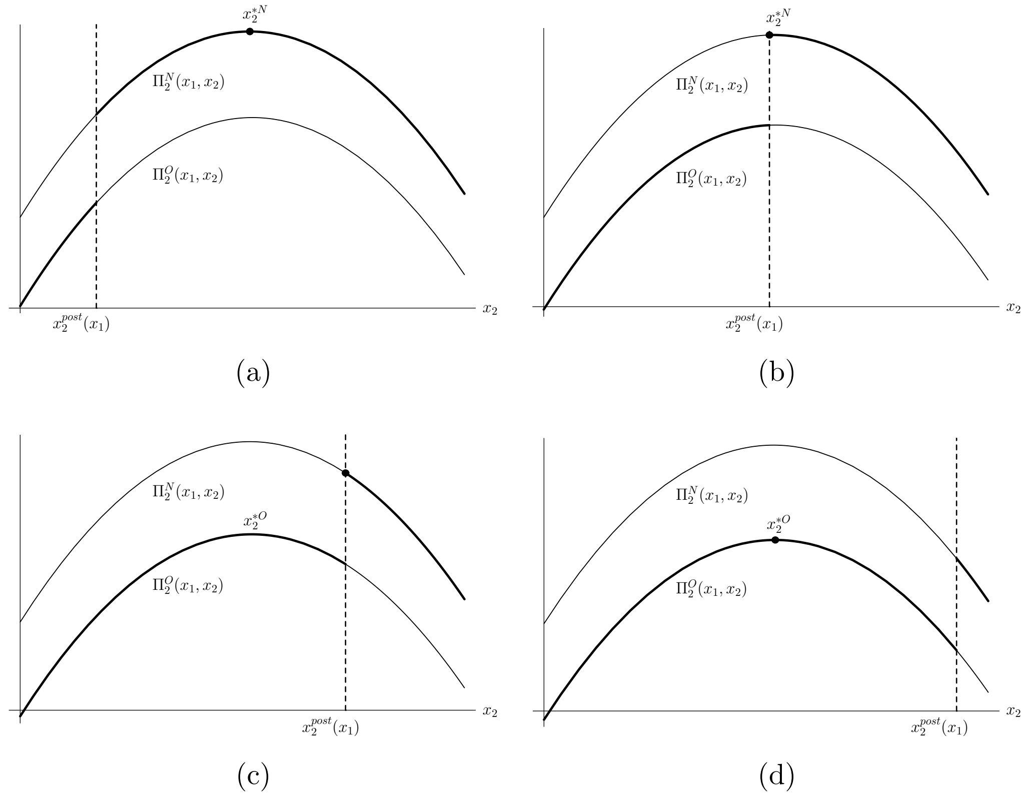

Figure 7: The profit function Π_2^N of firm 2 if firm 1 introduces the new product regardless of γ and the profit function Π_2^O if firm 1 launches the new product only for $\gamma = \gamma_h$. The profit of firm 2 if firm 1 acts optimally is drawn in bold face and the dot indicates the optimal choice $BR_2(x_1)$. (a) $x_1 < x_1^{T1}$, (b) $x_1 = x_1^{T1}$, (c) $x_1^{T1} < x_1 < x_1^{T2}$, (d) $x_1 > x_1^{T2}$

Appendix C

Expected welfare W is calculated in a standard way, namely as the sum of consumer surplus and producer profits. We denote again by $U(q_1, q_2; \gamma)$ the consumer preference function giving rise to the inverse demand functions (1). If $P_1^* = N$ expected welfare is given by

$$\begin{aligned} W &= p\,[U(q_{1h}, q_{2h}; \gamma_h) - p_{1h}q_{1h} - p_{2h}q_{2h} \\ &\quad +p_{1h}q_{1h} - c_n q_{1h} + p_{2h}q_{2h} - (c_o - x_2^e)q_{2h}] \\ &\quad +(1-p)\,[U(q_{1lN}, q_{2lN}; \gamma_l) - p_{1lN}q_{1lN} - p_{2lN}q_{2lN} \\ &\quad +p_{1lN}q_{1lN} - c_n q_{1lN} + p_{2lN}q_{2lN} - (c_o - x_2^e)q_{2lN}] - k(x_2^e), \end{aligned}$$

where $q_{1h} = q_1^*(\gamma_h, c_n, c_o - x_2^e), q_{2h} = q_2^*(\gamma_h, c_n, c_o - x_2^e), q_{1lN} = q_1^*(\gamma_l, c_n, c_o - x_2^e), q_{2lN} = q_2^*(\gamma_l, c_n, c_o - x_2^e)$ and $p_{ih} = a - q_{ih} - \gamma_h q_{jh}, p_{ilN} = a - q_{ilN} - \gamma_l q_{jlN}$. On the other hand, for $P_1^* = O$ we have

$$\begin{aligned} W &= p\,[U(q_{1h}, q_{2h}; \gamma_h) - p_{1h}q_{1h} - p_{2h}q_{2h} \\ &\quad + p_{1h}q_{1h} - c_n q_{1h} + p_{2h}q_{2h} - (c_o - x_2^e)q_{2h}] \\ &\quad + (1-p)\,[U(q_{1lO}, q_{2lO}; 1) - p_{1lO}q_{1lO} - p_{2lO}q_{2lO} \\ &\quad + p_{1lO}q_{1lO} - (c_o - x_1^e)q_{1lO} + p_{2lO}q_{2lO} - (c_o - x_2^e)q_{2lO}] - k(x_1^e) - k(x_2^e), \end{aligned}$$

where $q_{1lO} = q_1^*(1, c_o - x_1^e, c_o - x_2^e), q_{2lO} = q_2^*(1, c_o - x_1^e, c_o - x_2^e)$ and $p_{ilO} = a - q_{ilO} - q_{jlO}$.